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Estimating the Natural Yield Curve in Japan Using a VAR with Common Trends

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Abstract

This paper introduces a novel approach for simultaneously estimating nominal and real natural yield curves in Japan. Specifically, we employ macroeconomic variables (output gap and inflation rate) as observed variables, in addition to the nominal and real yield curves, and conduct an estimation combining the representative yield curve model, the Nelson-Siegel model (Nelson and Siegel, 1987), with a VAR with common trends (Del Negro et al., 2017). The results presented in this paper indicate that since the 1990s, both nominal and real natural yield curves have exhibited downward shifts, as a consequence of a decline in the natural rate of interest. Furthermore, both curves have flattened due to a trending decline in the term premium. The results also indicate that the extent of these changes differs between the nominal and real natural yield curves. However, it should be noted that the estimation of natural yield curves is still in the process of development. Consequently, the results should be interpreted with caution.

JEL classification: C32, E43, E52

Keywords: Natural rate of interest, Natural yield curve, Term structure

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1 Introduction

The natural rate of interest (r_t^*) is regarded as one of the most important theoretical benchmarks for the implementation of monetary policy. Consequently, there has been a sustained effort to refine the estimation of the natural rate of interest, with the academic community and central banks developing various estimation methods. The Bank of Japan also has a long history of estimating the natural rate of interest in Japan, beginning with Oda and Muranaga (2003), followed by the Kamada (2009), Fujiwara et al. (2016), and so on. Nakano et al. (2024) introduce various estimates of the natural rate of interest in Japan over the past around 25 years.

During this period, the adoption of unconventional monetary policies in Japan and other advanced economies has highlighted the need for assessing the degree of yield curve accommodation overall, rather than just the short-term aspects. In light of this policy transition, Imakubo et al. (2018), Dufrénot et al. (2022), and others have endeavored to estimate the natural yield curve in Japan, but the methodology for estimating it remains in the process of development.[1]

In this paper, we employ a simultaneous estimation approach to derive both the nominal and real natural yield curves and the trend inflation rate (π_t^*) for Japan, drawing on the methodology proposed in Goy and Iwasaki (2024). Specifically, we employ a representative yield curve model, the Nelson-Siegel model (Nelson and Siegel, 1987), in conjunction with macroeconomic variables, including the output gap and inflation rate, to estimate the nominal and real yield curves and the trend inflation rate. A time series model, designated a VAR with common trends (Del Negro et al., 2017), is employed to decompose the trends of the nominal and real yield curves into the natural rate of interest, the trend inflation rate, and the trends of

[1] This paper defines the "natural yield curve" similar to the concept in Imakubo et al. (2018), which differs from the "equilibrium yield curve" as a short-run equilibrium discussed in Piazzesi and Schneider (2007).

the slope components.

The estimation results indicate that since the 1990s, both nominal and real natural yield curves have exhibited downward shifts, primarily as a consequence of a decline in the natural rate of interest. Furthermore, both curves have flattened due to a trending decline in the term premium. The results also indicate that the extent of these changes differs between the nominal and real natural yield curves and that the nominal natural yield curve has shifted somewhat upward since the beginning of the 2020s, although it should be noted that the trend components near the end of the observed data period tend to be influenced by the observed data in the method used in this paper.

The following are the distinguishing features of this analysis compared with previous studies. First, this analysis estimates not only the real natural yield curve but also the nominal natural yield curve and π_t^* simultaneously. Considering that the target of monetary policy is the nominal interest rate, the capacity to derive the nominal natural yield curve and π_t^* represents a significant advantage of the method used in this paper. This is in contrast to Imakubo et al. (2018) and Dufrénot et al. (2022), which only estimate the real natural yield curve.[2] Second, this is the first study to estimate the natural yield curve in Japan using a time series approach. As is well documented in studies of the natural rate of interest, the estimation model selected can result in variation in the estimates produced. Compared to Imakubo et al. (2018), this paper makes fewer assumptions about the economic structure, and selects an approach that places more emphasis on letting the data speak for itself. To be more pricise, while Imakubo et al. (2018) incorporates the IS curve into the model and assumes a relationship between the trend components of the yield curve and the potential growth rate, this paper does not make these assumptions and uses a flexible estimation approach. This has the advantage of preventing estimation bias that could arise from model misspecification. However, it should be noted that this has the disadvantage that the estimated natural yield

[2] Bauer and Rudebusch (2020) estimated only the nominal natural yield curve in the U.S.

curves are more sensitive to observed data.

The structure of this paper is as follows. First, in Section 2, we provide an overview of the model used in the estimation after the conceptualization related to the natural yield curve. Section 3 explains the estimation method. The estimation results are presented in Section 4. Section 5 concludes.

2 Model

2.1 Definition of the natural yield curve

In preparing to define the natural yield curve, we first briefly review the natural rate of interest. The representative definitions of the natural rate of interest for long-term concepts in previous studies can be categorized into the following three types, as noted by Kiley (2020):[3]

- The level of the real interest rate consistent with output equal to its long-run potential (e.g., Laubach and Williams, 2003)
- The level to which real interest rates will converge in the long run absent shocks (e.g., Rachel and Smith, 2017)
- The level of the real interest rate consistent with long-run equilibration of savings and investment (e.g., Obstfeld, 2020)

Following standard macroeconomic theory, these three definitions are considered to be equivalent, but estimation approaches differ depending on which of these definitions is used.[4]

The estimation approach used for the natural yield curve can also differ depending on which

[3] Kiley (2020) also introduces the concept of a short-term natural rate of interest based on the New Keynesian model framework, as distinguished from the long-term concept of the natural rate of interest discussed here. The short-term concept of the natural rate of interest can fluctuate due to short-term demand shocks and other factors, and can diverge from the long-term concept of the natural rate of interest estimated in this paper.

