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What Prevents Productivity Gains from Translating into Wages: Lessons from Japan's Deflationary Period

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What Prevents Productivity Gains from Translating into Wages: Lessons from Japan's Deflationary Period *

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Abstract

What prevents productivity gains from translating into wages? This paper examines the roles of deflation and labor market frictions, with particular emphasis on job security, in driving the widening wage-productivity gap. Our empirical analysis shows that wage-setting in Japan was rigid in both upward and downward directions during the deflationary period, with upward rigidity being more pronounced. These wage freezes suppressed wage growth and contributed to the widening wage-productivity gap. Using a heterogeneous-firm general equilibrium model, we show that downward nominal wage rigidity, together with deflation and an emphasis on job security, suppresses wage growth and widens the wage-productivity gap by inducing firms to impose larger wage markdowns. Alleviating these factors raises not only wages but also aggregate productivity through labor reallocation across firms.

Keywords: Wage-Productivity Gap, Nominal Wage Rigidity, Trend Inflation, Reallocation Effect

JEL classification: E24, E52, J30

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1 Introduction

Comparing trends in labor productivity and real wages across advanced economies, the gap between the two has widened much more markedly in Japan than in Western European countries over the past 30 years (Figure 1).[1] Why has the gap between real wages and productivity—that is, the wage–productivity gap—widened in Japan? Under what conditions might this wage–productivity gap narrow? This paper examines the factors that account for differences between Japan and Western European countries in the evolution of the wage–productivity gap.

The literature has identified two potential factors, both highly relevant to Japan, that may have contributed to the widening of the wage–productivity gap. The first is the role of inflation in shaping wage-setting rigidities, namely the "greasing-the-wheels" effect (Tobin, 1972; Elsby, 2009). In a low-inflation or deflationary environment, nominal wage rigidities are more likely to impede wage adjustment and generate a persistent divergence between wages and productivity. The second is the decline in workers' bargaining power in the labor market (e.g., Grossman and Oberfield (2022), Berger et al. (2022)). In the Japanese context, this force may also be reflected in labor–management bargaining that places greater weight on employment stability than on wage growth. Motivated by these considerations, we investigate whether low inflation and the prioritization of employment stability over wage growth can account for the expansion of Japan's wage–productivity gap.[2]

As a first step, we conduct an empirical fact-finding exercise on wage-setting behavior in Japan. We apply an estimation framework that jointly evaluates upward and downward nominal wage rigidities to long-term Japanese worker-level microdata covering 1990–2024. We first show that both upward and downward nominal wage rigidities have been present in Japan over the past 30 years. In other words, periods of wage freezes have occurred during which wage setting was rigid both upward and downward (e.g., Daly and Hobijn (2014), Schaefer and Singleton (2023)). More importantly, we show that, since Japan entered a deflationary period in the late 1990s, upward rigidity has become more influential, exerting downward pressure on actual wages. In this regard, previous work has focused only on downward rigidity (e.g., Grigsby et al. (2021)), but to the best of our knowledge, no study has jointly evaluated upward rigidity using long-term microdata. In recent years, however, as inflation has risen, these wage-setting rigidities have weakened. That is, wage setting has become more flexible.

We then develop a tractable framework that allows us to analyze, under downward nominal wage rigidity (DNWR), the effects of (1) lower inflation and (2) the prioritization of employment stability in labor–management bargaining or, equivalently, an emphasis on relative wages. Specifi-

[1] We show real wages based on real wages per employee, while productivity is based on real productivity per employee in the main text. However, even if we examine these measures per hour, the main intuition remains unchanged: the gap between real wages and productivity in Japan has widened markedly compared to other advanced economies. See Appendix A.1 for more details on the description of the gap.

[2] See Bank of Japan (2024) for a detailed narrative on these forces in the Japanese economy.

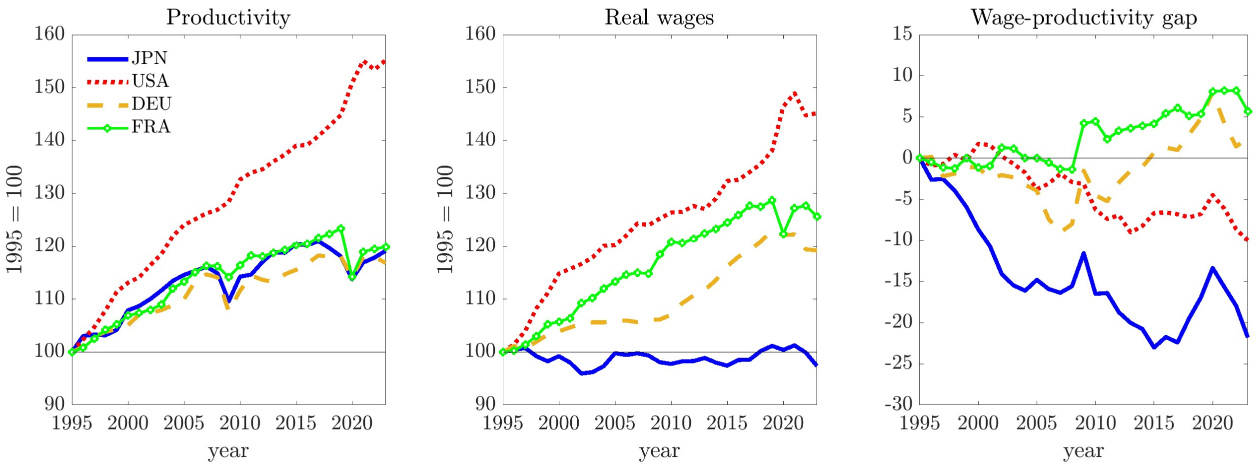

Figure 1: Productivity, real wages, and wage-productivity gap in advanced economies

cally, we extend the heterogeneous-firm partial-equilibrium model developed by Ehrlich and Montes (2024) into a general-equilibrium framework.[3] An important feature of our model is that, under DNWR, idiosyncratic shocks to firms generate endogenous upward rigidity in wage-setting behavior. Because wage cuts entail an additional cost, firms restrain wage increases today in anticipation of future negative idiosyncratic shocks. Separation elasticity provides a reduced-form measure of how strongly workers value relative wages. In our framework, it governs how sensitively separation responds to the firm's wage relative to the aggregate wage. A higher separation elasticity therefore corresponds to a stronger emphasis on relative wages, while a lower separation elasticity captures a greater prioritization of employment stability.

In the steady state, under DNWR, higher inflation raises both aggregate wages and endogenous labor productivity. The key mechanism is that higher inflation mitigates the costs that DNWR imposes on firms. By easing nominal wage rigidity, higher inflation strengthens worker mobility based on wage signals and improves the efficiency of labor allocation.[4] In an environment in which relative wages matter more—that is, when separation elasticity is higher—workers have a stronger incentive to move to high-productivity firms that offer higher wages, thereby raising both aggregate wages and aggregate productivity.

[3]Ehrlich and Montes (2024) propose a "quasi-monopsony" structure in which firms set wages in the absence of worker bargaining power, but workers can leave the firm when their relative wage is low. Using this partial-equilibrium model, they analyze the relationship between DNWR and employment in the German labor market. See Section 4 for details of this model.

[4]In Japan, the literature discusses this mechanism relating deflation and labor allocation (e.g., Ueda (2023), Ito (2024a), Ito (2024b), Sugioka et al. (2024), and Watanabe (2025)). In the U.S., the recent literature points out the labor allocation forces driven by post-COVID-19 inflation. Afrouzi et al. (2026), Hajdini et al. (2025), Pilossoph and Ryngaert (2024), and Pilossoph et al. (2024) all highlight how increased inflation can lead workers to search more for other jobs. Afrouzi et al. (2026) shows theoretically that unexpectedly high inflation induces workers to search more on the job, providing an incentive for firms to post more vacancies during periods of declining real wages. Pilossoph et al. (2024) show that unexpected inflation can also have allocative consequences by inducing workers to move to better matches, although these allocative effects were quantitatively small during the recent inflation period.

