Monetary Policy and Macroeconomic Stability Revisited

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Monetary Policy and Macroeconomic Stability Revisited*

Yasuo Hirose‡  Takushi Kurozumi‡  Willem Van Zandweghe§

Abstract

A large literature has established the view that the Fed’s change from a passive to an active policy response to inflation led to U.S. macroeconomic stability after the Great Inflation of the 1970s. We revisit this view by estimating a generalized New Keynesian model using a full-information Bayesian method that allows for indeterminacy of equilibrium and adopts a sequential Monte Carlo algorithm. The estimated model empirically outperforms canonical New Keynesian models that confirm the literature’s view. It also points to substantial uncertainty about whether the policy response to inflation was active or passive during the Great Inflation. More importantly, a more active policy response to inflation alone does not suffice for explaining the U.S. macroeconomic stability, unless it is accompanied by a change in either trend inflation or policy responses to the output gap and output growth. This extends the literature by emphasizing the importance of the changes in other aspects of monetary policy in addition to its response to inflation.

JEL Classification: C11, C52, C62, E31, E52

Keywords: Monetary policy, Great Inflation, Indeterminacy, Trend inflation, Sequential Monte Carlo

*The authors are grateful to Kosuke Aoki, Olivier Coibion, Taeyoung Doh, Ippei Fujiwara, Mark Gertler, Yuriy Gorodnichenko, Jonathan Heathcote, Edward Herbst, Alejandro Justiniano, Munechika Katayama, Christian Matthes, Sophocles Mavroeidis, Taisuke Nakata, Frank Schorfheide, Mototsugu Shintani, Sanjay Singh, Carl Walsh, Tsutomu Watanabe, Alexander Wolman, Raf Wouters, and participants at CEF 2017, CEF 2019, EEA-ESEM 2017, MMM 2018, SED 2018, SETA 2019, SWET 2018, 1st Keio-Waseda Macro Workshop, 13th Dynare Conference, 30th (EC)’2 Conference, and seminars at the Bank of Canada, the Bank of Japan, the Federal Reserve Bank of Kansas City, the Federal Reserve Board, the National Bank of Belgium, National Chengchi University, and University of Hawaii at Manoa for comments and discussions. The views expressed in this paper are those of the authors and do not necessarily reflect the official views of the Bank of Japan, the Federal Reserve Bank of Cleveland, or the Federal Reserve System.

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1 Introduction

What led to macroeconomic stability in the United States after the Great Inflation of the 1970s? A large literature has regarded the Great Inflation as a consequence of self-fulfilling expectations in indeterminate equilibrium, which lasted until determinacy was restored by changes in the Fed’s policy under the chairmanship of Paul Volcker and his successors.\footnote{Following the literature, this paper explains the U.S. macroeconomic stability from the perspective of monetary policy. Other explanations emphasize a decline in the volatility of shocks to the U.S. economy (e.g., Sims and Zha, 2006; Justiniano and Primiceri, 2008) or the development of inventory management (e.g., Kahn, McConnell, and Perez-Quirós, 2002). Without relying on indeterminate equilibrium, Ascari, Bonomolo, and Lopes (2019) account for U.S. macroeconomic instability during the Great Inflation by allowing temporarily unstable dynamics that converge eventually to stable ones.}

In particular, the literature has established the view that the U.S. economy’s shift from indeterminacy to determinacy was achieved by the Fed’s change from a passive to an active policy response to inflation. A monetary policy response to inflation is called active if it satisfies the Taylor principle that the nominal interest rate should be raised by more than the increase in inflation. Otherwise, it is called passive. Clarida, Galí, and Gertler (2000) demonstrate the literature’s view by estimating a Taylor (1993)-type monetary policy rule during two periods, before and after Volcker’s appointment as Fed Chairman, and combining the estimated rule with a calibrated New Keynesian (henceforth NK) model to analyze determinacy. Lubik and Schorfheide (2004) confirm the view by estimating a Taylor-type rule and an NK model jointly during similar periods using a full-information Bayesian approach that allows for indeterminacy and sunspot fluctuations.\footnote{See also Boivin and Giannoni (2006), Kimura and Kurozumi (2010), and Lubik and Matthes (2016) among others for the monetary-policy explanation of U.S. macroeconomic stability after the Great Inflation.}

This paper revisits the literature’s view by estimating a generalized NK (henceforth GNK) model jointly with a Taylor-type rule.\footnote{For a literature review on GNK models, see, e.g., Ascari and Sbordone (2014).} This model differs from canonical NK (henceforth CNK) models used in the literature mainly in that some prices remain unchanged in each period in line with micro evidence.\footnote{For the micro evidence on price setting during and after the Great Inflation, see, e.g., Klenow and Kryvtsov (2008), Nakamura and Steinsson (2008), and Nakamura et al. (2017).} Consequently, instead of a canonical one, a generalized NK Phillips curve appears in the GNK model, with the distinct features that its coefficients...
depend on the level of trend inflation and that it includes additional forward-looking terms through which inflation responds to expected changes in future demand and discount rates on future profits under nonzero trend inflation. These features cause the GNK model to be more susceptible to indeterminacy than CNK models, as indicated by Hornstein and Wolman (2005), Kiley (2007), Ascari and Ropele (2009), and Coibion and Gorodnichenko (2011). Indeed, even an active policy response to inflation that generates determinacy in CNK models can induce indeterminacy in the GNK model.

Our estimation is performed using a full-information Bayesian approach based on Lubik and Schorfheide (2004). In their approach, however, when a model is estimated over both determinacy and indeterminacy regions of the model’s parameter space, its likelihood function is possibly discontinuous at the boundary of each region. As a consequence, the Random-Walk Metropolis-Hastings (henceforth RWMH) algorithm—which has been the most widely used in Bayesian estimation—can get stuck near a local mode and fail to find the entire posterior distribution for the model’s parameters. To deal with this difficulty, our paper adopts the sequential Monte Carlo (henceforth SMC) algorithm developed by Herbst and Schorfheide (2014, 2015). As they illustrate, the SMC algorithm can produce more reliable estimates of model parameters than the RWMH algorithm when the parameters’ posterior distribution is multimodal. This is particularly the case when the likelihood function of a model to be estimated exhibits discontinuity as in our paper.

Our empirical analysis makes three main contributions to the literature. First of all, the GNK model empirically outperforms CNK models during both periods before and after the

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5See also Kurozumi (2014, 2016) and Kurozumi and Van Zandwaghe (2016, 2017).
6Our GNK model extends the model of Coibion and Gorodnichenko (2011) that assumes firm-specific labor. In Appendix A, we also consider another type of GNK model, which extends, in a similar fashion, the model of Ascari and Ropele (2009) that assumes homogeneous labor. The different specifications of labor yield distinct implications for the GNK Phillips curve. For instance, our model has no effect of relative price distortion on the Phillips curve, whereas there is such an effect in the other model. For this point, see Kurozumi and Van Zandwaghe (2017). The present paper estimates the two types of GNK models and shows that our model empirically outperforms the other.
7The full-information Bayesian approach of Lubik and Schorfheide (2004) has been used in previous studies, such as Benati and Surico (2009), Bhattacharai, Lee, and Park (2012, 2016), Doko Tchatoka et al. (2017), and Hirose (2007, 2008, 2013, 2020).
Volcker disinflation of 1979–1982. We consider two types of CNK models. One type is a CNK counterpart to the GNK model and assumes that prices that remain unchanged in the GNK model are updated by indexing to trend inflation as in Yun (1996). The GNK model and its CNK counterpart are both augmented with backward-looking rule-of-thumb price-setters as in Galí and Gertler (1999) to take into account the possibility of intrinsic inertia in inflation. The other type of CNK model instead incorporates, as in Smets and Wouters (2007), price indexation to past and trend inflation, which has been extensively used in empirical studies. The superior empirical performance of the GNK model relative to the two CNK models indicates that the GNK model’s features that are more consistent with the micro evidence on price setting also contribute to a better fit of the model to U.S. macroeconomic time series, and thus the GNK model is more suitable for the analysis of what led to U.S. macroeconomic stability after the Great Inflation.

Second, the U.S. economy was likely in the indeterminacy region of the GNK model’s parameter space before 1979, while it likely entered the determinacy region after 1982, in line with the result obtained in the literature. However, the estimated GNK model points to substantial uncertainty about whether the policy response to inflation was active or passive during the pre-1979 period in the Taylor-type rule, which adjusts the interest rate for contemporaneous values of inflation, the output gap, and output growth in the presence of interest-rate smoothing. In the GNK model even an active policy response to inflation possibly fails to ensure determinacy, as noted above. The ambiguous result contrasts with the literature’s finding that the policy response to inflation was surely passive during the Great

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8This implies that the GNK model and its CNK counterpart coincide only when trend inflation is zero, so that the GNK model does not literally generalize the CNK counterpart. Thus, we also consider an NK model that nests both the GNK model and the CNK counterpart, and shows that the GNK model empirically outperforms the nested model as well.

9Note that embedding such price-setters in the GNK model is still consistent with the micro evidence that some prices remain unchanged in each period.

10Orphanides (2004) shows an active policy response to expected future inflation even before Volcker’s appointment as Fed Chairman by estimating a Taylor-type rule using real-time data on the Federal Reserve Board’s Greenbook forecast. A similar empirical result is obtained by Coibion and Gorodnichenko (2011), who also argue for ambiguity about whether the Taylor principle was satisfied before the Volcker disinflation, by presenting large standard errors of the estimated policy response to inflation.
Inflation and that the subsequent change to an active response led to the U.S. economy’s shift from indeterminacy to determinacy (in CNK models).\footnote{The CNK models considered in this paper confirm the literature’s view; that is, the policy response to inflation was passive and the U.S. economy was likely in the indeterminacy region before 1979, while the policy response became active and the economy likely entered the determinacy region after 1982.}

Last but not least, the increase in the policy response to inflation from the pre-1979 to the post-1982 estimate alone does not suffice for explaining the U.S. economy’s shift to determinacy, unless it is accompanied by either the estimated decline in trend inflation or the estimated change in policy responses to the output gap and output growth. This finding reveals that a lower rate of trend inflation (or equivalently a lower inflation target), a more dampened response to the output gap, and a more aggressive response to output growth play a key role in accounting for the U.S. economy’s shift, along with a more active response to inflation. Therefore, our finding extends the literature by emphasizing the importance of the changes in other aspects of monetary policy in addition to its response to inflation.

This paper is an extension of Lubik and Schorfheide (2004) and a complementary study to Coibion and Gorodnichenko (2011). It strengthens the analysis of Lubik and Schorfheide by adopting the SMC algorithm in their full-information Bayesian approach and estimating the GNK model (jointly with the Taylor-type rule) as well as the CNK models, which are similar to their model. While Lubik and Schorfheide estimate their model separately for the determinacy and indeterminacy regions of the model’s parameter space, the SMC algorithm enables us to conduct our estimation for both of the regions in one step. Coibion and Gorodnichenko revisit the literature’s view by using a calibrated GNK model in an approach analogous to Clarida, Gali, and Gertler (2000).\footnote{Arias et al. (2020) extend the analysis of Coibion and Gorodnichenko (2011) by employing a medium-scale GNK model based on Christiano, Eichenbaum, and Evans (2005), which is estimated during a post-1984 period within the determinacy region of the model’s parameter space.} They offer the alternative view that the U.S. economy’s shift to determinacy after the Great Inflation is due to their estimated change in a Taylor-type rule and their calibrated fall in trend inflation.\footnote{In the estimation of the Taylor-type rule by Coibion and Gorodnichenko (2011), its constant term contains not only trend inflation but also other factors. Thus they calibrate the level of trend inflation.} An advantage of our analysis is that we estimate both trend inflation and the Taylor-type rule’s coefficients as well as other structural model parameters under cross-equation restrictions and show that
our GNK model empirically outperforms the CNK models, giving strong support to our view on the shift from indeterminacy to determinacy.\footnote{Regarding the GMM estimation of the Taylor-type rule by Clarida, Gali, and Gertler (2000), Mavroeidis (2010) points to a weak-identification issue and emphasizes the need to make use of identifying assumptions that can be derived from the full structure of their model.}

The remainder of the paper proceeds as follows. Section 2 presents a GNK model with a Taylor-type rule. Section 3 explains the estimation strategy and data. Section 4 shows the results of the empirical analysis. Section 5 concludes.