[4] The Bank of Japan has estimated the natural rate of interest using various approaches. For an overview, see Nakano et al. (2024).

definition of the natural rate of interest is used as the basis for the extension. For example, Imakubo et al. (2018) do an estimation that extends the first of the above definitions to the entire yield curve. That is, an IS curve is incorporated into the estimation model, as in Laubach and Williams (2003), and the trend components of the factors that comprise the yield curve are assumed to be linked to potential output. In contrast, this paper attempts to estimate the natural yield curve based on the second definition of the natural rate of interest, which is the level to which real interest rates will converge in the long run absent shocks. Although our approach has the disadvantage that the natural yield curve estimates are easily influenced by the observed data, when compared with the semi-structural approach used in Imakubo et al. (2018), it is less restrictive and allows the data to speak for itself.

Most studies of the natural rate of interest according to the second definition consider the natural rate of interest to be a permanent component in the fluctuation of short-term real interest rates and attempt to extract the random walk component. Formally, that is roughly similar to the definition of the trend in the trend-cycle decomposition by Beveridge and Nelson (1981). That is, the natural rate of interest r_t^* is defined as the infinite horizon forecast of the real short-term interest rate, when the temporary shock has disappeared, as follows:

$$r_t^* \equiv \lim_{j \to \infty} \mathbb{E}_t r_{t+j} \tag{1}$$

Extending this idea to the set $\{r_t(\tau)\}_{\tau=0}^{T}$ of the real interest rate $r_t(\tau)$ with maturities $0 < \tau \leq T$ gives the (real) natural yield curve:

$$r_t^*(\tau) \equiv \lim_{j \to \infty} \mathbb{E}_t r_{t+j}(\tau) \tag{2}$$

For an intuitive understanding of the feature of the natural yield curve based on such a definition, it is helpful to decompose $r_t(\tau)$ into the expected component of the real short-term interest rate, and the real term premium ($tp_t(\tau)$), as follows, based on the general liquidity premium hypothesis:

$$r_t(\tau) = \frac{1}{\tau}\sum_{j=0}^{\tau-1}\mathbb{E}_t r_{t+j} + tp_t(\tau) \tag{3}$$

Considering the trend component r^*_{t+j} of the short-term interest rate in period $t + j$ and the cyclical component r^c_{t+j}, which is the difference between a realized value and trend component, we can decompose $r_{t+j} = r^c_{t+j} + r^*_{t+j}$. In addition to this, using the fact that $\mathbb{E}_t r^*_{t+j} = r^*_t$, Equation (3) can be transformed as follows:

$$r_t(\tau) = r^*_t + \frac{1}{\tau}\sum_{j=0}^{\tau-1}\mathbb{E}_t r^c_{t+j} + tp_t(\tau) \tag{4}$$

Assuming that the term premium is also divided into a cyclical component and a trend component ($tp_t(\tau) = tp^c_t(\tau) + tp^*_t(\tau)$), the (real) natural yield curve defined by Equation (2) is $r^*_t(\tau) = r^*_t + tp^*_t(\tau)$, and is understood to consist of the movement in the natural rate of interest and the trend of the term premium in each maturity.[5]

2.2 Estimation model

Under the definitions in the previous section, the estimations in this paper are based on the model of Goy and Iwasaki (2024). The model combines a Nelson-Siegel model with macroeconomic variables as in Imakubo et al. (2018); however, as mentioned earlier, there are two main differences, as follows. First, the use of both nominal and real interest rate data allows simultaneous estimation of not only the real natural yield curve, but also the nominal natural yield curve and trend inflation rate. In this regard, Imakubo et al. (2018) use only the real interest rate. Second, Imakubo et al. (2018) attempt to capture "the level of the real

[5]There is no empirical consensus on whether or not the nominal and real term premia have a stochastic trend. Standard yield curve models assume that the factors comprising the yield curve are stationary, but in recent years, models have been proposed in which all of the factors comprising the yield curve have stochastic trends, such as in Bauer and Rudebusch (2020). If the term premia have no stochastic trends, the stochastic trend of the real long-term interest rate is the natural rate of interest plus a constant.

interest rate consistent with output equal to its long-run potential" as in Laubach and Williams (2003), assuming an IS curve and that the trends of the Nelson-Siegel factors of the real yield curve (in this paper, l^*, s^* and c^*) are linked to the potential growth rate. On the other hand, the model in this paper adopts a less restrictive and more data-driven approach by using a time series model called a VAR with common trends.

The details of the model will now be explained. First, we fit the Nelson-Siegel model for the yield curve of the real interest rates $\{r_t(\tau)\}_{\tau=0}^{T}$.

$$r_t(\tau) = l_t + \theta_s(\tau, \lambda^r)s_t + \theta_c(\tau, \lambda^r)c_t + e^r_{\tau,t} \tag{5}$$

$\theta_s(\tau, \lambda^r) = \frac{1-\exp(-\lambda^r\tau)}{\lambda^r\tau}$, $\theta_c(\tau, \lambda^r) = \left(\frac{1-\exp(-\lambda^r\tau)}{\lambda^r\tau} - \exp(-\lambda^r\tau)\right)$, λ^r is the parameter that governs the decay rate of the factor loadings in the second and third terms of the right-hand side of Equation (5). Under this formulation, given $\lim_{\tau\to\infty} r_t(\tau) = l_t$, $\lim_{\tau\to 0} r_t(\tau) = l_t + s_t$, and so on, we can interpret that l_t represents the level factor, and s_t represents the slope of the yield curve. Based on the characteristics of its coefficients, c_t can be interpreted to represent the curvature of the yield curve. $e^r_{\tau,t}$ is the error term.

Now, the nominal yield curve is represented as $\{y_t(\tau)\}_{\tau=0}^{T}$, and the Nelson-Siegel model is considered for $\pi_t(\tau)(\equiv y_t(\tau) - r_t(\tau))$, expressed as the difference between the nominal and real interest rates.

$$\pi_t(\tau) = l^\pi_t + \theta_s(\tau, \lambda^\pi)s^\pi_t + \theta_c(\tau, \lambda^\pi)c^\pi_t + e^\pi_{\tau,t} \tag{6}$$

The shape of θ_s and θ_c is the same as in the case of Equation (5), but the parameter representing the decay rate is λ^π, which is assumed to be different from the real yield curve. The interpretations of l^π_t, s^π_t and c^π_t are the level factor, slope factor and curvature factor, respectively, as in the case of the real yield curve. $e^\pi_{\tau,t}$ is likewise the error term.