In the dynamic analysis, we examine how exogenous productivity shocks pass through to aggregate wages and endogenous aggregate productivity, focusing on the roles of trend inflation and separation elasticity. First, an increase in trend inflation lowers firms' wage adjustment costs, which strengthens wage responses to productivity shocks and firms' incentives to keep up with wage increases at other firms, thereby raising pass-through to aggregate wages. This stronger wage response also promotes resource reallocation through worker movement toward high-productivity firms, which in turn raises pass-through to aggregate productivity.

By contrast, a higher separation elasticity increases pass-through to aggregate wages by intensifying separation pressure when aggregate wages rise and thereby inducing firms to respond more strongly to aggregate wage movements. At the same time, however, a greater sensitivity of separations to relative wages allows firms to retain employment without raising wages as aggressively, which dampens firm-level wage responses to productivity shocks. As a result, wage differentials become compressed and labor flows more easily toward relatively low-productivity firms, so pass-through from productivity shocks to aggregate productivity declines. Taken together, these analyses also contribute to the literature on heterogeneous firms and households (e.g., Ottonello and Winberry (2020), Auclert et al. (2021)).

To quantify the implications of our mechanism for Japan's long-term wage dynamics, we conduct a counterfactual analysis based on the historical path of labor productivity over the past three decades. We first construct a series of aggregate productivity shocks under the assumption that Japan's observed labor productivity path can be interpreted as the cumulative outcome of persistent productivity disturbances. We then feed this shock sequence into our DSGE model to generate endogenous paths for aggregate real wages and labor productivity under three alternative scenarios: the benchmark economy, an economy with higher trend inflation, and an economy with both higher trend inflation and a higher separation elasticity. We map the model-implied differences in real wages across these scenarios into counterfactual wage histories by adding them to the observed real wage series. This exercise allows us to assess how much of Japan's prolonged real wage stagnation can be attributed not only to weak underlying productivity growth itself, but also to the way productivity gains are transmitted to wages in a low-inflation and low-mobility environment.

Finally, we empirically examine the relationship between worker reallocation and wage-setting rigidity, which is central to the mechanism emphasized in the theoretical model. Specifically, after matching Japanese worker and firm data from multiple sources, we follow Decker et al. (2020) and analyze the relationship between firm-level employment growth and productivity, as well as the effect of wage-setting rigidity on that relationship. The empirical results first confirm that more productive firms tend to exhibit higher employment growth. They also show that when the share of wage freezes is high—that is, when wage setting is more rigid—worker reallocation in response to productivity is relatively weaker than it otherwise would be. These findings are consistent with the results of the theoretical analysis. In other words, under DNWR, when inflation remains low and wage setting is rigid, the signaling function of wages becomes less effective. As a result, worker mobility in response to firm productivity becomes more limited, and the efficiency of labor allocation deteriorates. Conversely, when inflation rises, the signaling role of wages may recover, improving the allocation of labor across firms. Through this analysis, the paper also contributes to the literature on the efficiency of resource allocation across firms (e.g., Baqaee and Farhi (2020)).

Why focus on deflation and the prioritization of employment stability? Nominal wage rigidity is one of the main factors underlying the wage–productivity gap. When nominal wages do not adjust flexibly either upward or downward and are effectively fixed, wage adjustment in line with worker productivity is impeded, and the wage–productivity gap widens. In such an environment, any force that suppresses nominal wage growth can increase the number of workers affected by nominal wage rigidity. This problem is particularly pronounced in Japan, where the wage–productivity gap has widened more substantially than in other advanced economies.

For this reason, this study focuses on two Japan-specific factors that suppress nominal wage growth and amplify the effects of wage freezes: (1) deflation and (2) the tendency in Japanese labor-market practices to prioritize employment stability over wage increases. First, regarding the relationship between inflation or deflation and nominal wage rigidity, it has long been argued that when inflation is positive, wages can adjust through a decline in real wages even without cuts in nominal wages, thereby mitigating the effects of nominal rigidity. This phenomenon is known as the "greasing-the-wheels" effect (Tobin (1972)). In Japan, however, unlike in other advanced economies, deflation persisted from the late 1990s for an extended period. This suggests that the "greasing-the-wheels" effect did not operate strongly enough in Japan, and that prolonged deflation may have contributed to the widening of the wage–productivity gap.

Second, the prioritization of employment stability in labor–management bargaining—or, more broadly, the decline in workers' bargaining power—has also been identified in previous studies as a structural factor depressing the labor share (e.g., Grossman and Oberfield (2022), Bergholt et al. (2022), Karabarbounis (2024)). Several studies focusing on Japan argue that the decline in workers' bargaining power has contributed to the fall in the labor share (Dobbelaere and Kiyota (2018), Goodhart and Pradhan (2020)). In particular, since the late 1990s, amid worsening corporate profitability, firms have increasingly prioritized employment stability over wage increases (Ogura (2017), Watanabe (2025)). These Japan-specific employment practices may have further suppressed nominal wage growth and amplified the effects of nominal wage rigidity.

Structure. This paper is structured as follows. Section 2 reviews relevant previous studies and explains the positioning of this paper in relation to them. Section 3 describes the estimation framework for the empirical analysis and presents the estimation results. In Section 4, we describe our theoretical framework and calibration strategy. Section 5 discusses analytical results on how inflation and preference for job security affect macro wage and productivity. In Section 6, we discuss the dynamic implications of the model under a positive productivity shock and show the results of counterfactual simulations under higher inflation and lower job security preferences. Section 7 use firm-level microdata to investigate whether wage-setting rigidity hinders the efficient allocation of labor in line with firms' productivity. Section 8 concludes the paper.

2 Literature Review

The analysis in this paper is related to (1) the labor share of income and wage markdown, (2) nominal wage rigidity, and (3) productivity and resource allocation. Below, we survey previous studies in these fields and discuss this paper's contributions to each area.

2.1 Labor Share and Wage Markdown

Firstly, recent analyses have reported a global declining trend in the labor share, and a substantial body of research, primarily focusing on the United States, has examined its factors and underlying causes (Grossman and Oberfield (2022); Bergholt et al. (2022); Karabarbounis (2024)). As these studies point out, several factors such as technological innovation, globalization, the decline in workers' bargaining power, and demographic dynamics have collectively exerted downward pressure on the labor share. A representative recent study, Autor et al. (2020), points out that wages have been suppressed relative to productivity in highly productive large firms ("superstar firms") in the United States. In Europe, it has been observed that the increase in labor supply due to globalization has led to a decline in workers' bargaining power (Lodge et al. (2021)).

In Japan as well, the labor share of income has shown a declining trend, and several studies have reported that the decline in workers' bargaining power has contributed to this downward pressure. For instance, Fukao and Perugini (2021) highlight that the increasing proportion of non-regular workers has weakened the bargaining power of labor unions, exerting downward pressure on the labor income share. Also, the increase in foreign direct investment (FDI) has strengthened the wage bargaining power of multinational corporations (Dobbelaere and Kiyota (2018)). Additionally, some studies highlight the weakening of workers' wage bargaining power in the context of labor hoarding during economic recessions (Goodhart and Pradhan (2020)). Furthermore, particularly since the late 1990s, worsening corporate earnings have led to a shift in labor-management negotiations, prioritizing employment stability over wage increases, which has also been identified as a factor contributing to the weakening of workers' wage bargaining power (Ogura (2017); Watanabe (2025)).