2 Generalized New Keynesian Model

This paper investigates the source of the U.S. economy’s shift from indeterminacy of equilibrium to determinacy after the Great Inflation by estimating a GNK model jointly with a Taylor-type rule. This model differs from CNK models used in previous studies mainly in that each period a fraction of prices remains unchanged in line with micro evidence.

In the model there are a representative household, a representative final-good firm, a continuum of intermediate-good firms, and a central bank. The model extends that of Coibion and Gorodnichenko (2011) by introducing (external) habit formation in the household’s consumption preferences, backward-looking rule-of-thumb price-setters among intermediate-good firms as in Gali and Gertler (1999), and interest-rate smoothing in the Taylor-type rule so that the model has inertia in output, inflation, and the interest rate.\footnote{Note that incorporating the backward-looking rule-of-thumb price-setters enables us to embed inflation inertia without contradicting the micro evidence that some prices remain unchanged in each period.} This extension is made because our estimation is conducted with a full-information Bayesian approach based on Lubik and Schorfheide (2004), which may have a bias toward indeterminacy unless the model can generate sufficient persistence in endogenous variables, as argued by Beyer and Farmer (2007).

2.1 Households

The representative household consumes final goods $\tilde{C}_t$, supplies a set of labor services $\{l_t(i)\}$, each of which is specific to intermediate-good firm $i \in [0, 1]$, and purchases one-period riskless...
bonds $B_t$ so as to maximize the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \exp(z_{u,t}) \left[ \log(\tilde{C}_t - hC_{t-1}) - \frac{1}{1 + 1/\eta} \int_0^1 (l_t(i))^{1+1/\eta} di \right]$$

subject to the budget constraint

$$P_t \tilde{C}_t + B_t = \int_0^1 P_t W_t(i) l_t(i) di + r_{t-1} B_{t-1} + T_t,$$

where $E_t$ is the rational expectation operator conditional on information available in period $t$, $\beta \in (0, 1)$ is the subjective discount factor, $h \in [0, 1]$ is the degree of habit persistence in consumption preferences, $\eta \geq 0$ is the elasticity of labor supply, $C_t$ is aggregate consumption, $P_t$ is the price of final goods, $W_t(i)$ is the real wage rate paid by intermediate-good firm $i$, $r_t$ is the (gross) interest rate on bonds and is assumed to coincide with the monetary policy rate, $T_t$ consists of lump-sum taxes and transfers and firm profits received, and $z_{u,t}$ is a shock to current preferences.\(^\text{16}\)

Because the household’s consumption $\tilde{C}_t$ turns out to coincide with the aggregate consumption $C_t$, the first-order condition for utility maximization with respect to consumption becomes

$$\Xi_t = \frac{\exp(z_{u,t})}{C_t - hC_{t-1}},$$

(1)

where $\Xi_t$ is the marginal utility of consumption, and the first-order conditions regarding labor supply and bond holdings are given by

$$W_t(i) = \frac{(l_t(i))^{1/\eta} \exp(z_{u,t})}{\Xi_t},$$

(2)

$$1 = E_t \frac{\beta \Xi_{t+1}}{\Xi_t} \frac{r_t}{\pi_{t+1}},$$

(3)

where $\pi_t = P_t / P_{t-1}$ is the (gross) inflation rate of the final-good price.

### 2.2 Firms

The representative final-good firm produces homogeneous goods $Y_t$ by combining intermediate goods $\{Y_t(i)\}$ so as to maximize profit

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$$

\(^\text{16}\)Our GNK model considers firm-specific labor as in Coibion and Gorodnichenko (2011). Appendix A analyzes another type of GNK model, which assumes homogeneous labor as in Ascari and Ropele (2009), and shows that such a model empirically underperforms our GNK model. See also footnote 6.
subject to the constant elasticity of substitution (henceforth CES) aggregator

$$Y_t = \left[ \int_0^1 (Y_t(i))^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)},$$

where $P_t(i)$ is the price of intermediate good $i$ and $\theta > 1$ is the elasticity of substitution between intermediate goods.

The first-order condition for profit maximization yields the final-good firm’s demand curve for intermediate good $i$

$$Y_t(i) = Y_t \left( \frac{P_t(i)}{P_t} \right)^{-\theta},$$

and thus the CES aggregator leads to

$$P_t = \left[ \int_0^1 (P_t(i))^{1-\theta} di \right]^{1/(1-\theta)}.$$  \hspace{1cm} \text{(5)}

The final-good market clearing condition is given by

$$Y_t = C_t.$$  \hspace{1cm} \text{(6)}

Each intermediate-good firm $i$ produces one kind of differentiated good $Y_t(i)$ under monopolistic competition using the production technology

$$Y_t(i) = A_t l_t(i),$$

where $A_t$ denotes the technology level and follows the stochastic process

$$\log A_t = \log a + \log A_{t-1} + z_{a,t},$$

where $\log a$ is the steady-state rate of technological change, which turns out to coincide with the steady-state rate of output growth, and $z_{a,t}$ is a (non-stationary) technology shock.

The first-order condition for cost minimization yields firm $i$’s real marginal cost

$$mc_t(i) = \frac{W_t(i)}{A_t}.$$  \hspace{1cm} \text{(9)}

Prices of intermediate goods are set on a staggered basis as in Calvo (1983). In each period, a fraction $\lambda \in (0, 1)$ of firms keeps prices unchanged, while the remaining fraction $1 - \lambda$ sets prices in the following two ways. As in Galí and Gertler (1999), a fraction $\omega \in [0, 1)$ of price-setting firms uses a backward-looking rule of thumb, while the remaining fraction $1 - \omega$ optimizes prices.
The price set by the backward-looking rule of thumb is given by

\[ P_t = P_{t-1} \pi_{t-1}, \quad \text{or} \quad P_t^r = \frac{P_t^r}{P_t} = \frac{(P_{t-1}^a/P_t) \pi_{t-1}}{P_t/P_{t-1}} = \frac{P_{t-1}^a \pi_{t-1}}{P_t}, \]  

(10)

where

\[ P_t^a = (P_t^r)^\omega (P_t^r)^{1-\omega}, \quad \text{or} \quad P_t^a = \left( \frac{P_t^a}{P_t} \right)^\omega \left( \frac{P_t^r}{P_t} \right)^{1-\omega} = \left( P_t^r \right)^\omega \left( P_t^r \right)^{1-\omega}, \]  

(11)

and \( P_t^a \) is the price set by optimizing firms in period \( t \). The price \( P_t^a \) maximizes the relevant profit function

\[ E_t \sum_{j=0}^{\infty} \lambda^j Q_{t+j} \left( \frac{P_t(i)}{P_{t+j}} - m_{c_{t+j}}(i) \right) Y_{t+j} \left( \frac{P_t(i)}{P_{t+j}} \right)^{-\theta}, \]

where \( Q_{t+j} \) is the stochastic discount factor between period \( t \) and period \( t+j \).

The first-order condition for the optimized price \( P_t^a \) becomes

\[ E_t \sum_{j=0}^{\infty} (\beta \lambda)^j \frac{Q_{t+j}}{\Xi_t} \frac{Y_{t+j}}{Y_t} \prod_{k=1}^{j} \frac{1}{\pi_{t+k}} \left( \frac{P_t^a}{P_t} \right)^{1/\theta} \left( \frac{Y_{t+j}}{Y_t} \right)^{1/\theta} \left( \frac{A_{t+j}}{A_{t+j}} \right)^{1-\theta} - \theta \right) = 0, \]

(12)

where the equilibrium condition \( Q_{t,t+j} = \beta^j \Xi_{t+j}/\Xi_t \) is used and \( m_{c_{t+j}} \) denotes period-\( t+j \) real marginal cost of firms that optimize prices in period \( t \). From (1), (2), (4), (6), (7), and (9), it follows that the marginal cost is given by

\[ m_{c_{t+j}} = \left( P_t^a \prod_{k=1}^{j} \frac{1}{\pi_{t+k}} \right) \left( \frac{Y_{t+j}}{A_{t+j}} \right)^{1/\theta} \left( \frac{Y_{t+j}}{A_{t+j}} \right)^{-\theta} \left( \frac{A_{t+j}}{A_{t+j}} \right)^{1-\theta} \]  

(13)

Under staggered price-setting, the final-good price equation (5) can be rewritten as

\[ 1 = (1 - \lambda) \left[ (1 - \omega) \left( P_t^r \right)^{1-\theta} + \omega \left( P_t^r \right)^{1-\theta} \right] + \lambda \pi_t^{\theta-1}. \]  

(14)

### 2.3 Central bank

The central bank conducts monetary policy according to a Taylor (1993)-type rule. This rule adjusts the policy rate \( r_t \) in response to inflation \( \pi_t \), the output gap \( x_t \), and output growth \( Y_t/Y_{t-1} \) in the presence of policy-rate smoothing:

\[ \log r_t = \phi_t \log r_{t-1} + (1-\phi_t) \left[ \log r + \phi_x (\log \pi_t - \log \pi) + \phi_x \log x_t + \phi_{\Delta y} \left( \log \frac{Y_t}{Y_{t-1}} - \log a \right) \right] + z_{r,t}, \]

(15)
where the output gap is defined as
\[ x_t = \frac{Y_t}{Y^n_t}, \]  
(16)

\[ Y^n_t \] is the natural rate of output, \( z_{r,t} \) is a monetary policy shock, \( r \geq 1 \) is the steady-state (gross) policy rate, \( \pi \) is the steady-state value of \( \pi_t \) and represents the (gross) rate of trend inflation, \( \phi_r \in [0, 1) \) is the degree of policy-rate smoothing, and \( \phi_x, \phi_{\Delta y} \) are the degrees of policy responses to inflation, the output gap, and output growth.

By considering flexible prices (i.e., \( \lambda = \omega = 0 \)) in the intermediate-good price equation (12) and the final-good price equation (14) and combining the resulting two equations with the marginal cost equation (13), we can derive the law of motion for the natural rate of output
\[ \left( \frac{Y^n_t}{A_t} \right)^{1+1/\eta} = \frac{\theta - 1}{\theta} + h \left( \frac{Y^n_t}{A_t} \right)^{1/\eta} \frac{Y^n_{t-1}}{A_t}. \]  
(17)

### 2.4 Equilibrium conditions

The equilibrium conditions consist of (1), (3), (6), (8), and (10)–(17). For the steady state to be well defined, the following condition is assumed:
\[ \lambda \max(\pi^{\theta-1}, \beta \pi^{\theta(1+1/\eta)}) < 1. \]  
(18)

This assumption ensures that the two relevant discount factors \( \beta \lambda \pi^{\theta-1} \) and \( \beta \lambda \pi^{\theta(1+1/\eta)} \) in the optimal price-setting condition (12) combined with the real marginal cost equation (13) are less than one.