We assume that the Nelson-Siegel factors have stochastic trends (x^*_t) and cyclical components ($\tilde{x}_t$). However, the curvature factors are assumed to be stationary for simplicity, based on the fact that it makes almost no contribution to the natural yield curve in Imakubo et al.

(2018). Furthermore, assuming that the Fisher equation $i_t^* = r_t^* + \pi_t^*$ holds in the long run, and given that $\pi_t^* = l_t^{\pi,*} + s_t^{\pi,*}$ and $r_t^* = l_t^* + s_t^*$ from the properties of the Nelson-Siegel model, Equations (5) and (6) can be transformed as follows:

$$y_t(\tau) = r_t(\tau) + \underbrace{\pi_t^* + \left(\theta_s(\tau,\lambda^\pi) - 1\right)s_t^{\pi,*} + \tilde{l}_t^\pi + \theta_s(\tau,\lambda^\pi)\tilde{s}_t^\pi + \theta_c(\tau,\lambda^\pi)c_t^\pi + e_{\tau,t}^\pi}_{\pi_t(\tau)} \tag{7}$$

$$r_t(\tau) = r_t^* + \left(\theta_s(\tau,\lambda^r) - 1\right)s_t^* + \tilde{l}_t + \theta_s(\tau,\lambda^r)\tilde{s}_t + \theta_c(\tau,\lambda^r)c_t + e_{\tau,t}^r \tag{8}$$

That is, $y_t^*(\tau)$ can be expressed in terms of the natural rate of interest r_t^*, the trend inflation rate π_t^*, and the trend components of the slope s_t^* and $s_t^{\pi,*}$.[6] Also, $r_t^*(\tau)$ can be expressed in r_t^* and s_t^*.

All of the above stochastic trend components follow a random walk.

$$\begin{pmatrix} r_t^* \\ \pi_t^* \\ s_t^* \\ s_t^{\pi,*} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r_{t-1}^* \\ \pi_{t-1}^* \\ s_{t-1}^* \\ s_{t-1}^{\pi,*} \end{pmatrix} + \varepsilon_t, \quad \varepsilon_t \sim \text{i.i.d.}\mathcal{N}(0, \Sigma_\varepsilon) \tag{9}$$

The ε_t is an IID process following a multivariate normal distribution.[7] For the cyclical

[6] As Goy and Iwasaki (2024) calculate the real interest rates from the inflation swap rates and the nominal interest rates, Equation (6) is interpreted as an inflation compensation curve that also includes the inflation risk premia. On the other hand, since this paper follows Imakubo et al. (2018) and calculates the real interest rates from the inflation expectations and nominal interest rates, Equation (6) is interpreted as an inflation expectations curve. Although there is no reason to believe that the slope of the inflation expectations curve, which consists only of expected components when following the argument in Section 2, has a stochastic trend, this paper assumes that the slope of the inflation expectations curve has a stochastic trend based on the data characteristics of the inflation expectations in Japan. The results suggest that, as in the case of Osada and Nakazawa (2024), there is an upward bias for longer forecast horizons, and that such a bias may be diminishing in the near term. It should be noted that these characteristics of Japan's inflation expectations data may be due to the short estimation period.

[7] It should be noted that ε_t is not a structural shock. Similarly, η_t in Equation (10) is not a structural shock.

components, we assume the following first-order VAR structure:

$$\begin{pmatrix} \tilde{l}_t \\ \tilde{s}_t \\ c_t \\ \tilde{l}^{\pi}_t \\ \tilde{s}^{\pi}_t \\ c^{\pi}_t \\ \tilde{\pi}_t \\ \tilde{x}_t \end{pmatrix} = A^c \begin{pmatrix} \tilde{l}_{t-1} \\ \tilde{s}_{t-1} \\ c_{t-1} \\ \tilde{l}^{\pi}_{t-1} \\ \tilde{s}^{\pi}_{t-1} \\ c^{\pi}_{t-1} \\ \tilde{\pi}_{t-1} \\ \tilde{x}_{t-1} \end{pmatrix} + \eta_t, \quad \eta_t \sim \text{i.i.d.}\mathcal{N}(0, \Sigma_\eta) \tag{10}$$

In addition to the cyclical components of the yield curve, the equation includes the cyclical components of inflation rate $\tilde{\pi}_t$ and the output gap $\tilde{x}$. This makes it easier to capture the cyclical components of the Nelson-Siegel factors, which in turn helps to extract the trend components.

We describe the model above in a state-space representation. First, let the vector of observed variables, Z_t, and the vector of state variables, X_t, be represented as follows:[8]

$$Z_t = \Big(\underbrace{y_t(\tau_1) \dots y_t(\tau_J)}_{\text{Nominal Yields}} \quad \underbrace{r_t(\tau_1) \dots r_t(\tau_K)}_{\text{Real Yields}} \quad \underbrace{\pi_t \quad \tilde{x}_t}_{\text{Macroeconomic Variables}} \Big)' \tag{11}$$

$$X_t = \left(r^*_t \; \pi^*_t \; s^*_t \; s^{\pi,*}_t \; \tilde{l}_t \; \tilde{s}_t \; c_t \; \tilde{l}^{\pi}_t \; \tilde{s}^{\pi}_t \; c^{\pi}_t \; \tilde{\pi}_t \; \tilde{x}_t \right)' \tag{12}$$