The decline in workers' wage bargaining power, or the strengthening of firms' bargaining power, has been incorporated into theoretical models. When assuming that firms have stronger bargaining power compared to workers (a monopsony market), it becomes possible to model wage markdowns, where wages fall below workers' productivity (Berger et al. (2022)). In this regard, recent empirical analyses using microdata have also reported the existence of wage markdowns (Yeh et al. (2022), Aoki et al. (2023)). Aoki et al. (2023) estimate wage markdowns and price markups in Japan and point out that while price markups have been shrinking, wage markdowns have been expanding. These estimation results suggest the possibility that wage markdowns have expanded in response to intensified international competition, which has made it difficult to raise product prices, as firms strive to secure their profits.

Based on these previous studies, in this paper, we focus on the decline in workers' bargaining power as a factor contributing to the downward pressure on the labor share of income. Specifically, using a theoretical model, we quantitatively analyze the extent to which the practice of prioritizing job security over wage increases in labor-management negotiations has contributed to the gap between real wages and labor productivity.

2.2 Downward Nominal Wage Rigidity

Secondly, driven by concerns that reducing nominal wages could diminish workers' motivation, downward nominal wage rigidity (DNWR) has been highlighted in previous studies conducted across various countries and regions (Elsby (2009)). In recent years, analyses have been conducted using large-scale microdata at the worker level. Grigsby et al. (2021) analyze individual worker wage data in the United States and point out that fluctuations in wage aggregates are largely due to changes in wage levels from job transitions and bonuses, while the basic wages of individual workers remain rigid. Furthermore, Branten et al. (2018) analyze post-GFC individual wage surveys in EU countries and highlight that nominal wages were not reduced during economic recessions. Regarding Japan, studies by Yamamoto (2007) and Kambayashi (2011), among others, identify the existence of nominal wage rigidity based on the distribution patterns of individual workers' wage growth rates. Yamamoto (2007) compares the degree of rigidity (the proportion of workers whose wages have remained unchanged) across different types of wages internationally and reports that while the rigidity of total annual earnings, including bonuses, is low in Japan, the basic salary of regular employees is more rigid compared to other countries.

When DNWR exists, wage increases may also be suppressed, leading to wage rigidity in both upward and downward directions - a phenomenon known as wage freeze. Using theoretical models that incorporate DNWR, Elsby (2009) and Daly and Hobijn (2014) highlight a mechanism in which the fear of being unable to reduce wages during future periods when wage cuts might be necessary results in firms restraining current wage increases. While empirical studies on upward nominal wage rigidity (UNWR) are limited, there are some studies that quantitatively examine UNWR for specific time periods. For instance, in Japan, Yamamoto and Kuroda (2016) and Hirata et al. (2020) define workers who did not experience wage reductions during past economic downturns as having faced DNWR, and they point out that, during subsequent periods of economic recovery, the wage growth rate for workers facing DNWR is comparatively smaller than that for workers who do not experience such rigidity.

Furthermore, regarding nominal wage rigidity, as pointed out as early as Tobin (1972), it has been noted that when inflation rates are high, real wages decline without the need to lower nominal wages, thereby weakening the impact of nominal wage rigidity (the "greasing-the wheels" effect of inflation). In this regard, recent empirical studies have also highlighted that in periods of low inflation, wage increases are more likely to be suppressed due to increased prospects of facing future downward rigidity (Schaefer and Singleton (2023)). In terms of the relationship between DNWR and inflation, recent analyses employing theoretical models that incorporate DNWR highlight that moderate levels of inflation can lead to improvements in social welfare (Aktug (2025), Mineyama (2022)).

This paper examines nominal wage rigidity in Japan from both empirical and theoretical perspectives. In the empirical section, it extends the friction model used by studies such as Fehr and Goette (2005) and Kuroda and Yamamoto (2003), which quantitatively analyze the degree of downward rigidity, into a framework that evaluates both upward and downward rigidities. With this framework, we investigate whether wage freeze has occurred in Japan, and examines the impact of the recent rise in inflation on rigidity. In the theoretical section, based on the partial equilibrium model of the labor market with DNWR proposed by Ehrlich and Montes (2024), a general equilibrium model with heterogeneous firms of varying productivity is constructed. Under conditions of nominal wage rigidity, we quantitatively analyze the impact of inflation rates on the gap between real wages and productivity.

2.3 Allocative Efficiency

Thirdly, this paper is also related to the analysis of the impact of resource allocation efficiency on productivity. Generally, the rise in macro-level productivity can be decomposed into the contributions of technological progress and improvements in the resource allocation of micro-agents. As for methods to measure the contribution of resource allocation to macro-level productivity growth, both statistical approaches and structural model approaches have been proposed (Goldin et al. (2024)).

In the statistical approach, macro-level productivity is decomposed into the within-firm effect, which reflects changes in productivity within individual firms, and the allocation effect, which reflects changes in the shares of firms with different productivity levels (e.g., Foster et al. (2001)). Analyses based on the statistical approach focusing on Japan have point out that the reallocation effect has significantly influenced productivity trends (Ikeuchi et al. (2018); Fukao et al. (2021)).

The structural model approach is a method to measure resource allocation inefficiencies arising from imperfect competition. A seminal study by Baqaee and Farhi (2020) quantifies inefficiencies due to imperfect competition through price markups, decomposing total factor productivity (TFP) growth into contributions from technological progress and from resource allocation effects. Aoki et al. (2024), in applying a similar analysis to Japan, report that, compared to the United States, Japan exhibits a smaller contribution from technological progress and a larger contribution from resource allocation effects.

Based on previous studies that suggest resource allocation effects contribute to macro-level productivity, this paper also examines the mechanism through which the reduction in nominal wage rigidity improves worker allocation and impacts macro-level productivity. We confirm that if the impact of nominal wage rigidity is alleviated by factors such as a rise in trend inflation, macro-level productivity can improve through the mobility of workers driven by wage signals.

3 Empirical Analysis

In this section, we provide empirical evidence that Japanese workers have faced both downward and upward nominal wage rigidity. In particular, we show that upward nominal wage rigidity is observed to be greater during the deflationary period in Japan, which is new to the literature. We start the empirical section with a review of the empirical literature on wage rigidity and show our contribution. Then we describe the details of the datasets and the empirical strategy. Finally, we present the results and discuss the implications of our findings.

3.1 Review on Empirical Analysis on Nominal Wage Rigidity

Since the 1990s, empirical studies leveraging worker-level microdata have proliferated across various countries, shedding light on the rigidity of nominal wages, particularly in the context of downward rigidity. Seminal works by Altonji and Devereux (1999) and Fehr and Goette (2005) use a friction model. This framework posits that potential wage adjustments are subject to threshold constraints. These studies document downward nominal wage rigidity (DNWR) in the United States and Switzerland. Likewise, Dickens et al. (2007) examines worker-level microdata from 16 Western European countries to analyze the distribution of wage changes, finding significant size of clustering around zero percent and concluding that DNWR is prevalent to varying extents across these nations. More recently, Grigsby et al. (2021) develops a comprehensive dataset based on U.S. administrative payroll data, revealing that nominal base wage declines among job-stayers are much rarer than previously thought.

On the other hand, empirical analyses that leverage microdata to investigate the upward nominal wage rigidity have been relatively less developed. Schaefer and Singleton (2023) provides evidence for upward rigidity by showing that, during periods of low price inflation in the United Kingdom, the upper tail of the wage growth distribution became compressed. Similarly, Christofides and Li (2005) analyzes worker-level microdata from Canada, using an estimation framework that incorporated an upper threshold into the friction model. They reveal the presence of upward thresholds in addition to downward ones.