Combining the equilibrium conditions, rewriting the resulting conditions in terms of the detrended variables \( y_t = Y_t/A_t \) and \( y^n_t = Y^n_t/A_t \), and log-linearizing the conditions under assumption (18) yields the GNK Phillips curve
\[ \hat{\pi}_t = \gamma_b \hat{\pi}_{t-1} + \gamma_f E_t \hat{\pi}_{t+1} + \kappa \hat{y}_t + \frac{h \kappa \lambda}{a - h}(\hat{y}_t - \hat{y}_{t-1} + z_{a,t}) + \psi_t, \]  
(19)

where hatted variables denote log-deviations from steady-state values and \( \psi_t \) is an auxiliary variable that evolves according to the forward-looking equation
\[ \psi_t = \gamma' E_t \psi_{t+1} + \kappa' (E_t \hat{y}_{t+1} - \hat{y}_t + E_t \hat{z}_{a,t+1} + \theta E_t \hat{\pi}_{t+1} - \hat{r}_t), \]  
(20)

so that the variable \( \psi_t \) drives inflation in response to expected changes in future demand and discount rates on future profits under nonzero trend inflation. The coefficients in the
equations (19) and (20) are given by
\[ \gamma_b = \frac{\omega}{\phi}, \gamma_f = \beta\lambda\pi^{\theta(1+1/\eta)} / \phi, \kappa = \kappa(1 + 1/\eta), \]
\[ \kappa_\lambda = (1 - \lambda\pi^{\theta-1})(1 - \beta\lambda\pi^{\theta(1+1/\eta)})(1 - \omega)/(\phi(1 + \theta/\eta)), \gamma_\psi = \beta\lambda\pi^{\theta-1}, \kappa_\psi = \gamma_\psi(\pi^{1+\theta/\eta} - 1)(1 - \lambda\pi^{\theta-1})(1 - \omega)/(\phi(1 + \theta/\eta)), \]
and \[ \varphi = \lambda\pi^{\theta-1} + \omega(1 - \lambda\pi^{\theta-1} + \beta\lambda\pi^{\theta(1+1/\eta)}). \]
Thus all the coefficients in the GNK Phillips curve, \( \gamma_b, \gamma_f, \kappa, \) and \( \kappa_\lambda, \) depend on the level of trend inflation \( \pi. \)

The remaining log-linearized equilibrium conditions are the spending Euler equation
\[ \hat{y}_t = \frac{h}{a + h}(\hat{y}_{t-1} - z_{a,t}) + \frac{a}{a + h}(E_t\hat{y}_{t+1} + E_t z_{a,t+1}) - \frac{a - h}{a + h}(\hat{r}_t - E_t\hat{\pi}_{t+1} + E_t z_{u,t+1} - z_{u,t}), \]
the natural rate of output
\[ \hat{y}_n^n = \frac{h\eta}{a(1 + \eta) - h}(\hat{y}_{t-1} - z_{a,t}), \]
the output gap
\[ \hat{x}_t = \hat{y}_t - \hat{y}_n^n, \]
and the Taylor-type monetary policy rule
\[ \hat{r}_t = \phi_r\hat{r}_{t-1} + (1 - \phi_r)[\phi_x\hat{\pi}_t + \phi_x\hat{x}_t + \phi_\Delta\phi(\hat{y}_t - \hat{y}_{t-1} + z_{a,t})] + z_{r,t}. \]

Each of the three shocks \( z_{j,t}, j \in \{u,a,r\} \) is assumed to follow the stationary first-order autoregressive process
\[ z_{j,t} = \rho_j z_{j,t-1} + \varepsilon_{j,t}, \]
where \( \rho_j \in [0,1) \) is the autoregressive parameter and \( \varepsilon_{j,t} \sim i.i.d. N(0,\sigma_j^2) \) is the innovation to each shock.

### 2.5 Canonical New Keynesian models

The GNK model presented above is estimated and used for analyzing the source of the U.S. economy’s shift from determinacy of equilibrium to indeterminacy after the Great Inflation. Prior to the analysis, the GNK model is compared with two types of CNK models in terms of empirical performance.

One type of CNK model is a CNK counterpart to the GNK model. It is based on Galí and Gertler (1999) and thus called the GG-CNK model. This model can be derived by altering the GNK model so that firms that keep prices unchanged in the aforementioned
setting update prices using indexation to trend inflation $\pi$ as in Yun (1996). Consequently, the GG-CNk model consists of (21)–(25) and the NK Phillips curve

$$\hat{\pi}_t = \gamma_{b,cnk} \hat{\pi}_{t-1} + \gamma_{f,cnk} E_t \hat{\pi}_{t+1} + \kappa_{cnk} \hat{y}_t + \frac{h \kappa_{cnk}}{a - h} (\hat{y}_t - \hat{y}_{t-1} + z_{a,t}),$$  

(26)

where $\gamma_{b,cnk} = \omega / \varphi_1$, $\gamma_{f,cnk} = \beta \lambda_{gg} / \varphi_1$, $\kappa_{cnk} = \kappa_{ckn}(1 + 1 / \eta)$, $\kappa_{k,cnk} = (1 - \lambda_{gg})(1 - \beta \lambda_{gg})(1 - \omega) / [\varphi_1 (1 + \theta / \eta)]$, $\varphi_1 = \lambda_{gg} + \omega (1 - \lambda_{gg} + \beta \lambda_{gg})$, and $\lambda_{gg}$ represents the probability of price indexation to trend inflation. This implies that the GNK model and its CNK counterpart—the GG-CNk model—coincide only when trend inflation is zero (i.e., $\pi = 1$). Hence, the GNK model does not literally generalize its CNK counterpart. Therefore, we also consider an NK model that nests both the GNK and the GG-CNk models, by altering the GNK model so that firms that keep prices unchanged in the model update prices using indexation to trend inflation $\pi$ with the degree $\alpha \in [0, 1]$. This model, referred to as the nested model, differs from the GNK model only in the coefficients of the GNK Phillips curve (19) and the auxiliary-variable equation (20), which are given by $\gamma_b = \omega / \varphi$, $\gamma_f = \beta \lambda_{gg} \pi^{\theta(1+1/\eta)(1-\alpha)} / \varphi$, $\kappa_\lambda = (1 - \lambda_{gg} \pi^{\theta(1-1)(1-\alpha)})(1 - \beta \lambda_{gg} \pi^{\theta(1+1/\eta)(1-\alpha)}) / [\varphi(1 + \theta / \eta)]$, $\gamma_\psi = \beta \lambda_{gg} \pi^{\theta(1-1)(1-\alpha)} / \varphi$, $\kappa_\psi = \gamma_\psi (\pi^{(1+\theta/\eta)(1-\alpha)} - 1)(1 - \lambda_{gg} \pi^{\theta(1-1)(1-\alpha)}(1 - \omega) / [\varphi(1 + \theta / \eta)]$, and $\varphi = \lambda_{gg} \pi^{\theta(1-1)(1-\alpha)} + \omega (1 - \lambda_{gg} \pi^{\theta(1-1)(1-\alpha)} + \beta \lambda_{gg} \pi^{\theta(1+1/\eta)(1-\alpha)})$. The nested model includes the GNK model and the GG-CNk model as the special cases of $\alpha = 0$ and $\alpha = 1$, respectively.

The other type of CNK model incorporates price indexation to past and trend inflation as in Smets and Wouters (2007) and has been extensively used in empirical studies. This model, called the SW-CNk model, can be derived by altering the GNK model so that each period a fraction $\lambda_{sw}$ of firms updates prices using indexation to recent past inflation $\pi_{t-1}$ and trend inflation $\pi$ with the relative past-inflation weight $\omega_{sw} \in [0, 1]$, while the remaining fraction $1 - \lambda_{sw}$ sets prices optimally. The SW-CNk model differs from the GG-CNk model only in the coefficients of the NK Phillips curve (26), $\gamma_{b,cnk}$, $\gamma_{f,cnk}$, $\kappa_{cnk}$, and $\kappa_{k,cnk}$, which are given by $\gamma_{b,cnk} = \omega_{sw} / \varphi_{sw}$, $\gamma_{f,cnk} = \beta / \varphi_{sw}$, $\kappa_{cnk} = \kappa_{ckn}(1 + 1 / \eta)$, $\kappa_{k,cnk} = (1 - \lambda_{sw})(1 - \beta \lambda_{sw}) / [\lambda_{sw} \varphi_{sw}(1 + \theta / \eta)]$, and $\varphi_{sw} = 1 + \beta \omega_{sw}$, where $\omega_{sw}$ denotes the probability of price indexation to past inflation.
3 Estimation Strategy and Data

This section describes the strategy and data for estimating the GNK model, the two types of CNK models, and the nested model, which are all presented in the preceding section. These models are estimated using a full-information Bayesian approach based on Lubik and Schorfheide (2004). Specifically, each model’s likelihood function is constructed not only for the determinacy region of the model’s parameter space but also for the indeterminacy region.\textsuperscript{17} The likelihood function can then exhibit discontinuity at the boundary of each region.\textsuperscript{18} As a consequence, the posterior distribution for parameters in the model is possibly multimodal, and thus the widely used RWMH algorithm can get stuck near a local mode and fail to find the entire posterior distribution for the parameters. To deal with this problem, the SMC algorithm developed by Herbst and Schorfheide (2014, 2015) is adopted to generate the posterior distribution.\textsuperscript{19} The SMC algorithm can overcome the problem inherent in multimodality by building a particle approximation to the posterior distribution gradually through tempering the likelihood function.

In this section we begin by describing the method for solving linear rational expectations (henceforth LRE) models under indeterminacy. We then explain how Bayesian inferences over both determinacy and indeterminacy regions of the parameter space are made with the SMC algorithm. Moreover, we present the data and prior distributions used in estimation.

3.1 Rational expectations solutions under indeterminacy

Lubik and Schorfheide (2003) derive a full set of solutions to LRE models by extending the solution algorithm developed by Sims (2002).\textsuperscript{20} Any LRE model can be written in the

\textsuperscript{17}The full-information Bayesian approach of Lubik and Schorfheide (2004) allows for indeterminate equilibrium by including a sunspot shock and its related arbitrary coefficient matrix in solutions to linear rational expectations models. By estimating the coefficient matrix with a fairly loose prior, a set of particular solutions that are the most consistent with data can be selected from a full set of solutions.

\textsuperscript{18}With a univariate model, Lubik and Schorfheide (2004) illustrate discontinuity of the model’s likelihood function that is constructed for both determinacy and indeterminacy regions of its parameter space.

\textsuperscript{19}Creal (2007) is the first paper that uses an SMC algorithm in Bayesian estimation of a dynamic stochastic general equilibrium model.

\textsuperscript{20}Sims (2002) generalizes the solution algorithm of Blanchard and Kahn (1980) and characterizes one particular solution in the case of indeterminacy. In this solution, the contribution to forecast errors of
canonical form
\[ \Gamma_0(\vartheta)s_t = \Gamma_1(\vartheta)s_{t-1} + \Psi(\vartheta)\varepsilon_t + \Pi(\vartheta)\xi_t, \]  
\[ (27) \]
where \( \Gamma_0(\vartheta), \Gamma_1(\vartheta), \Psi(\vartheta), \) and \( \Pi(\vartheta) \) are coefficient matrices that depend on model parameters \( \vartheta \), \( s_t \) is a vector of endogenous variables including those expected at time \( t \), \( \varepsilon_t \) is a vector of fundamental shocks, and \( \xi_t \) is a vector of forecast errors. Specifically, in the GNK model, these vectors are given by
\[ s_t = \left[ \hat{y}_t, \hat{\pi}_t, \hat{r}_t, \hat{y}_{n,t}, \hat{x}_t, \psi_t, z_{u,t}, z_{a,t}, z_{r,t}, E_t\hat{y}_{t+1}, E_t\hat{\pi}_{t+1}, E_t\psi_{t+1} \right]', \]
\[ \varepsilon_t = \left[ \varepsilon_{u,t}, \varepsilon_{a,t}, \varepsilon_{r,t} \right]', \]
\[ \xi_t = \left[ (\hat{y}_t - E_t\hat{y}_t), (\hat{\pi}_t - E_t\hat{\pi}_t), (\psi_t - E_t\psi_t) \right]', \]

According to Lubik and Schorfheide (2003), a full set of solutions to the LRE model (27) is of the form
\[ s_t = \Phi_x(\vartheta)s_{t-1} + \Phi_\varepsilon(\vartheta, \tilde{M})\varepsilon_t + \Phi_\zeta(\vartheta)\zeta_t, \]  
\[ (28) \]
where \( \Phi_x(\vartheta), \Phi_\varepsilon(\vartheta, \tilde{M}), \) and \( \Phi_\zeta(\vartheta) \) are coefficient matrices, \( \tilde{M} \) is an arbitrary matrix, and \( \zeta_t \sim\) i.i.d. \( N(0, \sigma_\zeta^2) \) is a reduced-form sunspot shock, which is a non-fundamental disturbance.\(^{21}\)

The matrix \( \tilde{M} \) captures the correlation of the forecast errors \( \xi_t \) with the fundamental shocks \( \varepsilon_t \), and thus nonzero components in \( \tilde{M} \) allow for the correlation of \( \varepsilon_t \) with the sunspot shock \( \zeta_t \). In the case of determinacy, the solution (28) is reduced to
\[ s_t = \Phi_x^D(\vartheta)s_{t-1} + \Phi_\varepsilon^D(\vartheta)\varepsilon_t. \]  
\[ (29) \]

Two features distinguish the solution (28) under indeterminacy. First, the dynamics of the LRE model is driven not only by the fundamental shocks \( \varepsilon_t \) but also by the sunspot shock \( \zeta_t \). Second, the solution is not unique due to the presence of the arbitrary matrix \( \tilde{M} \), that is, the LRE model induces indeterminate solutions. Thus, to specify the law of motion of the endogenous variables \( s_t \), the matrix \( \tilde{M} \) must be pinned down.