Using these, the observation and state equations are,

$$Z_t = CX_t + e_t, \quad e_t \sim \mathcal{N}(0, \Sigma_e) \tag{13}$$

$$X_t = \begin{pmatrix} \mathrm{I}_4 & 0_{4\times 8} \\ 0_{8\times 4} & A^c \end{pmatrix} X_{t-1} + \begin{pmatrix} \mathrm{I}_4 & 0_{4\times 8} \\ 0_{8\times 4} & \mathrm{I}_8 \end{pmatrix} \begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix}, \quad \begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0_{4\times 1} \\ 0_{8\times 1} \end{pmatrix}, \begin{pmatrix} \Sigma_\varepsilon & 0_{4\times 8} \\ 0_{8\times 4} & \Sigma_\eta \end{pmatrix} \right) \tag{14}$$

[8] Goy and Iwasaki (2024) calculate the real yield curve using inflation swap rates, and they also use survey-based long-term inflation expectations as observed variables. In general, market-based and survey-based inflation expectations are known to diverge, and using both helps produce estimates of the trend inflation rate with less bias. However, since this paper uses survey-based inflation expectations to calculate the real yield curve from the view of comparability with previous studies and data availability, survey-based inflation expectations are not directly included as an observed variable.

where $0_{m\times n}$ denotes the $m \times n$ zero matrix, and $\mathrm{I_n}$ denotes the n order identity matrix. Also, C in Equation (13) is given by,

$$C = \begin{pmatrix} 1_J & 1_J & \Theta_s^J(\lambda^r) - 1_J & \Theta_s^J(\lambda^\pi) - 1_J & 1_J & \Theta_s^J(\lambda^r) & \Theta_c^J(\lambda^r) & 1_J & \Theta_s^J(\lambda^\pi) & \Theta_c^J(\lambda^\pi) & 0_J & 0_J \\ 1_K & 0_K & \Theta_s^K(\lambda^r) - 1_K & 0_K & 1_K & \Theta_s^K(\lambda^r) & \Theta_c^K(\lambda^r) & 0_K & 0_K & 0_K & 0_K & 0_K \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

where 1_n and 0_n denote n order column vectors with all elements 1 and 0, respectively, and $\Theta_s^n(\lambda^x) = \left(\theta_s(\tau_1, \lambda^x) \dots \theta_s(\tau_n, \lambda^x)\right)'$, $\Theta_c^n(\lambda^x) = \left(\theta_c(\tau_1, \lambda^x) \dots \theta_c(\tau_n, \lambda^x)\right)'$ ($x = r$ or π, $n = J$ or K). In Equation (13), $e_t = \left(e^y_{\tau_1,t} \dots e^y_{\tau_J,t} e^r_{\tau_1,t} \dots e^r_{\tau_K,t} 0\,0\right)'$ is an error vector ($e^y_{\tau,t} \equiv e^r_{\tau,t} + e^\pi_{\tau,t}$) and its variance-covariance matrix Σ_e is a diagonal matrix. This model is almost the same as the VAR with common trends used by Del Negro et al. (2017), and the state-space models estimated by Bauer and Rudebusch (2020), Johannsen and Mertens (2021), and so on.[9]

3 Estimation method

3.1 Data

In estimating the model, we used the nominal zero-coupon rates, the real zero-coupon rates, the inflation rate, and the output gap, as shown in Figure 1. The sample period is from the third quarter of 1992 to the first quarter of 2023. For the nominal zero-coupon rates, we used 3- and 6-month, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, and 20-year data calculated by Bloomberg. While the 3-month real zero-coupon rate is calculated by subtracting the actual inflation rate from the nominal zero-coupon rate, for maturities longer than 1 year, we use the nominal zero-coupon rates deflated by the inflation expectations for each maturity obtained from the

[9] In the Bauer and Rudebusch (2020), data for the period facing the effective lower bound constraint are not used in the estimation, but since it is difficult to do the same for Japan, which faces the constraint for most of the sample period, the constraint is discarded in the estimation in this paper. The method of Johannsen and Mertens (2021), which takes into account the shadow rate, is also not employed in this paper due to the large number of state variables and high computational burden of the model in this paper compared with Johannsen and Mertens (2021). It should also be noted that in the case of Japan, in addition to the effective lower bound, yield curve control may affect the estimated natural yield curves.