Turning to studies focused on Japan, Kuroda and Yamamoto (2003) analyzes data from the 1990s using a friction model and identified evidence of downward rigidity in base salaries for regular employees. Additionally, Kambayashi (2011), employing an extended dataset covering the period from 1993 to 2006, finds concentration around zero percent in the distribution of base salary changes. They find that this concentration around zero percent becoming more pronounced in the late 1990s. With respect to studies focusing upward nominal wage rigidity (UNWR) in Japan, Yamamoto and Kuroda (2016) and Hirata et al. (2020) find that the wage growth rates of workers experiencing downward wage rigidity lagged behind those of workers unaffected by such constraints.[5]

We contribute to the literature as we conduct a quantitative analysis using worker-level micro-data spanning a long period, from 1990 to the most recent data available in 2024, which includes both deflationary and non-deflationary periods. With our data sets, we are able to examine whether the extent of upward and downward wage rigidity has changed over time. Furthermore, the analysis adopts an estimation framework that accounts for not only downward but also upward rigidity in nominal wages.

3.2 Data Used for the Analysis

This study uses survey data from the Basic Survey on Wage Structure (BSWS), conducted by the Ministry of Health, Labour and Welfare, covering the period from 1989 to 2024. The dataset consists of approximately 45.2 million worker-level samples in total. This survey provides comprehensive, long-term data on individual workers' wages and working hours, alongside demographic information such as gender, age, and educational background, as well as characteristics of the establishments, including industry sector and firm size. Our focus is on the scheduled wages of full-time workers.

The survey employed in this study is a repeated cross-sectional survey, not a panel survey. It draws new samples each year and lacks recorded identifiers to track individual workers across multiple survey periods. Therefore, calculating worker-level wage growth rates using this survey requires the identification of the same workers across survey years. Following the methodology outlined by Kambayashi (2011) and Date et al. (2024), this study matches samples that are assumed to correspond to the same individuals based on workplace, gender, educational background, age, and tenure. This matching process enables us to construct a pseudo-panel dataset.

In detail, the same workplace was identified by matching establishments with identical prefectures, municipalities, basic survey areas, and establishment numbers between the previous year and the current year. It is important to note that in the years when the sampling frame of the BSWS was updated, the continuity of these identifiers was not always preserved. However, the linking of identifiers was facilitated by using corresponding tables between the old and new surveys, obtained from sources such as the Economic Census provided by the Ministry of Internal Affairs and Communications. Nonetheless, for the periods 1992-1993, 1995-1996, and 1997-1998, corresponding tables for the old and new surveys were unavailable. Therefore, in this study, connections for these years

[5] They define workers who do not experience wage reductions during economic downturns as facing downward rigidity.

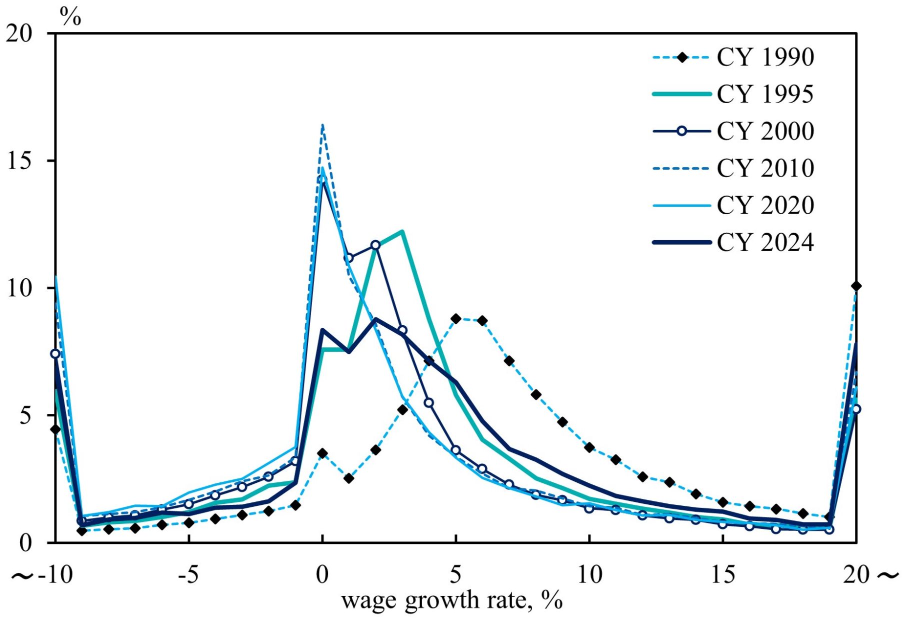

Figure 2: Distributions of nominal wage growth over the last three decades

could not be established, and the wage growth rates for the years 1993, 1996, and 1998 are excluded from the dataset.

Subsequently, among the worker samples extracted from identified matching establishments, those with matching gender and educational background between the previous year and the current year, and whose age and tenure had both increased by one year from the previous year, were identified and linked as the same worker. To minimize the likelihood of erroneously linking samples of different individuals, observations were excluded if multiple workers within the same establishment in either the previous or current year shared identical gender, educational background, age, and tenure. Furthermore, workers were excluded from the analysis if their wage growth rates or changes in working hours fell below -35% or exceeded +60%. Consequently, wage growth rates were calculated for a total of approximately 3.6 million samples, excluding the survey years 1993, 1996, and 1998. These calculated wage growth rates are illustrated in Figure 2.

Examining the distribution of wage growth rates in Figure 2, it is observed that since the 2000s, when price inflation stagnated, the distribution of wage growth rates shifted leftward and became concentrated around zero percent. This suggests the presence of workers facing nominal wage rigidity. Subsequently, throughout the 2010s, the concentration of wage growth rates around zero percent persisted without significant changes. However, from 2020 to the most recent data of 2024, while the overall wage growth rate distribution shifted to the right, the proportion around zero percent decreased, indicating a potential change in the extent of nominal wage rigidity. In the following sections, the degree of nominal wage rigidity and its potential changes will be analyzed using quantitative methods.

3.3 Empirical Analysis Framework

In this section, a quantitative analysis will be conducted using the constructed dataset of wage growth rates to examine the rigidity of wage growth rates, as well as potential changes in these rigidities over time.

Specifically, drawing on studies such as Christofides and Li (2005), this paper conducts an estimation using a friction model with upper and lower thresholds. The friction model assumes the existence of a potential wage growth rate, which is determined by the attributes of workers and firms. However, the observed wage growth rate is constrained by both upper and lower thresholds, as defined by (1) and (2).

$$\Delta w_i^* = x_i'\beta + \varepsilon_i, \quad \varepsilon_i \sim N\left(0, \tau^2\right) \tag{1}$$

$$\Delta w_i = \begin{cases} \Delta w_i^* - \theta_t^+ & \text{if} \quad \theta_t^+ \leq \quad \Delta w_i^* \\ 0 & \text{if} \quad \theta_t^- < \quad \Delta w_i^* < \theta_t^+ \\ \Delta w_i^* - \theta_t^- & \text{if} \quad \qquad\quad \Delta w_i^* \leq \theta_t^- \end{cases} \tag{2}$$

In (1), the dependent variable Δw_i^* represents the potential wage growth rate, and i denotes the worker. Regarding the explanatory variables x_i, they consist of data on corporate profitability (year-over-year changes in ordinary profits for the industry and firm size to which worker i belongs), inflation rate (prefecture-level consumer price index excluding fresh food), labor market conditions (Employment Condition DI by industry and company size), age, a dummy variable for workers aged 60 and over, working hours, a gender dummy, an education dummy, an external labor market dummy, and dummy variables indicating industry and firm size.[6]

Equation (2) illustrates the relationship between the potential wage growth rate Δw_i^* and the observed wage growth rate Δw_i. Specifically, it is assumed that when the potential wage growth rate falls within the threshold range (θ_t^-, θ_t^+), the observed wage growth rate is zero, meaning wages remain unchanged. On the other hand, if the potential wage growth rate exceeds the thresholds, only the portion exceeding the specified thresholds is observed as the actual wage growth rate. Figure 3 illustrates the relationship between the two. Moreover, the upper threshold θ_t^+ and lower threshold θ_t^- were estimated while allowing them to vary across different periods: (i) 1990-1994, (ii) 1995-2003, (iii) 2004-2012, (iv) 2013-2019, and (v) 2020-2024. The estimation method is based on the Bayesian estimation approach for Tobit models proposed by Chib (1992), and the estimation technique proposed by Omori and Miyawaki (2010) was adopted.