\(^{21}\)Lubik and Schorfheide (2003) originally express the last term in (28) as \( \Phi_\zeta(\theta, M_\zeta)\zeta_t \), where \( M_\zeta \) is an arbitrary matrix and \( \zeta_t \) is a vector of sunspot shocks. For identification, Lubik and Schorfheide (2004) impose the normalization \( M_\zeta = 1 \) with the dimension of the sunspot shock vector being unity. Such a normalized shock is referred to as a “reduced-form sunspot shock” in that it contains beliefs associated with all the expectational variables.
The arbitrary matrix $\tilde{M}$ is inferred from the data used in estimation, following Lubik and Schorfheide (2004). The prior distribution for $\tilde{M}$ is set so that it is centered around the matrix $M^*(\vartheta)$ given in a particular solution. That is, $\tilde{M}$ is replaced with $M^*(\vartheta) + M$, and $M$ is estimated with prior mean zero. The matrix $M^*(\vartheta)$ is selected so that the contemporaneous impulse responses of endogenous variables to fundamental shocks (i.e., $\partial s_t / \partial \varepsilon_t$) are continuous at the boundary between determinacy and indeterminacy regions of the parameter space. More specifically, for each set of $\vartheta$, the procedure searches for a vector $\vartheta^*$ that lies on the boundary of the determinacy region, and selects $M^*(\vartheta)$ that minimizes the discrepancy between $\partial s_t / \partial \varepsilon_t(\vartheta, M^*(\vartheta))$ and $\partial s_t / \partial \varepsilon_t(\vartheta^*)$ using a least-squares criterion. In the search for $\vartheta^*$, the procedure finds $\vartheta^*$ numerically by perturbing the parameter $\phi_\pi$ in the monetary policy rule (24), given the other parameters in $\vartheta$.

3.2 Bayesian inference with a sequential Monte Carlo algorithm

The LRE model is estimated using a full-information Bayesian approach that extends the model’s likelihood function to the indeterminacy region of the parameter space. Following Lubik and Schorfheide (2004), the likelihood function for a sample of observations $X_T = [X_1, ..., X_T]'$ is given by

$$p(X^T|\vartheta, M) = 1\{\vartheta \in \Theta^D\} p^D(X^T|\vartheta) + 1\{\vartheta \in \Theta^I\} p^I(X^T|\vartheta, M),$$

where $\Theta^D$ and $\Theta^I$ are the determinacy and indeterminacy regions of the parameter space; $1\{\vartheta \in \Theta^i\}, i \in \{D, I\}$ is the indicator function that equals one if $\vartheta \in \Theta^i$ and zero otherwise; and $p^D(X^T|\vartheta)$ and $p^I(X^T|\vartheta, M)$ are the likelihood functions of the state-space models that consist of observation equations and either the determinacy solution (29) or the indeterminacy solution (28). Then, by Bayes’ theorem, updating a prior distribution $p(\vartheta, M)$ with the sample $X^T$ leads to the posterior distribution

$$p(\vartheta, M|X^T) = \frac{p(X^T|\vartheta, M)p(\vartheta, M)}{p(X^T)} = \frac{p(X^T|\vartheta, M)p(\vartheta, M)}{\int p(X^T|\vartheta, M)p(\vartheta, M)d\vartheta \cdot dM}.$$  

\footnote{We also considered an alternative prior for the indeterminacy solution. As in Bhattarai, Lee, and Park (2016), this prior is centered at the solution proposed by Sims (2002) that is described in footnote 20. We obtained similar posterior estimates to those which are shown in the subsequent section and confirmed the robustness of our main results.}
To approximate the posterior distribution, we exploit the generic SMC algorithm with likelihood tempering described in Herbst and Schorfheide (2014, 2015). In the algorithm, a sequence of tempered posteriors are defined as
\[
\varpi_n(\vartheta) = \frac{[p(X^T|\vartheta, M)^{\tau_n}p(\vartheta, M)]}{\int[p(X^T|\vartheta, M)^{\tau_n}p(\vartheta, M)d\vartheta \cdot dM]}, \quad n = 0, ..., N_{\tau}.
\]
The tempering schedule \(\{\tau_n\}_{n=0}^{N_{\tau}}\) is determined by \(\tau_n = (n/N_{\tau})^\chi\), where \(\chi\) is a parameter that controls the shape of the tempering schedule. The SMC algorithm generates parameter draws \(\vartheta^{(i)}_n, M^{(i)}_n\) and associated importance weights \(w^{(i)}_n\)—which are called particles—from the sequence of posteriors \(\{\varpi_n\}_{n=1}^{N_{\tau}}\); that is, at each stage, \(\varpi_n(\vartheta)\) is represented by a swarm of particles \(\{\vartheta^{(i)}_n, M^{(i)}_n, w^{(i)}_n\}_{i=1}^N\), where \(N\) denotes the number of particles.\(^{24}\) For \(n = 0, ..., N_{\tau}\), the algorithm sequentially updates the swarm of particles \(\{\vartheta^{(i)}_n, M^{(i)}_n, w^{(i)}_n\}_{i=1}^N\) through importance sampling.\(^{24}\)

Posterior inferences about parameters to be estimated are made based on the particles \(\{\vartheta^{(i)}_{N_{\tau}}, M^{(i)}_{N_{\tau}}, w^{(i)}_{N_{\tau}}\}_{i=1}^N\) from the final importance sampling. The SMC-based approximation of the marginal data density is given by
\[
p(X^T) = \prod_{n=1}^{N_{\tau}} \left( \frac{1}{N} \sum_{i=1}^N \tilde{w}^{(i)}_n \right),
\]
where \(\tilde{w}^{(i)}_n\) is the incremental weight defined as \(\tilde{w}^{(i)}_n = [p(X^T|\vartheta^{(i)}_{n-1}, M^{(i)}_{n-1})]^{\tau_n-\tau_{n-1}}\). The posterior probability of equilibrium determinacy can be calculated as\(^{25}\)
\[
P\{\vartheta \in \Theta^D | X^T\} = \frac{1}{N} \sum_{i=1}^N 1\{\vartheta^{(i)}_{N_{\tau}} \in \Theta^D\}.
\]

In the subsequent empirical analysis, the SMC algorithm uses \(N = 10,000\) particles and \(N_{\tau} = 200\) stages. The parameter that controls the tempering schedule is set at \(\chi = 2\) following Herbst and Schorfheide (2014, 2015).

### 3.3 Data

Our estimation is performed using three U.S. time series on the quarterly frequency: the per-capita real GDP growth rate \(100\Delta \log Y_t\), the inflation rate of the GDP implicit price

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\(^{23}\)We make use of parallelization in the evaluation of the importance weights \(w^{(i)}_n\) for \(i = 1, ..., N\).

\(^{24}\)This process includes one step of a single-block RWMH algorithm.

\(^{25}\)Based on the prior draws, the prior probability of equilibrium determinacy can be calculated in the same manner.
deflator \( (100\log \pi_t) \), and the federal funds rate \( (100\log r_t) \). The observation equations that relate the data to model variables are given by

\[
\begin{bmatrix}
100\Delta \log Y_t \\
100 \log \pi_t \\
100 \log r_t
\end{bmatrix}
= 
\begin{bmatrix}
\bar{a} \\
\bar{\pi} \\
\bar{r}
\end{bmatrix}
+ 
\begin{bmatrix}
\tilde{y}_t - \tilde{y}_{t-1} + z_{a,t} \\
\tilde{\pi}_t \\
\tilde{r}_t
\end{bmatrix},
\]

where \( \bar{a} = 100(a - 1), \bar{\pi} = 100(\pi - 1), \) and \( \bar{r} = 100(r - 1) \).

To examine the U.S. economy’s shift from indeterminacy to determinacy, that is, U.S. macroeconomic stability after the Great Inflation of the 1970s, the estimation is conducted for two periods: the pre-1979 period from 1966:Q1 to 1979:Q2 and the post-1982 period from 1982:Q4 to 2008:Q4. Following Lubik and Schorfheide (2004), the Volcker disinflation period from 1979:Q3 to 1982:Q3 is excluded.

### 3.4 Fixed parameters and prior distributions

Before the estimation, the elasticity of labor supply and the elasticity of substitution between intermediate goods are fixed at \( \eta = 1 \) and \( \theta = 9.32 \) to avoid an identification issue. The former value is a standard one in the macroeconomic literature, while the latter is the estimate of Ascani and Sbordone (2014). All the other parameters are estimated; their prior distributions are shown in Table 1.

The prior mean of the steady-state (quarterly) rates of output growth, inflation, and nominal interest \( \bar{a}, \bar{\pi}, \bar{r} \) is set at their respective averages over the period from 1966:Q1 to 2008:Q4. The prior distributions for the structural and policy parameters—\( h \) (spending habit persistence); \( \omega \) (fraction of backward-looking rule-of-thumb price-setters) or \( \omega_{sw} \) (relative weight on past inflation in price indexation); \( \lambda \) (probability of no price change), \( \lambda_{gg} \) (probability of price indexation to trend inflation), or \( \lambda_{sw} \) (probability of price indexation to past inflation); \( \phi_r \) (policy-rate smoothing); \( \phi_\pi \) (policy response to inflation); \( \phi_x \) (policy response to the output gap); and \( \phi_{\Delta y} \) (policy response to output growth)—are based on Smets

\[\text{Note:} \quad \text{Because the post-1982 period ends before the nominal interest rate reached its effective lower bound, the non-linearity arising from the lower bound is not a critical issue for our estimation strategy.}\]

\[\text{Note:} \quad \text{For the subjective discount factor } \beta, \text{ the steady-state condition } \beta = \pi a/r \text{ is used in estimation.}\]
and Wouters (2007). For the GNK model, these distributions lead to the prior probability of equilibrium determinacy of 0.482, which is almost even, thus indicating that there is a priori no substantial bias toward determinacy or indeterminacy. In the same vein, for the SW-CNK model, the GG-CNK model, and the nested model, the prior mean of \( \phi_\pi \) is set at 1.125, 1.1, and 1.245, so that the prior probability of determinacy is 0.481, 0.485, and 0.484, respectively.