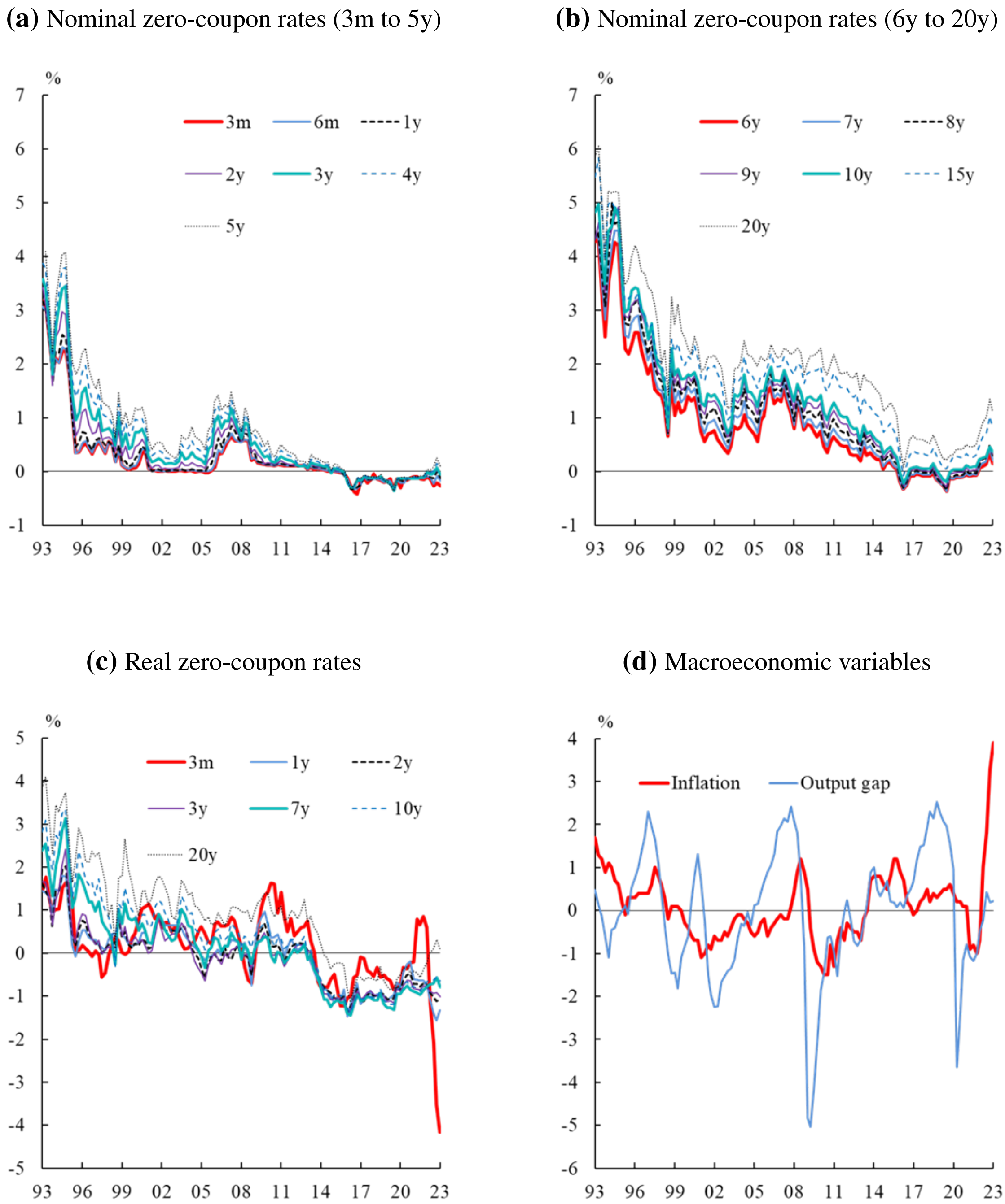
"Consensus Forecasts," following Imakubo et al. (2018). However, since the period covered by the survey is limited to the 10 years ahead, we assume that inflation expectations for more than 10 years ahead are the same as those for 6-10 years ahead. The real zero-coupon rates used in the estimation are 3-month, 1, 2, 3, 7, 10, and 20-year rates. For the inflation rate, we use the CPI (all items less fresh food and energy) published by the Ministry of Internal Affairs and Communications after excluding temporary factors such as the consumption tax. For the output gap, we use a series calculated by the Bank of Japan and centered by subtracting the average of the sample period.

3.2 Estimation procedure and prior distribution

This paper uses an algorithm that is generally similar to the Gibbs sampler MCMC proposed in Del Negro et al. (2017), so we provide only an overview. However, in this analysis, we add a part to estimate the error term. The estimation consists of following three steps: in the first step, the parameters constituting A^c are generated by the Metropolis-Hasting method with the state variables and other parameters given, and then, with all parameters given, we obtain the state variables using the Durbin and Koopman (2002) simulation smoother. In the second step, the parameters of the state equation are generated by Gibbs sampling. In the third step, Gibbs sampling is performed for the variance of the error term. In this paper, these steps were repeated 100,000 times for the four chains, and the results are retained for every tenth of the last 10,000 iterations.

The following explains the prior distribution. The parameters included in the coefficient matrix C of the observation equation (13) are λ^r and λ^π, which govern the factor loadings of the Nelson-Siegel model. For both of these prior distributions, we set independent normal distributions with mean 0.04 and standard deviation 0.01, respectively. For the variance-covariance matrix Σ_e of the error vector e_t, an inverse-gamma distribution with shape parameter $\alpha = 2$ and scale parameter $0.1^2 * (\alpha + 1)$ was set with reference to Bauer and Rudebusch (2020).

Figure 1. Data used for estimation



Source: Bloomberg, Consensus Economics Inc., "Consensus Forecasts," Ministry of Internal Affairs and Communications, Bank of Japan.

For the parameters of the state equation (14), we set the prior distribution used as standard in Bayesian VARs, as in Del Negro et al. (2017). First, for the prior distribution of A^c, we assumed a Minnesota prior, $p(vec(A^c)|\Sigma_\eta) = \mathcal{N}(vec(\underline{A^c}), \Sigma_\eta \otimes \underline{\Omega})I(vec(A^c))$. We note that $\underline{A^c}$ is centered around zero, given that the VAR part captures the stationary process component. In addition, the hyperparameters are set to 0.1. Note that $I(vec(A^c))$ is an indicator function that takes 0 when the VAR diverges and 1 otherwise. Then, for the prior distribution of the variance-covariance matrix, an inverse-Wishart distribution ($\mathcal{IW}(\kappa, (\kappa + n + 1)\underline{\Sigma})$, where $\underline{\Sigma}$ is the mode and κ is the degrees of freedom) is set. First, the prior distribution of Σ_η is set such that the mean is a diagonal matrix with the diagonal components 1 and κ_η is equal to the number of cyclical components (8) + 2, based on the same idea as in Del Negro et al. (2017). For the prior distribution of Σ_ε, $\underline{\Sigma}$ was set to be a diagonal matrix, which has diagonal components 0.002, 0.003, 0.003, and 0.002.[10] In setting these, the variance of the HP trends for each series was used as a reference. For the degrees of freedom, we set $\kappa_\varepsilon = 100$, following previous studies analyzing trend components using methods similar to those in this paper, which set a tight prior distribution for the variance of trend innovations when extracting trend components.