[6] Using a finite mixture model, as in Date et al. (2024), full-time workers are categorized into two classes with different wage structures. Date et al. (2024) confirm that those two classes correspond to external labor market class, characterized by a relatively flat wage curve as indicated in previous studies, and internal labor market class.

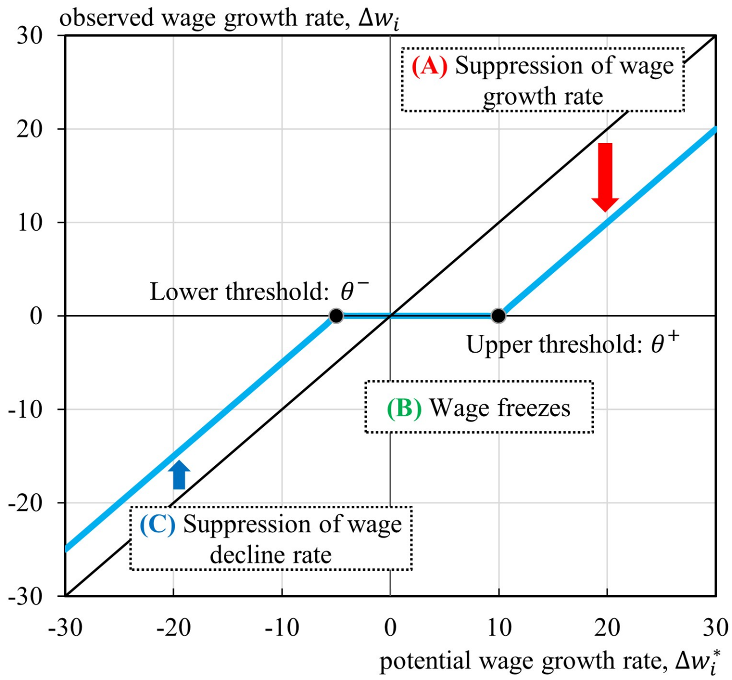

Figure 3: Conceptual diagram illustrating observed and potential wage growth

3.4 Results of the Empirical Analysis

First, the determinants of the potential wage growth rate, as estimated from (1), were examined in Table 1. Among the macroeconomic factors, it was observed that improvements in corporate profitability, higher inflation rates, and tighter labor market conditions contributed to increases in the potential wage growth rate. With regard to worker attributes, the findings indicate that younger age, longer scheduled working hours, and being female were associated with significant increases in wage growth rates. Regarding educational background, the results suggest that holding a university or graduate degree positively influenced wage growth rates. Furthermore, it was confirmed that workers affiliated with the external labor market experienced significant downward pressure on their wage growth rates.

In this study, it is assumed that the mechanism governing potential wage determination remains stable over the estimation period. However, the observed wage growth rate is subject to upward and downward thresholds, which vary across different periods. As shown in Figure 4, the estimated thresholds for each period show different trends for the upper and lower bounds. The lower threshold remained relatively steady, experiencing only minor fluctuations across all periods. In contrast, the upper threshold exhibited a noticeable upward trend from the early 1990s (Period I) to the early 2000s (Period III), followed by a modest decline as the economy transitioned into the early 2010s (Period IV) and subsequently into the early 2020s (Period V).

The proportions of workers facing upward and downward rigidity are shown in Figure 5. Here,

Table 1: Estimated factors affecting the potential wage

Variable	Median	95% CI
Ordinary profit ratio (y/y change, by firm industry and size)	0.09	[0.08, 0.09]
CPI inflation rate (by prefecture, excluding fresh foods)	0.64	[0.62, 0.65]
Employment conditions DI (by firm industry and size)	−0.03	[−0.03, −0.03]
Age	−0.11	[−0.12, −0.11]
Aged 60 or above	−0.58	[−0.64, −0.53]
Scheduled working hours (y/y growth rate)	0.13	[0.13, 0.13]
Female	0.17	[0.14, 0.20]
Senior high school graduates	0.00	[−0.04, 0.04]
Junior college graduates and above	0.00	[−0.06, 0.05]
University graduates and above	0.21	[0.16, 0.26]
External labor market	−1.24	[−1.28, −1.21]
Constant	4.88	[4.78, 4.97]

NOTE: The table reports median estimates and 95% credible intervals.

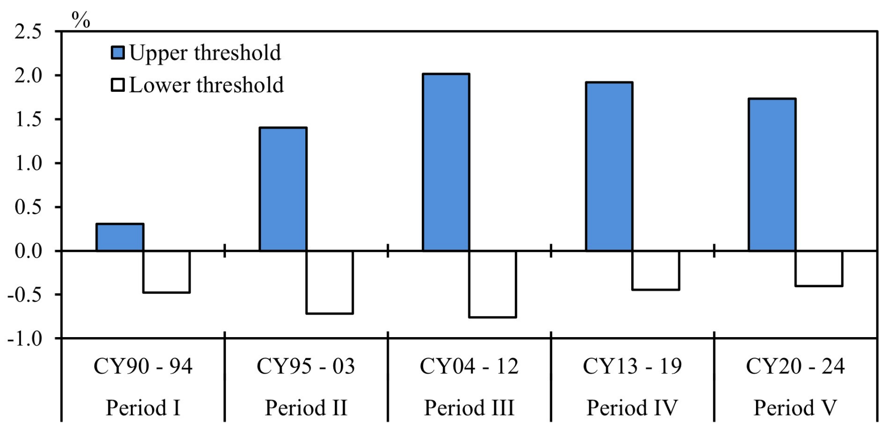

Figure 4: Estimation results of upper and lower thresholds

NOTE: The figures are medians of the posterior distributions.

based on the estimated thresholds, workers experiencing upward rigidity were defined as those whose potential wage growth rates were positive, but whose wages did not increase, while workers experiencing downward rigidity were defined as those whose potential wage growth rates were negative, but whose wages remained unchanged. In the early 1990s (Period I), the proportion of workers encountering either form of wage rigidity was less than 1%, indicating that potential wage growth rates were largely reflected in actual wage adjustments during this period. However, the proportion of workers experiencing rigidity increased steadily, peaking in the early 2000s (Period III) at 7% for upward rigidity and 4% for downward rigidity.