Regarding the structural shocks, the prior distributions for the autoregressive parameters \( \rho_i, i \in \{u, a, r\} \) are beta distributions with mean of 0.5 and standard deviation of 0.2, while those for the standard deviations of the shock innovations \( \sigma_i, i \in \{u, a, r\} \) are inverse gamma distributions with mean of 0.63 and standard deviation of 0.33. As for the indeterminacy solution, the priors for the coefficients \( M_i, i \in \{u, a, r\} \) are normal distributions with mean zero and standard deviation of unity, while that for the standard deviation of the sunspot shock \( \sigma_\zeta \) is the same as those for the standard deviations of the structural shock innovations.

4 Results of Empirical Analysis

This section presents the results of the empirical analysis. First, we discuss the estimation results. We then address the paper’s main question of what led to the U.S. economy’s shift from indeterminacy of equilibrium to determinacy after the Great Inflation.

4.1 Estimation results

This subsection begins by comparing the empirical performance among the GNK model, the two types of CNK models, and the nested model. Tables 2 and 3 report the posterior estimates of these four models in the pre-1979 and the post-1982 periods, respectively. The second to last row of each table presents the log marginal data densities \( \log p(X^T) \) and shows that the value for the GNK model (i.e., −128.05) is the largest in the pre-1979 period, while that for the SW-CNK model (i.e., −64.43) is the largest in the post-1982 period. Besides, in both periods, the GG-CNK model has the smallest values, and the values for the nested

\[28\text{For } \alpha \text{ (the degree of price indexation to trend inflation in the nested model), the prior is the uniform distribution between zero and unity.}\]
model are between those for the GNK model and for the GG-CNk model. Thus we focus on the GNK model and the SW-CNk model in the subsequent analysis.

In light of the empirical result of Cogley and Sbordone (2008) that there is no need for backward-looking components in an NK Phillips curve when drift in trend inflation is taken into account, we estimate the GNK and the SW-CNk models with no inertia in inflation (i.e., \( \omega = 0 \) in the GNK model and \( \omega_{sw} = 0 \) in the SW-CNk model). Table 4 shows the posterior estimates of the GNK and the SW-CNk models with no inflation inertia in the pre-1979 and the post-1982 periods. The log marginal data densities \( \log p(X^T) \) shown in the second to last row of the table indicate two findings. First, the GNK and the SW-CNk models without inflation inertia exhibit higher densities than those with it in both periods: for the GNK (SW-CNk) model, \(-121.23 > -128.05 \) (\(-124.62 > -130.43 \)) in the pre-1979 period and \(-53.66 > -65.98 \) (\(-56.87 > -64.43 \)) in the post-1982 period. Second, the GNK model with \( \omega = 0 \) has larger densities than the SW-CNk model with \( \omega_{sw} = 0 \) in both periods. Therefore, the GNK model with no inertia of inflation is more suitable than any other models considered for the analysis of what led to U.S. macroeconomic stability after the Great Inflation, which has been addressed using CNk models in previous literature. In other words, the feature of the GNK model that some prices remain unchanged in each quarter is not only more consistent with micro evidence on price setting, but also contributes to a better fit of the model to the U.S. macroeconomic time series.

The posterior probability of equilibrium determinacy \( \mathbb{P}\{\theta \in \Theta^D | X^T\} \) is reported in the last row of Table 4. For both the GNK model with \( \omega = 0 \) and the SW-CNk model with \( \omega_{sw} = 0 \), the probability of determinacy is almost zero in the pre-1979 period, whereas it is unity in the post-1982 period. Hence, both models share the estimation result that the U.S. economy was likely in the indeterminacy region of the parameter space before 1979, while the economy likely entered the determinacy region after 1982, in line with the result obtained in previous literature. However, there is an important difference between the estimation results of the two models. In the CNk model, the policy response to inflation \( \phi_\pi \)—its posterior mean of 0.44 and the 90 percent highest posterior density (HPD) interval of [0.07, 0.74]—was definitely passive in the pre-1979 period and then became active—the posterior mean of 2.85 and the 90% HPD interval of [1.93, 3.65]—in the post-1982 period. This result is consistent with that obtained in the literature, and thus the CNk model
confirms the literature’s view that ascribes the U.S. economy’s shift from indeterminacy to
determinacy after the Great Inflation to the Fed’s change from a passive to an active policy
response to inflation. On the other hand, the GNK model exhibits substantial uncertainty
about whether the policy response to inflation was active or passive—the posterior mean
of 1.25 and the 90% HPD interval of [0.22, 2.23]—during the pre-1979 period, in contrast
with the literature’s view.\footnote{According to the posterior distribution, the posterior probability of the policy response to inflation being active during the pre-1979 period in the GNK model is 0.58.}
It is worth noting that, despite such uncertainty, the posterior probability of indeterminacy is almost unity in the pre-1979 period, because even an active policy response to inflation possibly fails to ensure determinacy in the GNK model, as indicated by Ascari and Ropele (2009) and Coibion and Gorodnichenko (2011).\footnote{For an estimated Taylor-type rule, Orphanides (2004) obtains an active response to expected future inflation during the pre-1979 period and thus claims that self-fulfilling expectations cannot be the source of U.S. macroeconomic instability during the Great Inflation. This claim, however, does not necessarily hold for the GNK model because an active policy response to inflation—the Taylor principle—is not a sufficient condition for determinacy. In Appendix B, we derive a long-run version of the Taylor principle—that in the long run the nominal interest rate should be raised by more than the increase in inflation—for the GNK model. This version of the Taylor principle serves as a necessary condition for determinacy in GNK models, as shown by Kurozumi (2014, 2016) and Kurozumi and Van Zandweghe (2016, 2017).}

Because the GNK model outperforms the CNK model during both periods in terms of the fit to the
data, our finding is more compelling than the literature’s view.

In the GNK model with $\omega = 0$, the second and sixth columns of Table 4 show that four
of the estimated parameters changed their posterior mean estimates substantially between
the pre-1979 and the post-1982 periods.\footnote{We conducted the (local) identification analysis proposed by Iskrev (2010) and confirmed that all the estimated parameters of the GNK model with no inflation inertia (i.e., $\omega = 0$) are identified.}
First, trend inflation fell by more than half from $\bar{\pi} = 1.44$ to $\bar{\pi} = 0.69$ in quarterly terms. Second, the policy response to inflation more
than doubled from $\phi_x = 1.25$ in the pre-1979 period to $\phi_x = 3.00$ in the post-1982 period.
Third, the policy response to the output gap decreased by more than half from $\phi_x = 0.29$
to $\phi_x = 0.10$. Fourth, the policy response to output growth increased by more than three
times from $\phi_{\Delta y} = 0.14$ to $\phi_{\Delta y} = 0.54$. These four changes suggest that the Fed in the post-
1982 period was inclined not only to conduct a disinflation policy by lowering its implicit
inflation target to a moderate level and raising the policy response to inflation, but also to
disregard the output gap and put more emphasis on output growth as an indicator of real economic activity. The last finding is compatible with the argument of Orphanides (2001), who suggests that monetary policy should put less emphasis on the output gap because such a gap involves great uncertainty about the measurement of unobservable potential output.

Comparing the estimated standard deviations of the structural shock innovations in the GNK model with $\omega = 0$ between the pre-1979 and the post-1982 periods, one may wonder why those of the preference and technology shock innovations are smaller in the pre-1979 period than in the post-1982 period, although the U.S. economy was much more volatile in the former period. The reasons are twofold. First, the economy during the pre-1979 period is estimated as in indeterminate equilibrium, where the sunspot shock can arise and generate greater macroeconomic volatility. Table 5 reports the variances of output growth, inflation, and the interest rate implied by the model with and without the sunspot shock, as well as those in the data. In the presence of the shock, the variances of the three variables implied by the model in the pre-1979 period are, respectively, 1.72, 0.47, and 0.44, which are all larger than their counterparts in the post-1982 period, 0.64, 0.12, and 0.28. Then, if the sunspot shock were absent from the model, the variance of inflation would decrease from 0.47 to 0.03 during the pre-1979 period, indicating the importance of the sunspot shock in explaining the high variability of inflation during that period.

Second, the propagation of shocks is altered under indeterminacy with the weaker monetary policy responses to inflation and output growth during the pre-1979 period.\textsuperscript{32} Figures 1 and 2 display the impulse responses of the three observed variables (i.e., output growth, inflation, and the interest rate) to an estimated one-standard-deviation innovation of each shock in the GNK model with $\omega = 0$, as well as the SW-CN model with $\omega_{sw} = 0$, during the pre-1979 and the post-1982 periods, respectively, using the posterior mean estimates of model parameters.\textsuperscript{33} The solid lines show that the responses of output growth to the

\textsuperscript{32}Technically, the solution under indeterminacy can generate richer dynamics and induce higher volatilities of endogenous model variables than that under determinacy, because fewer roots of the matrix $\Phi_{x}(\vartheta)$ in the solution (28) are suppressed.

\textsuperscript{33}Figure 2 has no panels for impulse responses to the sunspot shock. This is because in both models, the posterior probability of equilibrium determinacy during the post-1982 period is unity, and thus there is no role of the sunspot shock in the period.
structural shocks in the GNK model are larger in the pre-1979 period than in the post-1982 period, even though the estimated standard deviations of the preference and technology shock innovations are smaller in the former period. This contributes to the larger variance of output growth in the pre-1979 period (i.e., 1.72) than that in the post-1982 period (i.e., 0.64).

Figure 1 also illustrates some crucial differences between the GNK and the SW-CNK models in the impulse responses during the pre-1979 period (while Figure 2 exhibits little substantial difference between them during the post-1982 period). In the SW-CNK model, the technology shock generates not only a negative comovement between inflation and output growth but also a positive one between inflation and the interest rate during the pre-1979 period. This can account for the Great Inflation, where high inflation and low economic growth—stagflation—occurred with an accommodative monetary policy (i.e., the passive monetary policy). As shown in Table 4, the estimated standard deviation of the technology shock innovation $\sigma_a$ is larger in the SW-CNK model than in the GNK model, which suggests that the shock plays a greater role in the former model. By contrast, in the GNK model, the technology shock brings about a weak response of inflation and a negative comovement between inflation and the interest rate, which are both ascribed to the weak policy response to inflation. Instead of the technology shock, the sunspot shock generates a strong response of inflation and a positive comovement between inflation and the interest rate, as well as a negative one between inflation and output growth. Thus, in the GNK model, the sunspot shock can successfully replicate the stagflation as observed in the Great Inflation.

### 4.2 Source of the U.S. economy’s shift from indeterminacy to determinacy

This subsection addresses the paper’s main question of what led to the U.S. economy’s shift from indeterminacy to determinacy after the Great Inflation. In light of the estimation results in the preceding subsection, the present analysis examines the source of the shift by focusing on the changes in trend inflation and policy responses to inflation, the output gap, and output growth from the pre-1979 to the post-1982 estimates in the GNK model with no inflation inertia (i.e., $\omega = 0$).
Figure 3 illustrates how the determinacy region of the GNK model’s parameter space for the annualized trend inflation rate $\bar{\pi}$ and the policy response to inflation $\phi_\pi$ expands with changes in the other model parameters. In each panel of the figure, the marks “×”, “∗”, and “o” respectively represent the pairs of $(\bar{\pi}_{pre79}, \phi_{pre79})$, $(\bar{\pi}_{pre79}, \phi_{post82})$, and $(\bar{\pi}_{post82}, \phi_{post82})$, where $\bar{\pi}_{pre79}$ and $\phi_{pre79}$ denote the posterior mean estimates of the trend inflation rate and the policy response to inflation during the pre-1979 period presented in the second column of Table 4, and $\bar{\pi}_{post82}$ and $\phi_{post82}$ denote those during the post-1982 period presented in the sixth column of the table.\(^{34}\)

Panel (a) shows the case in which all the model parameters (except trend inflation and the policy response to inflation) are fixed at the pre-1979 estimates (presented in the second column of Table 4). In this panel, the pair of the pre-1979 estimates of trend inflation and the policy response to inflation $(\bar{\pi}_{pre79}, \phi_{pre79})$—which is represented by “×”—lies in the indeterminacy region of the parameter space, in line with the estimation result that the posterior probability of determinacy during the pre-1979 period is almost zero. The panel also demonstrates that the pair of the pre-1979 estimate of trend inflation and the post-1982 estimate of the policy response to inflation $(\bar{\pi}_{pre79}, \phi_{post82})$—which is denoted by “∗”—is also located within the indeterminacy region. This indicates that the increase in the policy response to inflation from the pre-1979 estimate $\phi_{pre79}$ to the post-1982 estimate $\phi_{post82}$ alone does not suffice for explaining the shift from indeterminacy to determinacy. Moreover, the pair of the post-1982 estimates of trend inflation and the policy response to inflation $(\bar{\pi}_{post82}, \phi_{post82})$—which is represented by “o”—lies inside the determinacy region. This finding suggests that the shift can be explained by the fall in trend inflation from the pre-1979 estimate $\bar{\pi}_{pre79}$ to the post-1982 estimate $\bar{\pi}_{post82}$ along with the increase in the policy response to inflation.