4 Estimation results

4.1 Natural yield curve components

First, trajectories of r_t^*, s_t^*, and π_t^* are shown in Figure 2. What is noteworthy here is that not only the traditionally noted downward trend in r_t^*, but also an upward trend in s_t^* has been observed since the 1990s. As pointed out in Section 2, if s_t^* has no trend, the natural interest rates for the long maturity are consistent with those of the short-term, so there is no need

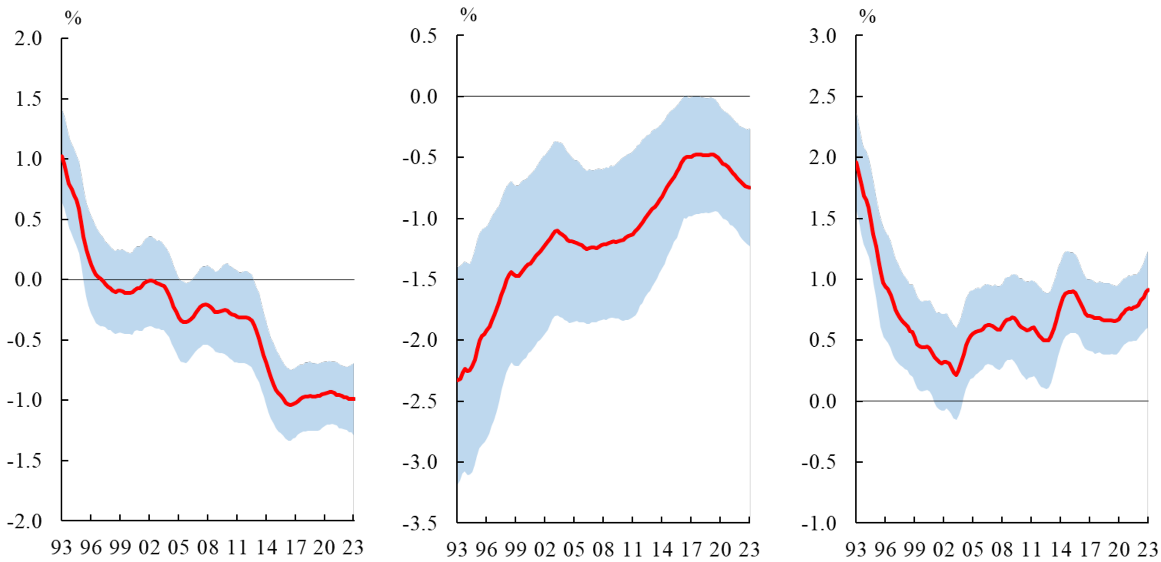
[10]These settings imply that the standard deviation of the changes in each trend component over the 100-year period should be around 1 percent.

to consider the natural yield curve, and we just have to estimate the natural rate of interest. The results here show that the slope of the natural yield curve changes throughout the period, suggesting the usefulness of considering the natural yield curve. As noted above, such a trend in the slope component has also been observed in Imakubo et al. (2018), but in their estimation, it was assumed to be linked to the potential growth rate, whereas in this paper, it is driven only by exogenous shocks. Even with this change in estimation methodology, the slope trend is robustly confirmed.

The trend inflation rate (π_t^*), after declining rapidly in the 1990s, has remained in the mid- to upper-0% range. Since the pandemic, the trend inflation rate has been gradually increasing, which is similar to the tendency in several other estimation methods (BOJ, 2024).

Figure 2. Estimated components of the natural yield curve

(a) r_t^* **(b)** s_t^* **(c)** π_t^*



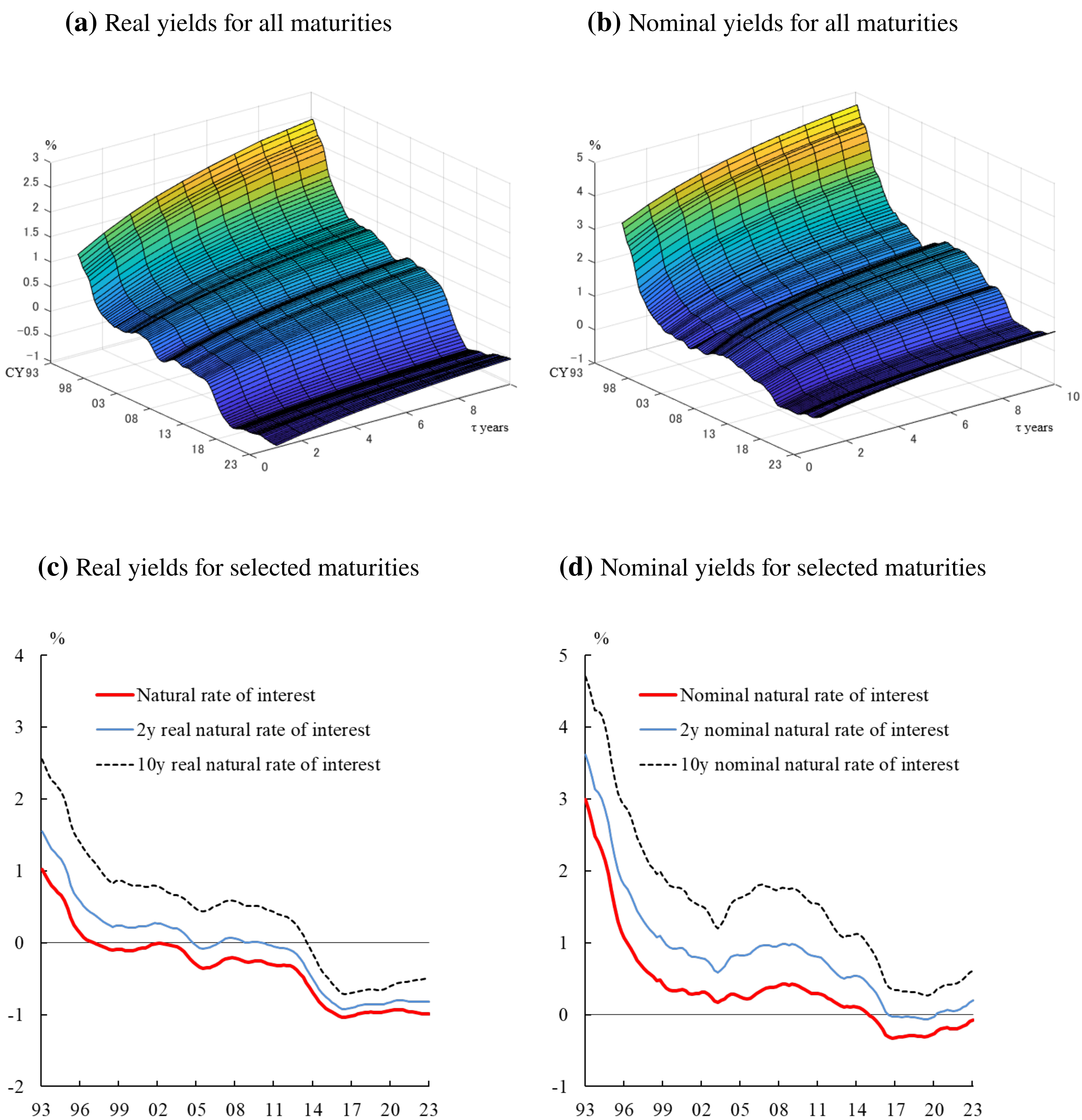
Note: The shaded areas represent 95% credible sets.