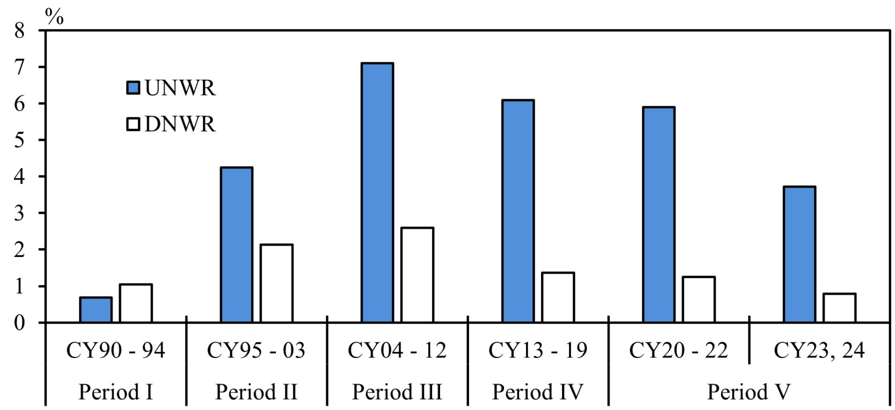

Figure 5: Proportion of workers facing upward and downward rigidity
NOTE: Workers facing UNWR (DNWR) are defined as those whose potential wage growth rate is positive (negative), but whose observed wages remain unchanged.

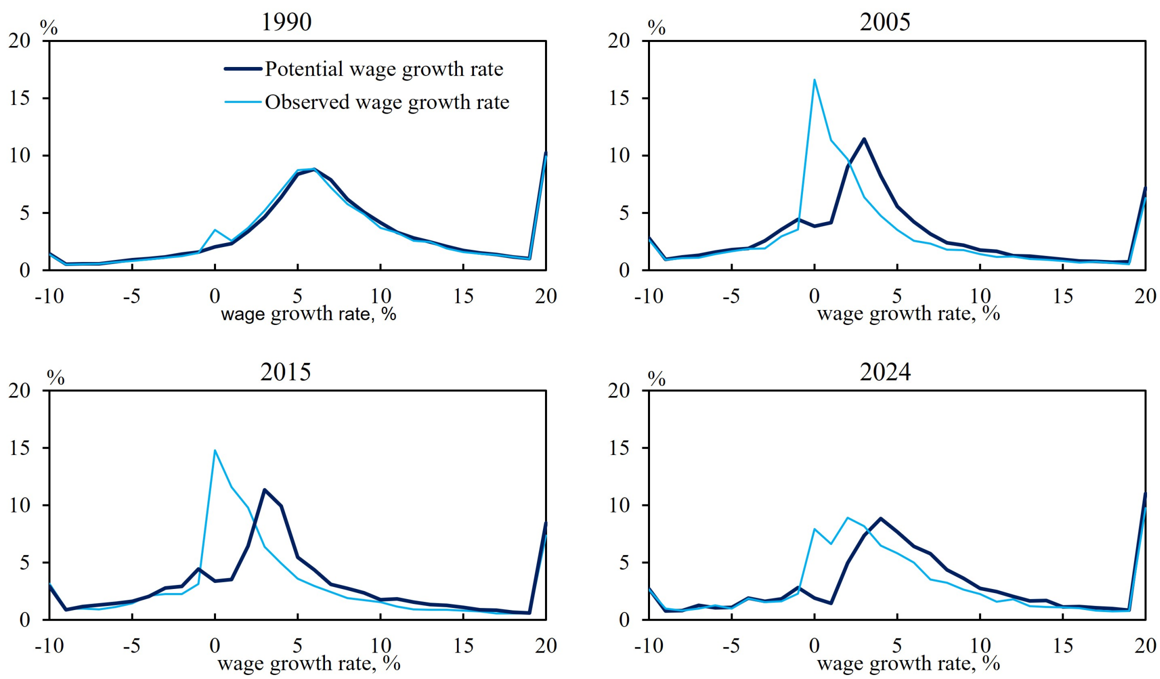

Figure 6: Comparison of potential and observed wage growth distributions

3.5 Comparison Between Observed Data and Estimated Potential Wages

In this section, the potential wage growth rate for each worker is calculated based on the estimation results derived from the aforementioned framework. Then, the distribution of potential wage growth rates is constructed and compared to the distribution of observed wage growth rates, as illustrated in Figure 6.

First, in 1990, the influence of both the upper and lower estimated thresholds was minimal, resulting in the distributions of potential and observed wage growth rates being approximately similar. Next, focusing on 2005 and 2015 – years within the deflationary period – the following key trends were identified: (i) an expansion of the upper threshold, and (ii) a decline in potential wage growth rates, driven by factors such as decreasing inflation rates. Consequently, the distribution of observed wage growth rates shifted leftward relative to the distribution of potential wage growth rates, forming a pronounced peak near zero percent.

For 2024, while the thresholds themselves remained relatively unchanged, rising inflation rates led to an increase in potential wage growth rates. As a result, the proportion of workers experiencing upward wage rigidity declined, narrowing the gap between potential wage growth rates and observed wage growth rates. This trend indicates a mitigated impact of nominal wage rigidity.

The results of the empirical analyses suggest the following key findings. First, the analysis quantitatively confirms that a significant proportion of workers in Japan have experienced nominal wage rigidity, both upward and downward. Second, the influence of upward wage rigidity intensified notably after the late 1990s, coinciding with the onset of deflation in the Japanese economy. During this period, observed wage growth rates consistently fell below potential wage growth rates. Third, in the most recent period since 2020, the impact of wage rigidity appears to have eased. This mitigation is likely attributable to rising inflation rates and other contributing factors, which have collectively driven an increase in potential wage growth rates.

4 Theoretical Framework

4.1 Model

In our theoretical analysis, to investigate the factors behind the gap between productivity and wages, we construct a framework that incorporates not only the empirically observed downward nominal wage rigidity (DNWR) but also the trade-off between wage increases and job security in labor-management negotiations. When examining macro-level productivity, the effects of resource allocation among firms also become crucial. Thus, we extend the standard New Keynesian model by introducing firms with heterogeneous productivity levels following Ehrlich and Montes (2024).

The model features two distinct margins of employment adjustment: retention of incumbent workers through wage-setting, and hiring of new workers through vacancy posting. DNWR makes wage-setting history-dependent and shifts the relative cost of using these margins. These mechanisms jointly determine the effective marginal cost of labor and the endogenous wage markdown, which are the focus of the subsequent analysis.

The economy consists of heterogeneous firms that produce intermediate goods and face idiosyncratic productivity shocks, wholesale firms that supply these intermediate goods monopolistically, and firms that combine the intermediate goods to produce final goods. Workers supply labor to firms producing intermediate goods and consume final goods. The remaining agents are the government and the central bank. The government conducts lump-sum taxation or transfers, while the central bank adjusts the nominal interest rate in response to the inflation rate. Time is discrete and indexed by $t = 0, 1, 2, \ldots$, and the unit of the time horizon for the model is annual.

4.1.1 Heterogeneous intermediate-goods producing firms

Intermediate goods-producing firms hire workers to conduct production. The only input is labor, and it is assumed that each worker supplies a fixed unit of labor. Thus, neither working hours nor capital is considered in the analysis.

There is a unit measure of intermediate-goods producing firms indexed by $j \in [0, 1]$. Productivity differs across firms, and the index j represents an individual firm. Given aggregate prices and quantities $\{P_t^w, W_t, \pi_t, u_t\}$, firm j chooses its current real wage $w_{j,t}$ and vacancies $v_{j,t}$ to maximize the present discounted value of profits. Firms sell their output at a relative price P_t^w, which corresponds to the real price of intermediate goods faced by wholesale producers. The Bellman equation for firm j is as follows:

$$V_{j,t}\left(a_{j,t}, w_{j,t-1}, n_{j,t-1}\right) = \max_{w_{j,t}, v_{j,t}} \Pi_{j,t} + \beta \mathbb{E}_t\left[V_{j,t+1}\left(a_{j,t+1}, w_{j,t}, n_{j,t}\right)\right]. \tag{3}$$

The firm's state variables are current productivity $a_{j,t}$, last period's wage $w_{j,t-1}$, and last period's employment $n_{j,t-1}$.