Panel (b) displays the case in which the policy responses to the output gap and output growth, $\phi_x$ and $\phi_{\Delta y}$, are set at the post-1982 estimates (presented in the sixth column of Table 4), keeping the other model parameters fixed at the pre-1979 estimates. As the difference between panels (a) and (b) shows, the change in the policy responses to the output gap and output growth from the pre-1979 to the post-1982 estimates significantly

\(^{34}\)In each panel of Figure 3, the boundary between determinacy and indeterminacy regions coincides with that given by the long-run version of the Taylor principle presented in Appendix B.
expands the determinacy region. As a consequence, in panel (b), while the pair of the pre-1979 estimates of trend inflation and the policy response to inflation \((\bar{\pi}_{\text{pre}79}, \phi_{\text{pre}79})\) is still located in the indeterminacy region, that of the pre-1979 estimate of trend inflation and the post-1982 estimate of the policy response to inflation \((\bar{\pi}_{\text{pre}79}, \phi_{\text{post}82})\) lies inside the determinacy region. This finding indicates that the decrease in the policy response to the output gap and the increase in the response to output growth, along with the rise in the response to inflation, can account for the shift from indeterminacy to determinacy, regardless of the fall in trend inflation.\(^{35}\)

Panel (c) presents the case in which all the model parameters are set at the post-1982 estimates. In this panel, the pair of the post-1982 estimates of trend inflation and the policy response to inflation \((\bar{\pi}_{\text{post}82}, \phi_{\text{post}82})\) is located inside the determinacy region, in line with the estimation result that the posterior probability of determinacy during the post-1982 period is one. Panel (c) is not so different from panel (b), suggesting that the change from the pre-1979 to the post-1982 estimates of all the model parameters other than trend inflation and the policy responses to inflation, the output gap, and output growth plays a minor role in accounting for the shift from indeterminacy to determinacy.

These panels demonstrate that the increase in the policy response to inflation from the pre-1979 to the post-1982 estimate alone does not suffice for explaining the U.S. economy’s shift from indeterminacy to determinacy after the Great Inflation, unless it is accompanied by either the estimated fall in trend inflation or the estimated change in policy responses to the output gap and output growth. Taking into consideration that trend inflation is equivalent to the central bank’s inflation target in the model, this finding indicates that the changes in the Fed’s implicit inflation target and policy responses to real economic activity have played a key role in the shift to determinacy, in addition to its more active response to inflation.

\(^{35}\)In a GNK model with a Taylor-type rule, the destabilizing role of the policy response to the output gap is indicated by Ascari and Ropele (2009), while the stabilizing role of the policy response to output growth is pointed out by Coibion and Gorodnichenko (2011).
5 Conclusion

This paper has revisited a large literature’s view that U.S. macroeconomic stability after the Great Inflation of the 1970s was achieved by the Fed’s change from a passive to an active policy response to inflation. We have estimated a GNK model jointly with a Taylor-type rule during two periods, before and after the Volcker disinflation of 1979–1982, by adopting an SMC algorithm in a full-information Bayesian approach based on Lubik and Schorfheide (2004). Our estimation results have shown that, in both periods, the GNK model (with no inertia in inflation) empirically outperforms two types of CNK models used in previous literature. This indicates that the GNK model is more suitable than the two CNK models for analyzing the source of the U.S. macroeconomic stability.

According to the estimated GNK model, the U.S. economy was likely in the equilibrium-indeterminacy region of the model’s parameter space before 1979, while it likely entered the determinacy region after 1982, in line with the result obtained in the literature. However, there is considerable uncertainty as to whether the policy response to (current) inflation was active or passive during the pre-1979 period, which contrasts with the literature’s view that the policy response was surely passive during the Great Inflation and that the subsequent change to an active response led to the U.S. economy’s shift from indeterminacy to determinacy. Moreover, we have demonstrated that the increase in the policy response to inflation from the pre-1979 to the post-1982 estimate alone does not suffice for explaining the shift, unless it is accompanied by the change from the pre-1979 to the post-1982 estimates of either trend inflation or the policy responses to the output gap and output growth. This finding extends the literature on the role of monetary policy in achieving U.S. macroeconomic stability after the Great Inflation, by emphasizing the importance of the changes in the Fed’s implicit inflation target and policy responses to real economic activity.
Appendix

A Another GNK Model (with Homogeneous Labor)

The GNK model employed in this paper considers firm-specific labor, as in Coibion and Gorodnichenko (2011). In this section we analyze another type of GNK model, which assumes homogeneous labor as in Ascari and Ropele (2009), and compare it with our GNK model in terms of empirical performance.

A.1 Households

In the GNK model with homogeneous labor, the representative household supplies such labor services $l_t$. The utility function is of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t \exp(z_{u,t}) \left[ \log(\hat{C}_t - hC_{t-1}) - \frac{1}{1 + 1/\eta} \frac{l_t^{1+1/\eta}}{t_0} \right],$$

and the budget constraint is given by

$$P_t \hat{C}_t + B_t = P_t W_t l_t + r_{t-1} B_{t-1} + T_t,$$

where $W_t$ is the real wage rate of homogeneous labor.

The first-order conditions for utility maximization with respect to consumption and bond holdings turn out to coincide with those in our GNK model (i.e., (1) and (3)), while that regarding labor supply is given by

$$W_t = \frac{l_t^{1/\eta} \exp(z_{u,t})}{\Xi_t}.$$  \hfill (30)

A.2 Firms

As for firms, there is no change in the setting of final-good firms, whereas all intermediate-good firms’ first-order conditions for cost minimization lead to identical real marginal cost

$$mc_t(i) = \frac{W_t}{A_t} = mc.$$  \hfill (31)

Moreover, the first-order condition for the optimized price $P^o_t$ becomes

$$E_t \sum_{j=0}^{\infty} (\beta \lambda)^j \frac{\Xi_{t+j}}{\Xi_t} \frac{Y_{t+j}}{Y_t} \prod_{k=1}^{j} \pi_t \left( p_t \prod_{k=1}^{j} \frac{1}{\pi_{t+k}} - \frac{\theta}{\theta - 1} mc_{t+j} \right) = 0.$$  \hfill (32)
The labor market clearing condition, along with the demand curve (4), yields

\[ l_t = \int_0^1 l_t(i)di = \frac{Y_t}{A_t} \Delta_t, \]  

(33)

where \( \Delta_t \) denotes relative price distortion and is given by

\[ \Delta_t = \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\theta} di. \]  

(34)

Using (1), (6), (30), and (33), the real marginal cost (31) becomes

\[ mc_t = \left( \frac{Y_t}{A_t} \right)^{\frac{1}{\eta}} \left( \frac{Y_t}{A_t} - h \frac{Y_{t-1}}{A_t} \right) \Delta_t^{\frac{1}{\eta}}. \]  

(35)

Under Calvo-style staggered price-setting, the relative price distortion equation (34) can be rewritten as

\[ \Delta_t = \lambda \pi \theta \Delta_{t-1} + (1 - \lambda) \left[ (1 - \omega)(\hat{p}_t^{\theta})^{-\theta} + \omega (\hat{p}_t^{\theta})^{-\theta} \right]. \]  

(36)

### A.3 Equilibrium conditions

There are no changes in the settings of the central bank and the natural rate of output, and thus the equilibrium conditions consist of (1), (3), (6), (8), (10), (11), (14), (16), (17), (32), (35), and (36). For the steady state to be well defined, the following condition is assumed:

\[ \lambda \max(\pi^{\theta-1}, \pi^{\theta}) < 1. \]  

(37)

Combining the equilibrium conditions, rewriting the resulting conditions in terms of the detrended variables \( y_t = Y_t/A_t \) and \( y^n_t = Y^n_t/A_t \), and log-linearizing the conditions under the assumption (37) yields (21)–(22) as well as

\[ \hat{\pi}_t = \gamma_{b,h} \hat{\pi}_{t-1} + \gamma_{f,h} E_t \hat{\pi}_{t+1} + \kappa_h \hat{y}_t + \frac{h \kappa_h}{a - \hat{r}_t} (\hat{y}_t - \hat{y}_{t-1} + z_{a,t}) + \frac{\kappa_h}{\eta} \hat{\Delta}_t + \psi_t, \]  

(38)

\[ \hat{\Delta}_t = \lambda \pi^{\theta} \hat{\Delta}_{t-1} + \frac{\theta \lambda \pi^{\theta-1}(\pi - 1)}{1 - \lambda \pi^{\theta-1}} \hat{\pi}_t, \]  

(39)

\[ \psi_t = \gamma_{\psi,h} E_t \psi_{t+1} + \kappa_{\psi,h} (E_t \hat{y}_{t+1} - \hat{y}_t + E_t \hat{z}_{a,t+1} + \theta E_t \hat{\pi}_{t+1} - \hat{r}_t), \]  

(40)

where the coefficients are given by \( \gamma_{b,h} = \omega / \varphi_h, \gamma_{f,h} = \beta \lambda \pi^{\theta}/\varphi_h, \kappa_h = \kappa_{\lambda,h}(1 + 1/\eta), \kappa_{\lambda,h} = (1 - \lambda \pi^{\theta-1})(1 - (1 - \lambda \pi^{\theta-1})/\varphi_h, \gamma_{\psi,h} = \beta \lambda \pi^{\theta-1}, \kappa_{\psi,h} = \gamma_{\psi,h}(\pi - 1)(1 - \lambda \pi^{\theta-1})(1 - \omega)/\varphi_h, \) and \( \varphi_h = \lambda \pi^{\theta-1} + \omega (1 - \lambda \pi^{\theta-1} + \beta \lambda \pi^{\theta}). \)

The GNK model with homogeneous labor differs from our GNK model (with firm-specific labor) in that the GNK Phillips curve (19) depends additionally on the relative price distortion \( \hat{\Delta}_t \).
A.4 Empirical performance

The GNK model with homogeneous labor is also estimated, using the same estimation strategy and data as described in the paper. Table 6 reports the posterior estimates of the GNK model with homogeneous labor in the pre-1979 and the post-1982 periods. The second to last row of the table presents the log marginal data densities \( \log p(X^T) \) and shows that the model without inertia of inflation has a larger value than that with it in both periods: \(-126.13 > -130.31\) in the pre-1979 period and \(-55.89 > -62.80\) in the post-1982 period. Thus, there is no need for inflation inertia in the GNK model with homogeneous labor, in line with our GNK model. Turning to the comparison of the two types of GNK models (with no inflation inertia, i.e., \( \omega = 0 \)), our GNK model has larger values of the log marginal data density than the other in both periods: \(-121.23 > -126.13\) in the pre-1979 period and \(-53.66 > -55.89\) in the post-1982 period. Therefore, our GNK model empirically outperforms the GNK model with homogeneous labor.