4.2 Trajectory of the natural yield curve

Figure 3 shows the real and nominal natural yield curves calculated using r_t^*, s_t^*, π_t^*, etc. as shown in the previous section. First, for the real natural yield curve, looking at the trend over the entire estimation period, we observed a trend decline and flattening as a whole since the 1990s, as in Imakubo et al. (2018). As is clear from the model equation, a decline in r_t^* affects the natural yield curve for all maturities equally, and thus the decline in the overall natural yield curve is interpreted as being due to a decline in the natural rate of interest. The flattening is due to an increase in s_t^* (a decrease in the trend component of the term premium). Considering these impacts on the 10-year real natural rate of interest, this also suggests that the impact of the decline in the natural rate of interest was more significant than the flattening. While it should be noted that, in examining recent trends, as with other trend-cycle decomposition methods, the analysis in this paper is affected by the "sample endpoint problem," in which the estimated value of the most recent trend component is easily influenced by the actual data, it is noteworthy that the further decline in the natural rate of interest and the flattening of the natural yield curve has been halted.

The next result for the nominal natural yield curve indicates that, as in the case of the real terms, there has been a trend decline and flattening since the 1990s. However, there is a difference in extent, with the nominal natural yield curve declining more, reflecting the decline in π_t^*. On the other hand, the flattening of the nominal natural yield curve has been somewhat slower. Although the "sample endpoint problem" should be kept in mind, the nominal natural rate of interest has been rising, reflecting the upward trend of π_t^*, indicating that the overall medium- to long-term natural yield curve has also been rising somewhat as a result. As described above, the real and nominal natural yield curves do not necessarily move in parallel, reflecting the movement of the trend components of the inflation expectations curve. Policymakers need to pay close attention to this point when the trend of the inflation

Figure 3. The natural yield curve

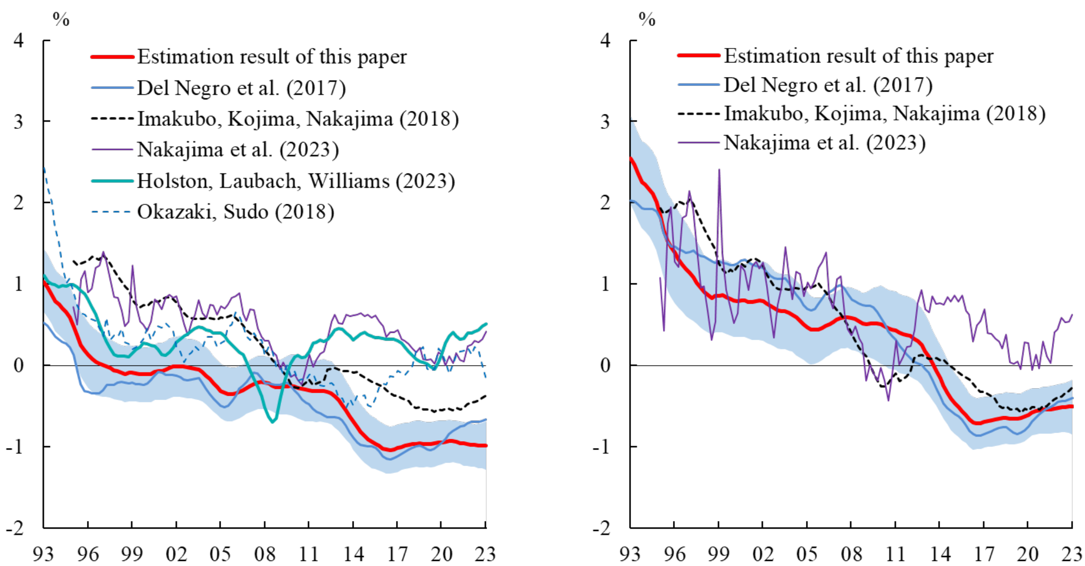


expectations curve moves significantly.

In Figure 4, we compare our estimates of the natural rate of interest and the real natural rate for the long maturity with those of previous studies. First, it can be seen that the estimates for the natural rate of interest are generally within the range of the estimates of various previous studies. Compared with the semi-structural model, which explicitly takes into account the relationship with the potential growth rate, it is estimated slightly lower, but this is thought to be partly due to the fact that the potential growth rate has been higher relative to the trend of the real interest rate. In Davis et al. (2019), it is pointed out that in some advanced economies, estimates of the natural rate of interest based on yield curves are estimated lower than those based on semi-structural models, and this phenomenon is referred to as the "natural rate puzzle." There is a possibility that a "natural rate puzzle" exists in Japan as well. The results for the real natural rate for the long maturity are also generally consistent with other studies.

Figure 4. Comparison with previous studies

(a) Natural rates of interest **(b)** Real natural rates for the long maturity



Note: The shaded areas represent 95% credible sets.

5 Conclusion

This paper introduces a novel approach for simultaneously estimating nominal and real natural yield curves in Japan. Specifically, we employ macroeconomic variables such as the output gap and inflation rate as observed variables, in addition to the nominal and real yield curves, and extract stochastic trends of the factors that constitute the natural yield curves, combining the Nelson-Siegel model with a VAR with common trends estimation method.