The firm's period-t profit is given by:

$$\Pi_{j,t} = P_t^w y_{j,t} - w_{j,t} n_{j,t} - \underbrace{\mathcal{D}\left(w_{j,t}, w_{j,t-1}\right) n_{j,t}}_{\text{DNWR}} - \underbrace{\mathcal{C}\left(v_{j,t}, n_{j,t-1}\right)}_{\text{Vacancy-posting cost}}. \tag{4}$$

Revenue equals output multiplied by the relative price P_t^w. Costs consist of the wage bill, downward nominal wage adjustment costs that are proportional to current employment, and vacancy posting costs that depend on current vacancies and past employment.

Production function Firm j produces output using labor as the sole input according to

$$y_{j,t} = a_{j,t} n_{j,t}^{\alpha}, \tag{5}$$

where $y_{j,t}$ denotes firm-level output, $n_{j,t}$ denotes employment, and $\alpha \in (0, 1)$ implies decreasing returns to scale. This assumption ensures an interior firm size distribution and prevents unbounded expansion of highly productive firms.

Firm-level productivity $a_{j,t}$ consists of an aggregate component and an idiosyncratic component.

Specifically, the logarithm of productivity satisfies

$$
\begin{aligned}
\log a_{j,t} &= \log z_t + \log u_{j,t}, \\
\log z_t &= \rho^z \log z_{t-1} + \varepsilon^z_t, \quad \varepsilon^z_t \sim N(0, \sigma^2_z), \\
\log u_{j,t} &= (1-\rho^u) \log \bar{u} + \rho^u \log u_{j,t-1} + \varepsilon^u_{j,t}, \quad \varepsilon^u_{j,t} \sim N(0, \sigma^2_u).
\end{aligned}
\tag{6}
$$

Here, z_t captures aggregate productivity common to all firms, while $u_{j,t}$ captures firm-specific productivity. The parameters ρ^z and ρ^u govern persistence, and ε^z_t and $\varepsilon^u_{j,t}$ are mean-zero innovations.

Downward nominal wage rigidity Following Ehrlich and Montes (2024), DNWR is introduced through an asymmetric adjustment cost:

$$
\mathcal{D}\left(w_{j,t}, w_{j,t-1}\right) = \left[\lambda \left(\frac{w_{j,t-1}}{w_{j,t}} - (1+\pi_t)\right)\right] \mathbb{1}_{(1+\pi_t)w_{j,t} < w_{j,t-1}}. \tag{7}
$$

where π_t denotes the inflation rate and $\lambda \geq 0$ governs the severity of DNWR. The indicator function equals one whenever the firm attempts to cut the nominal wage. In such cases, the firm incurs an additional cost proportional to the size of the wage cut. Higher inflation allows real wages to decline mechanically even if nominal wages are unchanged, thereby relaxing the effective rigidity.

Vacancy-posting cost Following Bilal et al. (2022) and Bloesch et al. (2024), we assume:

$$
\mathcal{C}(v_{j,t}, n_{j,t-1}) = \frac{c^v}{1+\chi} \left(\frac{v_{j,t}}{n_{j,t-1}}\right)^{\chi} v_{j,t}. \tag{8}
$$

where $c^v > 0$ scales the level of hiring costs. $\chi \geq 0$ determines the degree of convexity of vacancy-posting costs with respect to the number of vacancies. Hence, per-vacancy costs increase with the ratio of vacancies to incumbent workers, $v_{j,t}/n_{j,t-1}$. This formulation implies that hiring is more costly, in marginal terms, for firms that expand rapidly relative to their existing workforce.

Firms' employment The law of motion for firm j's employment is given by:

$$
n_{j,t} = \left[1 - \delta\left(\frac{w_{j,t}}{W_t}\right)\right] n_{j,t-1} + m_{j,t}. \tag{9}
$$

The first term represents workers retained from the previous period, while the second term represents newly hired workers. The separation rate $\delta(\cdot)$ depends on the firm's wage relative to the aggregate wage index W_t, so that workers are more likely to leave firms that pay relatively low wages.

The separation rate is defined as:

$$\delta\left(\frac{w_{j,t}}{W_t}\right) = \bar{\delta}\left(\frac{w_{j,t}}{W_t}\right)^{-\epsilon^{\delta}}, \tag{10}$$

where $\bar{\delta}$ denotes the average separation rate. The parameter ϵ^{δ} is the elasticity of separations with respect to a firm's relative wage. A higher value makes separations more wage-sensitive and therefore corresponds to a lower attachment to job security, a more elastic labor supply to the firm, and weaker monopsony power.

Matching function in the labor market New matches are formed through a matching technology

$$m_{j,t} = v_{j,t}^{1-\mu} u_{t-1}^{\mu}, \tag{11}$$

where $m_{j,t}$ denotes effective annual hires contributing to the firm's end-of-period employment. $v_{j,t}$ denotes the number of vacancies posted by firm j and u_{t-1} denotes aggregate unemployment. The parameter $\mu \in (0,1)$ controls the elasticity of matches with respect to unemployment.

Because the model is formulated at an annual frequency, we interpret the matching technology as a time-aggregated hiring technology. Firms choose their recruiting intensity at the beginning of each year and maintain their recruiting activities throughout the year, while worker separations, unemployment spells, and re-employment occur at a higher frequency.[7]

4.1.2 Workers

The economy is populated by a representative household (worker) consisting of a continuum of members of measure one. In each period, a member is either employed or unemployed. An employed member supplies one unit of labor inelastically, while an unemployed member supplies no labor. The household provides perfect consumption insurance across its members, so that consumption is identical for all individuals, regardless of their employment status.

Preferences The preferences of the representative household are defined as the equally weighted average of its members' utilities (CRRA):

$$U_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s u(C_{t+s}). \tag{12}$$

where C_t denotes aggregate consumption, and $\beta \in (0,1)$ is the discount factor.

[7] Under this interpretation, u_{t-1} represents the stock of job seekers available to recruiting firms during the year, rather than a fixed cohort of workers who can be matched only once. $m_{j,t}$ denotes effective annual hires that contribute to end-of-year employment, thereby incorporating the time aggregation of within-year hiring and separation flows. Aggregate annual hires may therefore exceed the unemployment stock measured at a particular point in time.

Budget constraint The household can save in risk-free bonds and receives profits from firms, Ψ_t:

$$P_t C_t + \frac{B_t}{R_t} = \int_0^1 P_t w_{j,t} n_{j,t} dj + B_{t-1} + \Psi_t. \tag{13}$$

Here, P_t is the aggregate price level of the final good, B_t denotes nominal holdings of a one-period risk-free bond chosen at time t, and R_t is the gross nominal interest rate. Labor income is obtained from firms, where $w_{j,t}$ denotes the real wage paid by firm j per worker and $n_{j,t}$ denotes firm j's employment level. The term Ψ_t represents aggregate firm profits rebated lump-sum to the household.

4.1.3 Price Setting; final goods producing firms and wholesale firms

Final-good prices are set by monopolistically competitive wholesale firms subject to Rotemberg adjustment costs. Monopolistic wholesale goods firm i chooses its price $p_t(i)$ to maximize the sequence of discounted future profits:

$$\mathbb{E}_t \left[\sum_{s=t}^{\infty} q_s \left(\frac{p_s(i)}{P_s} y_s(i) - P_s^w y_s(i) - AC_s(i) \right) \right], \tag{14}$$

where q_s is the real stochastic discount factor of households. The adjustment cost is:

$$AC_t(i) = \frac{\varphi}{2} \left[\frac{p_t(i)}{p_{t-1}(i)} - (1+\pi) \right]^2 \frac{p_t(i)}{P_t} y_t(i). \tag{15}$$

where $\varphi > 0$ governs price rigidity, and π denotes steady-state inflation. The final good is aggregated according to a CES technology:

$$Y_t = \left(\int_0^1 y_t(i)^{\frac{\epsilon^p - 1}{\epsilon^p}} di \right)^{\frac{\epsilon^p}{\epsilon^p - 1}}. \tag{16}$$

where $\epsilon^p > 1$ is the elasticity of substitution across varieties.