B Long-run Version of the Taylor Principle

This section presents the long-run version of the Taylor principle in the GNK model of the paper. To obtain it, the long-run inflation elasticity of output is derived. The GNK Phillips curve (19), the variable \( \psi_t \)'s equation (20), and the spending Euler equation (21) imply that the elasticity is given by

\[
\epsilon_y = \frac{1 - \gamma_b - \gamma_f - \kappa \psi_t (\theta - 1)}{1 - \beta \lambda \pi^{\theta - 1}}. \tag{41}
\]

Then, from the Taylor-type rule (24), it follows that the long-run version of the Taylor principle is represented as

\[
\phi_\pi + \phi_x \epsilon_y > 1. \tag{42}
\]
References


Table 1: Prior distributions for parameters of the GNK model, the two types of CNK models, and the nested model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Mean</th>
<th>St. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{a}$</td>
<td>Normal</td>
<td>0.370</td>
<td>0.150</td>
</tr>
<tr>
<td>$\bar{\pi}$</td>
<td>Normal</td>
<td>0.985</td>
<td>0.750</td>
</tr>
<tr>
<td>$\bar{r}$</td>
<td>Gamma</td>
<td>1.597</td>
<td>0.250</td>
</tr>
<tr>
<td>$h$</td>
<td>Beta</td>
<td>0.700</td>
<td>0.100</td>
</tr>
<tr>
<td>$\omega/\omega_{sw}$</td>
<td>Beta</td>
<td>0.500</td>
<td>0.150</td>
</tr>
<tr>
<td>$\lambda/\lambda_{gg}/\lambda_{sw}$</td>
<td>Beta</td>
<td>0.500</td>
<td>0.050</td>
</tr>
<tr>
<td>$\phi_r$</td>
<td>Beta</td>
<td>0.750</td>
<td>0.100</td>
</tr>
<tr>
<td>$\phi_z$</td>
<td>Gamma</td>
<td>1.5/1.125/1.1/1.245</td>
<td>0.750</td>
</tr>
<tr>
<td>$\phi_x$</td>
<td>Gamma</td>
<td>0.125</td>
<td>0.100</td>
</tr>
<tr>
<td>$\phi_{\Delta y}$</td>
<td>Gamma</td>
<td>0.125</td>
<td>0.100</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Uniform</td>
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<td>0.289</td>
</tr>
<tr>
<td>$\rho_u$</td>
<td>Beta</td>
<td>0.500</td>
<td>0.200</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Beta</td>
<td>0.500</td>
<td>0.200</td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>Beta</td>
<td>0.500</td>
<td>0.200</td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>Inverse gamma</td>
<td>0.627</td>
<td>0.328</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>Inverse gamma</td>
<td>0.627</td>
<td>0.328</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>Inverse gamma</td>
<td>0.627</td>
<td>0.328</td>
</tr>
<tr>
<td>$\sigma_{\zeta}$</td>
<td>Inverse gamma</td>
<td>0.627</td>
<td>0.328</td>
</tr>
<tr>
<td>$M_u$</td>
<td>Normal</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$M_a$</td>
<td>Normal</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$M_r$</td>
<td>Normal</td>
<td>0.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Notes: The prior mean of the policy response to inflation $\phi_z$ is set at 1.5 for the GNK model, 1.125 for the SW-CNK model, 1.1 for the GG-CNK model, and 1.245 for the nested model. The prior probability of equilibrium determinacy is then 0.482 for the GNK model, 0.481 for the SW-CNK model, 0.485 for the GG-CNK model, and 0.484 for the nested model. Inverse gamma distributions are of the form $p(\sigma|\nu, s) \propto \sigma^{\nu-1}e^{-\nu s^2/2\sigma^2}$, where $\nu = 4$ and $s = 0.5$. 


Table 2: Posterior estimates of the GNK model, the two types of CNK models, and the nested model in the pre-1979 period

<table>
<thead>
<tr>
<th>Parameter</th>
<th>GNK model</th>
<th>SW-CNK model</th>
<th>GG-CNK model</th>
<th>Nested model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>90% interval</td>
<td>Mean</td>
<td>90% interval</td>
</tr>
<tr>
<td>$a$</td>
<td>0.326</td>
<td>[0.122, 0.544]</td>
<td>0.382</td>
<td>[0.170, 0.602]</td>
</tr>
<tr>
<td>$\pi$</td>
<td>1.478</td>
<td>[1.100, 1.819]</td>
<td>1.350</td>
<td>[0.917, 1.760]</td>
</tr>
<tr>
<td>$h$</td>
<td>0.563</td>
<td>[0.440, 0.691]</td>
<td>0.550</td>
<td>[0.427, 0.683]</td>
</tr>
<tr>
<td>$\omega/\omega_{sw}$</td>
<td>0.158</td>
<td>[0.071, 0.257]</td>
<td>0.213</td>
<td>[0.071, 0.342]</td>
</tr>
<tr>
<td>$\lambda/\lambda_{gg}/\lambda_{sw}$</td>
<td>0.541</td>
<td>[0.469, 0.607]</td>
<td>0.496</td>
<td>[0.411, 0.585]</td>
</tr>
<tr>
<td>$\phi_r$</td>
<td>0.704</td>
<td>[0.598, 0.829]</td>
<td>0.680</td>
<td>[0.554, 0.810]</td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>1.101</td>
<td>[0.255, 2.012]</td>
<td>0.483</td>
<td>[0.051, 0.826]</td>
</tr>
<tr>
<td>$\phi_x$</td>
<td>0.291</td>
<td>[0.069, 0.475]</td>
<td>0.157</td>
<td>[0.003, 0.307]</td>
</tr>
<tr>
<td>$\phi_{\Delta y}$</td>
<td>0.121</td>
<td>[0.001, 0.249]</td>
<td>0.118</td>
<td>[0.002, 0.231]</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$\rho_u$</td>
<td>0.430</td>
<td>[0.112, 0.714]</td>
<td>0.497</td>
<td>[0.162, 0.815]</td>
</tr>
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<td>$\rho_a$</td>
<td>0.766</td>
<td>[0.616, 0.936]</td>
<td>0.399</td>
<td>[0.097, 0.697]</td>
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<td>$\rho_r$</td>
<td>0.419</td>
<td>[0.208, 0.601]</td>
<td>0.432</td>
<td>[0.244, 0.617]</td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>1.079</td>
<td>[0.239, 2.537]</td>
<td>0.919</td>
<td>[0.247, 2.024]</td>
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<tr>
<td>$\sigma_a$</td>
<td>0.711</td>
<td>[0.317, 1.095]</td>
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<td>[0.889, 2.522]</td>
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<td>[0.225, 0.335]</td>
<td>0.276</td>
<td>[0.224, 0.321]</td>
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<td>$\sigma_\zeta$</td>
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<td>[0.283, 0.510]</td>
<td>0.479</td>
<td>[0.305, 0.621]</td>
</tr>
<tr>
<td>$M_u$</td>
<td>–0.048</td>
<td>[−0.612, 0.467]</td>
<td>–0.054</td>
<td>[−0.844, 0.784]</td>
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<td>–0.041</td>
<td>[−0.945, 0.707]</td>
<td>–0.288</td>
<td>[−0.835, 0.077]</td>
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<tr>
<td>$M_r$</td>
<td>–0.062</td>
<td>[−0.619, 0.638]</td>
<td>1.032</td>
<td>[0.107, 2.006]</td>
</tr>
</tbody>
</table>

| $\log p(X^T)$ | −128.046 | −130.434 | −133.240 | −128.446 |
| $P\{\vartheta \in \Theta^D|X^T\}$ | 0.000 | 0.070 | 0.002 | 0.000 |

Notes: This table shows the posterior mean and 90 percent highest posterior density intervals based on 10,000 particles from the final importance sampling in the SMC algorithm. In the table, $\log p(X^T)$ represents the SMC-based approximation of log marginal data density and $P\{\vartheta \in \Theta^D|X^T\}$ denotes the posterior probability of equilibrium determinacy.
Table 3: Posterior estimates of the GNK model, the two types of CNK models, and the nested model in the post-1982 period

<table>
<thead>
<tr>
<th>Parameter</th>
<th>GNK model</th>
<th>SW-CNK model</th>
<th>GG-CNK model</th>
<th>Nested model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>90% interval</td>
<td>Mean</td>
<td>90% interval</td>
</tr>
<tr>
<td>¯a</td>
<td>0.374</td>
<td>[0.196, 0.556]</td>
<td>0.410</td>
<td>[0.238, 0.574]</td>
</tr>
<tr>
<td>¯π</td>
<td>0.724</td>
<td>[0.560, 0.898]</td>
<td>0.692</td>
<td>[0.533, 0.837]</td>
</tr>
<tr>
<td>h</td>
<td>0.653</td>
<td>[0.570, 0.728]</td>
<td>0.590</td>
<td>[0.510, 0.665]</td>
</tr>
<tr>
<td>ω/ω_sw</td>
<td>0.069</td>
<td>[0.027, 0.115]</td>
<td>0.136</td>
<td>[0.051, 0.220]</td>
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<tr>
<td>λ/λ_gg/λ_sw</td>
<td>0.490</td>
<td>[0.411, 0.558]</td>
<td>0.434</td>
<td>[0.367, 0.497]</td>
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<tr>
<td>φ_r</td>
<td>0.680</td>
<td>[0.608, 0.765]</td>
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<td>[0.590, 0.764]</td>
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<td>φ_Δy</td>
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<td>0.115</td>
<td>[0.002, 0.236]</td>
</tr>
<tr>
<td>α</td>
<td>– – –</td>
<td>– – –</td>
<td>– – –</td>
<td>– – –</td>
</tr>
<tr>
<td>ρ_u</td>
<td>0.909</td>
<td>[0.874, 0.954]</td>
<td>0.915</td>
<td>[0.882, 0.949]</td>
</tr>
<tr>
<td>ρ_a</td>
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<td>[0.014, 0.246]</td>
<td>0.088</td>
<td>[0.014, 0.156]</td>
</tr>
<tr>
<td>ρ_r</td>
<td>0.674</td>
<td>[0.577, 0.762]</td>
<td>0.629</td>
<td>[0.543, 0.726]</td>
</tr>
<tr>
<td>σ_r</td>
<td>0.229</td>
<td>[0.177, 0.302]</td>
<td>0.232</td>
<td>[0.183, 0.281]</td>
</tr>
<tr>
<td>σ_ζ</td>
<td>1.019</td>
<td>[0.321, 1.771]</td>
<td>– – –</td>
<td>– – –</td>
</tr>
<tr>
<td>M_u</td>
<td>-0.500</td>
<td>[-1.854, 0.857]</td>
<td>– – –</td>
<td>– – –</td>
</tr>
<tr>
<td>M_a</td>
<td>0.277</td>
<td>[-0.818, 1.478]</td>
<td>– – –</td>
<td>– – –</td>
</tr>
<tr>
<td>M_r</td>
<td>0.654</td>
<td>[-0.999, 2.246]</td>
<td>– – –</td>
<td>– – –</td>
</tr>
<tr>
<td>log (p(X^T))</td>
<td>-65.977</td>
<td>– – –</td>
<td>– – –</td>
<td>– – –</td>
</tr>
<tr>
<td>(P{\theta \in \Theta^D</td>
<td>X^T})</td>
<td>0.988</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Notes: This table shows the posterior mean and 90 percent highest posterior density intervals based on 10,000 particles from the final importance sampling in the SMC algorithm. In the table, log \(p(X^T)\) represents the SMC-based approximation of log marginal data density and \(P\{\theta \in \Theta^D|X^T\}\) denotes the posterior probability of equilibrium determinacy.
Table 4: Posterior estimates of the GNK and the SW-CNK models with no inertia in inflation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Pre-1979 period</th>
<th>Post-1982 period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GNK model: $\omega = 0$</td>
<td>SW-CNK model: $\omega_{sw} = 0$</td>
</tr>
<tr>
<td>$\bar{a}$</td>
<td>0.391 $[0.161, 0.616]$</td>
<td>0.381 $[0.159, 0.606]$</td>
</tr>
<tr>
<td>$\bar{\pi}$</td>
<td>1.442 $[1.115, 1.760]$</td>
<td>1.365 $[0.970, 1.778]$</td>
</tr>
<tr>
<td>$h$</td>
<td>0.551 $[0.432, 0.678]$</td>
<td>0.540 $[0.422, 0.669]$</td>
</tr>
<tr>
<td>$\lambda_{sw}/\lambda_{sw}$</td>
<td>0.527 $[0.459, 0.598]$</td>
<td>0.502 $[0.419, 0.585]$</td>
</tr>
<tr>
<td>$\phi_r$</td>
<td>0.701 $[0.580, 0.826]$</td>
<td>0.686 $[0.562, 0.816]$</td>
</tr>
<tr>
<td>$\phi_{\pi}$</td>
<td>1.252 $[0.223, 2.232]$</td>
<td>0.436 $[0.069, 0.737]$</td>
</tr>
<tr>
<td>$\phi_{\Delta \eta}$</td>
<td>0.286 $[0.082, 0.498]$</td>
<td>0.167 $[0.002, 0.334]$</td>
</tr>
<tr>
<td>$\rho_u$</td>
<td>0.456 $[0.119, 0.745]$</td>
<td>0.469 $[0.142, 0.795]$</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>0.717 $[0.519, 0.916]$</td>
<td>0.432 $[0.148, 0.724]$</td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>0.444 $[0.240, 0.639]$</td>
<td>0.441 $[0.253, 0.633]$</td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>1.014 $[0.271, 2.082]$</td>
<td>0.717 $[0.232, 1.345]$</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>0.786 $[0.339, 1.208]$</td>
<td>1.565 $[0.909, 2.300]$</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>0.295 $[0.228, 0.363]$</td>
<td>0.275 $[0.225, 0.318]$</td>
</tr>
<tr>
<td>$\sigma_{\zeta}$</td>
<td>0.385 $[0.272, 0.481]$</td>
<td>0.431 $[0.307, 0.545]$</td>
</tr>
<tr>
<td>$M_u$</td>
<td>0.003 $[−0.475, 0.521]$</td>
<td>−0.061 $[−0.765, 0.620]$</td>
</tr>
<tr>
<td>$M_a$</td>
<td>−0.107 $[−0.953, 0.594]$</td>
<td>−0.204 $[−0.519, 0.054]$</td>
</tr>
<tr>
<td>$M_r$</td>
<td>0.170 $[−0.436, 0.932]$</td>
<td>1.096 $[0.228, 2.032]$</td>
</tr>
<tr>
<td>$\log p(X_T)$</td>
<td>−121.225</td>
<td>−124.619</td>
</tr>
<tr>
<td>$P{\vartheta \in \Theta^D</td>
<td>X_T}$</td>
<td>0.010</td>
</tr>
</tbody>
</table>