The estimation results indicate that since the 1990s, both nominal and real natural yield curves have exhibited downward shifts, primarily as a consequence of a decline in the natural rate of interest, with flattening. Furthermore, the results also indicate that while the overall decline for the nominal yield curve is larger than for the real yield curve, reflecting the decline in the trend inflation rate, the extent of flattening of the nominal yield curve is smaller than in real terms.

There are some points to note about the analysis in this paper. First, since the value of the natural rate of interest is highly dependent on the estimation method, the results of the analysis in this paper must also be interpreted carefully. Second, it is necessary to keep in mind the "sample endpoint problem," which is a particularly significant problem with the method used in this paper. In this regard, the extent of the upward shift in the nominal natural yield curve after the 2020s, which is indicated by the estimation results of this paper, should also be interpreted with caution.

Finally, we discuss future issues. First, the estimates in this paper do not fully take into account the effects of the effective lower bound and yield curve control, but we cannot rule out the possibility that estimation biases may have arisen from not taking these constraints into account. In the future, it would be beneficial to conduct estimations that take these constraints into account by referring to Bauer and Rudebusch (2020) or Johannsen and Mertens (2021), and so on. Second, we treat all factors that could cause the natural yield curves to fluctuate as exogenous. Therefore, questions such as why the natural rate of interest has declined and why the trend of the slope component of the yield curve has fluctuated are outside the scope of this paper's analysis. In the future, a deeper understanding of the factors that cause fluctuations in factors such as the natural rate of interest, from both a theoretical and an empirical perspective, is expected to lead to the development of more sophisticated estimation methods and, by extension, to the solution of the "natural rate puzzle." It would also be useful to analyze the position and characteristics of the natural yield curve in terms of monetary policy from a theoretical perspective.

References

Bauer, Michael D. and Glenn D. Rudebusch (2020) "Interest rates under falling stars," *American Economic Review*, Vol. 110, No. 5, pp. 1316–54.

Beveridge, Stephen and Charles R. Nelson (1981) "A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the 'business cycle'," *Journal of Monetary Economics*, Vol. 7, No. 2, pp. 151–174.

BOJ (2024) "Outlook for economic activity and prices April 2024."

Davis, Josh, Cristian Fuenzalida, and Alan M. Taylor (2019) "The natural rate puzzle: global macro trends and the market-implied r^*," NBER Working Papers 26560, National Bureau of Economic Research, Inc.

Del Negro, Marco, Domenico Giannone, Marc P. Giannoni, and Andrea Tambalotti (2017) "Safety, liquidity, and the natural rate of interest," *Brookings Papers on Economic Activity*, Vol. 48, No. 1, pp. 235–316.

Dufrénot, Gilles, Meryem Rhouzlane, and Etienne Vaccaro-Grange (2022) "Potential growth and natural yield curve in Japan," *Journal of International Money and Finance*, Vol. 124.

Durbin, James and Siem J. Koopman (2002) "A simple and efficient simulation smoother for state space time series analysis," *Biometrika*, Vol. 89, No. 3, pp. 603–616.

Fujiwara, Shigeaki, Yuto Iwasaki, Ichiro Muto, Kenji Nishizaki, and Nao Sudo (2016) "Supplementary paper series for the "comprehensive assessment"(2): developments in the natural rate of interest in japan," Bank of Japan Review Series 2016-E-12, Bank of Japan.

Goy, Gavin and Yuto Iwasaki (2024) "From the natural rate towards a natural curve: a first step to benchmarking the term structure," mimeo.

Imakubo, Kei, Haruki Kojima, and Jouchi Nakajima (2018) "The natural yield curve: its concept and measurement," *Empirical Economics*, Vol. 55, No. 2, pp. 551–572.

Johannsen, Benjamin K. and Elmar Mertens (2021) "A time-series model of interest rates with the effective lower bound," *Journal of Money, Credit and Banking*, Vol. 53, No. 5, pp. 1005–1046.

Kamada, Koichiro (2009) "Japan's equilibrium real interest rate," in Fukao, Kyoji ed. *Macroeconomy and Industrial Structures*, Chap. 12: Keio University Press (only in Japanese).

Kiley, Michael T. (2020) "The global equilibrium real interest rate: concepts, estimates, and challenges," *Annual Review of Financial Economics*, Vol. 12, No. 1, pp. 305–326.

Laubach, Thomas and John C. Williams (2003) "Measuring the natural rate of interest," *The Review of Economics and Statistics*, Vol. 85, No. 4, pp. 1063–1070.

Nakano, Shogo, Yu Sugioka, and Hiroki Yamamoto (2024) "Recent developments in measuring the natural rate of interest," Bank of Japan Working Paper Series 24-E-12, Bank of Japan.

Nelson, Charles R. and Andrew F. Siegel (1987) "Parsimonious modeling of yield curves," *Journal of Business*, Vol. 60, No. 4, pp. 473–489.

Obstfeld, Maurice (2020) "Global dimensions of U.S. monetary policy," *International Journal of Central Banking*, Vol. 16, No. 1, pp. 73–132.

Oda, Nobuyuki and Jun Muranaga (2003) "On the natural rate of interest: theories and estimates," Bank of Japan Working Paper Series 03-J-5, Bank of Japan (only in Japanese).

Osada, Mitsuhiro and Takashi Nakazawa (2024) "Assessing measures of inflation expectations: a term structure and forecasting power perspective," Bank of Japan Review Series 2024-E-4, Bank of Japan.

Piazzesi, Monika and Martin Schneider (2007) “Equilibrium yield curves,” in *NBER Macroeconomics Annual 2006, Volume 21*, pp. 389–472: National Bureau of Economic Research, Inc.

Rachel, Lukasz and Thomas D. Smith (2017) “Are low real interest rates here to stay?,” *International Journal of Central Banking*, Vol. 13, No. 3, pp. 1–42.