Notes: This table shows the posterior mean and 90 percent highest posterior density intervals based on 10,000 particles from the final importance sampling in the SMC algorithm. In the table, $\log p(X_T)$ represents the SMC-based approximation of log marginal data density and $P\{\vartheta \in \Theta^D | X_T\}$ denotes the posterior probability of equilibrium determinacy.
Table 5: Variances of observed variables in the data and implied by the GNK model with no inflation inertia

<table>
<thead>
<tr>
<th></th>
<th>Output growth</th>
<th>Inflation</th>
<th>Interest rate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pre-1979 period:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>1.031</td>
<td>0.299</td>
<td>0.284</td>
</tr>
<tr>
<td>GNK model: $\omega = 0$</td>
<td>1.717</td>
<td>0.473</td>
<td>0.436</td>
</tr>
<tr>
<td>GNK model: $\omega = 0$ and no sunspot shock</td>
<td>1.692</td>
<td>0.026</td>
<td>0.173</td>
</tr>
<tr>
<td><strong>Post-1982 period:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>0.420</td>
<td>0.068</td>
<td>0.391</td>
</tr>
<tr>
<td>GNK model: $\omega = 0$</td>
<td>0.638</td>
<td>0.120</td>
<td>0.275</td>
</tr>
</tbody>
</table>

Note: This table shows the variances of the three observed variables—output growth, inflation, and the interest rate—in the data and those implied by the GNK model with no inflation inertia (i.e., $\omega = 0$) using the posterior mean estimates of parameters.
Table 6: Posterior estimates of the GNK model with homogeneous labor

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Pre-1979 period</th>
<th>Post-1982 period</th>
<th>Pre-1979 period</th>
<th>Post-1982 period</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{a}$</td>
<td>0.483 [0.271, 0.665]</td>
<td>0.380 [0.179, 0.608]</td>
<td>0.425 [0.285, 0.558]</td>
<td>0.425 [0.290, 0.563]</td>
</tr>
<tr>
<td>$\bar{\pi}$</td>
<td>1.143 [1.112, 1.574]</td>
<td>1.493 [1.210, 1.840]</td>
<td>0.684 [0.573, 0.803]</td>
<td>0.679 [0.580, 0.777]</td>
</tr>
<tr>
<td>$\bar{h}$</td>
<td>0.521 [0.418, 0.608]</td>
<td>0.496 [0.386, 0.614]</td>
<td>0.458 [0.366, 0.543]</td>
<td>0.453 [0.361, 0.538]</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.175 [0.099, 0.295]</td>
<td>0</td>
<td>0.164 [0.073, 0.256]</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.491 [0.441, 0.572]</td>
<td>0.555 [0.486, 0.636]</td>
<td>0.531 [0.457, 0.604]</td>
<td>0.575 [0.514, 0.646]</td>
</tr>
<tr>
<td>$\phi_r$</td>
<td>0.579 [0.486, 0.716]</td>
<td>0.657 [0.488, 0.801]</td>
<td>0.710 [0.624, 0.789]</td>
<td>0.703 [0.622, 0.788]</td>
</tr>
<tr>
<td>$\phi_{\pi}$</td>
<td>1.058 [0.282, 2.155]</td>
<td>0.929 [0.009, 2.111]</td>
<td>3.579 [2.598, 4.540]</td>
<td>3.843 [2.800, 4.878]</td>
</tr>
<tr>
<td>$\phi_x$</td>
<td>0.118 [0.005, 0.233]</td>
<td>0.188 [0.002, 0.353]</td>
<td>0.124 [0.002, 0.256]</td>
<td>0.130 [0.000, 0.255]</td>
</tr>
<tr>
<td>$\phi_{\Delta y}$</td>
<td>0.089 [0.000, 0.210]</td>
<td>0.139 [0.015, 0.249]</td>
<td>0.454 [0.206, 0.675]</td>
<td>0.391 [0.161, 0.616]</td>
</tr>
<tr>
<td>$\rho_u$</td>
<td>0.523 [0.312, 0.808]</td>
<td>0.594 [0.246, 0.880]</td>
<td>0.925 [0.896, 0.956]</td>
<td>0.933 [0.904, 0.962]</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>0.119 [0.024, 0.236]</td>
<td>0.230 [0.057, 0.395]</td>
<td>0.098 [0.022, 0.161]</td>
<td>0.145 [0.072, 0.216]</td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>0.428 [0.294, 0.544]</td>
<td>0.361 [0.222, 0.521]</td>
<td>0.504 [0.396, 0.606]</td>
<td>0.448 [0.351, 0.547]</td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>0.793 [0.302, 1.362]</td>
<td>0.652 [0.254, 1.135]</td>
<td>2.200 [1.486, 2.914]</td>
<td>2.287 [1.424, 3.156]</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>1.685 [1.303, 2.093]</td>
<td>1.624 [1.240, 2.020]</td>
<td>0.959 [0.796, 1.111]</td>
<td>0.923 [0.788, 1.072]</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>0.301 [0.218, 0.415]</td>
<td>0.307 [0.219, 0.430]</td>
<td>0.249 [0.195, 0.304]</td>
<td>0.265 [0.206, 0.328]</td>
</tr>
<tr>
<td>$\sigma_\zeta$</td>
<td>0.449 [0.278, 0.691]</td>
<td>0.410 [0.257, 0.577]</td>
<td>0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>$M_u$</td>
<td>0.193 [-1.016, 1.143]</td>
<td>0.239 [-0.556, 1.076]</td>
<td>0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>$M_a$</td>
<td>-0.122 [-1.275, 0.876]</td>
<td>-0.022 [-0.894, 0.591]</td>
<td>0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>$M_r$</td>
<td>-1.265 [-3.005, 0.331]</td>
<td>-1.349 [-3.316, 0.795]</td>
<td>0 0 0 0</td>
<td></td>
</tr>
</tbody>
</table>

log $p(X^T)$ | -130.314 | -126.128 | -62.802 | -55.888 |
$P\{\vartheta \in \Theta^D|X^T\}$ | 0.375 | 0.229 | 1.000 | 1.000 |

Notes: This table shows the posterior mean and 90 percent highest posterior density intervals based on 10,000 particles from the final importance sampling in the SMC algorithm. In the table, log $p(X^T)$ represents the SMC-based approximation of log marginal data density and $P\{\vartheta \in \Theta^D|X^T\}$ denotes the posterior probability of equilibrium determinacy. The prior mean of the policy response to inflation $\phi_{\pi}$ is set at 1.175 for both models with inflation inertia (i.e., $\omega \neq 0$) and without it (i.e., $\omega = 0$). The prior probabilities of equilibrium determinacy are then 0.489 for the model with $\omega \neq 0$ and 0.488 for the model with $\omega = 0$. 
Figure 1: Impulse responses during the pre-1979 period in the GNK and the SW-CNK models with no inflation inertia

(a) Preference shock $\varepsilon_{u,t}$

(b) Technology shock $\varepsilon_{a,t}$

(c) Monetary policy shock $\varepsilon_{r,t}$

(d) Sunspot shock $\zeta_{t}$

Note: This figure shows the impulse responses of output growth, inflation, and the interest rate in terms of deviations from steady-state values, to a one-standard-deviation innovation to each of the preference, technology, monetary policy, and sunspot shocks, using the posterior mean estimates of parameters in the GNK model with $\omega = 0$ and the SW-CNK model with $\omega_{sw} = 0$ during the pre-1979 period.
Figure 2: Impulse responses during the post-1982 period in the GNK and the SW-CNK models with no inflation inertia

(a) Preference shock $\varepsilon_{u,t}$

(b) Technology shock $\varepsilon_{a,t}$

(c) Monetary policy shock $\varepsilon_{r,t}$

Note: This figure shows the impulse responses of output growth, inflation, and the interest rate in terms of deviations from steady-state values, to a one-standard-deviation innovation to each of the preference, technology, and monetary policy shocks, using the posterior mean estimates of parameters in the GNK model with $\omega = 0$ and the SW-CNK model with $\omega_{sw} = 0$ during the post-1982 period.
Figure 3: Equilibrium-determinacy region of the GNK model’s parameter space

(a) Pre-1979 estimates of all model parameters

(b) Post-1982 estimates of $\phi_x$ and $\phi_{\Delta y}$

(c) Post-1982 estimates of all model parameters

Notes: For the annualized trend inflation rate $4\bar{\pi}$ and the policy response to inflation $\phi_\pi$, the figure illustrates the equilibrium-determinacy region of the GNK model’s parameter space. In each panel, the marks “$\times$”, “$*$”, and “$\circ$” respectively represent the pairs of $(4\bar{\pi}_{pre79}, \phi_{\pi_{pre79}})$, $(4\bar{\pi}_{pre79}, \phi_{\pi_{post82}})$, and $(4\bar{\pi}_{post82}, \phi_{\pi_{post82}})$, where $\bar{\pi}_{pre79}$ ($\bar{\pi}_{post82}$) and $\phi_{\pi_{pre79}}$ ($\phi_{\pi_{post82}}$) denote the mean estimates of the trend inflation rate and the policy response to inflation in the pre-1979 (post-1982) period.