The New Keynesian Phillips Curve (NKPC) in this economy is given by:

$$\pi_t = \beta \mathbb{E}_t [\pi_{t+1}] + \frac{\epsilon^p - 1}{\varphi (1+\pi)^2} p_t^w. \tag{17}$$

where π_t and p_t^w denote deviations of inflation and real marginal cost from their steady-state values, respectively.

4.1.4 Central bank

Monetary policy follows a Taylor rule:

$$r_t = \rho^{mp} r_{t-1} + (1 - \rho^{mp})\phi_\pi \pi_t. \tag{18}$$

where $\widehat{r}_t$ denotes the deviation of the policy rate from its steady-state value, $\rho^{mp} \in [0, 1)$ captures interest-rate smoothing, and $\phi_\pi > 0$ governs the response to inflation.

4.1.5 Equilibrium definition

Equilibrium System. *Given a sequence of productivity shocks $\{\epsilon_t^z\}$ and an initial firm distribution $\Phi_0(a, w, n)$, a competitive equilibrium is a sequence of policies $\{w(a, w_-, n_-), v(a, w_-, n_-)\}$ for intermediate-goods-producing firms, the employment law of motion implied by these policies, distributions $\Phi_t(a, w_-, n_-)$, prices $\{P_t, P_t^W, W_t, R_t\}$, and aggregate quantities $\{Y_t, N_t, C_t\}$ such that all workers optimize, all firms optimize, and the goods market clears.*

4.2 The Mechanism of Wage Freeze Driven by DNWR

Figure 7 illustrates the policy function for wage setting in the current period for heterogeneous firms. The case where DNWR exists ($\lambda > 0$) is represented by the blue surface, while the case without DNWR ($\lambda = 0$) is shown by the yellow surface. Comparing the left side of the figure (firms with low wages in the previous period), it can be observed that wage levels are lower when DNWR exists than when it does not. This indicates that, in the presence of DNWR, firms have an incentive not to raise wages in the current period due to the high costs associated with reducing wages in the future. In other words, the existence of DNWR indicates the emergence of upward nominal wage rigidity, making it more difficult to raise wages in the current period.

Conversely, comparing the right side of the figure (firms with high wages in the previous period), it can be observed that wage levels are higher when DNWR exists than when it does not. This is because, in the presence of DNWR, there are costs associated with reducing the previous period's wages in the current period. As a result, current wages remain higher than in the case without DNWR, reflecting the difficulty of reducing nominal wages.

Figure 8 illustrates the policy functions for wage growth in cases where DNWR exists and does not exist. When DNWR is present, it can be observed that some firms face a *wage freeze*, where they are unable to either raise or lower wages.

Furthermore, if we consider the case with DNWR as the observed wages and the case without DNWR as the potential wages, a relationship between the two can be identified, as shown on the right in Figure 8. In other words, the theoretically derived policy functions for wage growth are consistent with the assumptions made in the empirical analysis.

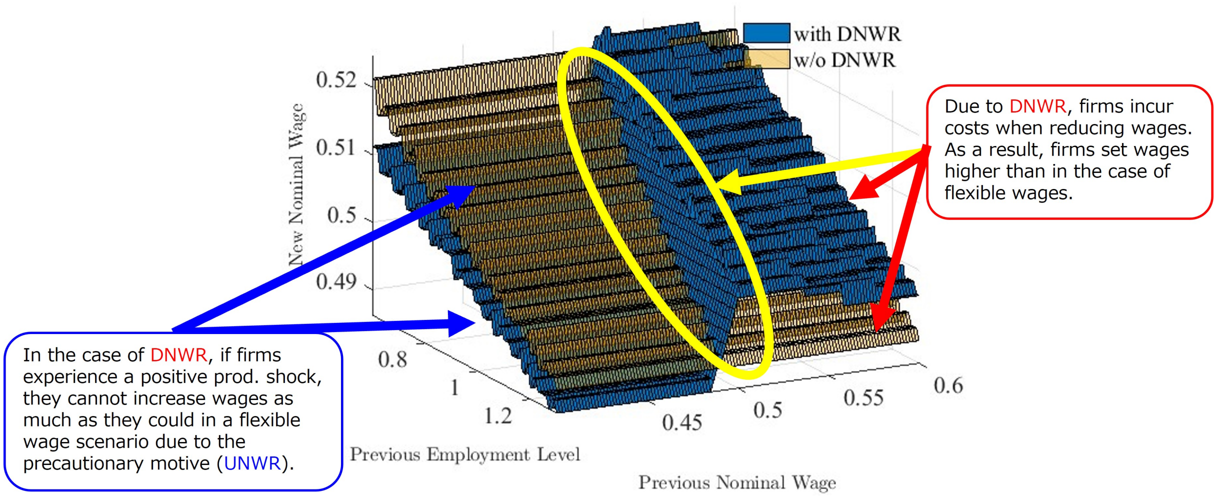

Figure 7: Firms' policy functions for wages in the next period

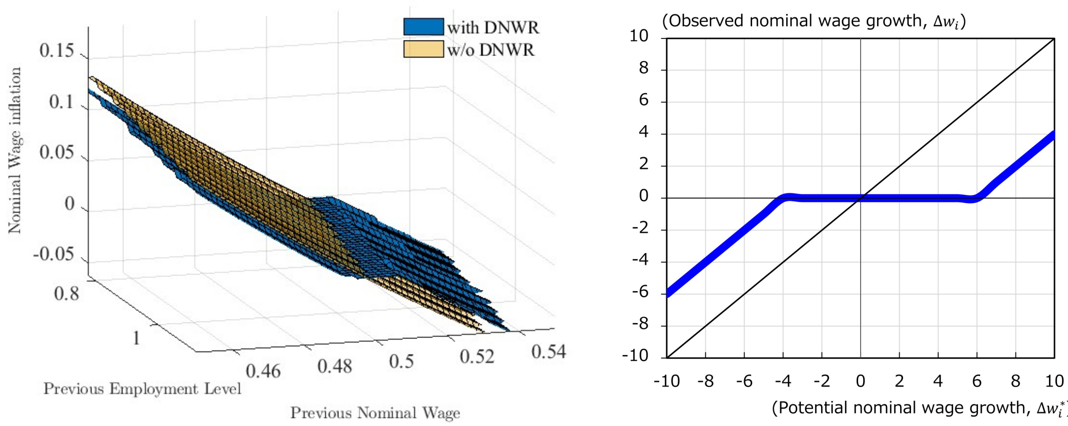

Figure 8: Potential and observed nominal wage growth

4.3 Calibration

This section describes the calibration of the model parameters. The parameters can be grouped into two broad categories: (i) parameters whose values are taken from the literature or empirical data, and (ii) key parameters that are calibrated to align the model with the data. The latter govern labor market frictions, particularly those related to separations and wage-setting behaviors, including mechanisms governing DNWR.

4.3.1 Parameter calibration from the literature

Table 2 summarizes the baseline parameter values and their empirical or theoretical justifications.

Preferences and production The discount factor β is chosen to be consistent with a growth rate and an annual real interest rate. Production depends solely on labor and features decreasing

Table 2: *Calibrated* parameters

Parameter		Value	Reason
Heterogeneous firms			
Discount rate	β	0.99	$\beta = 1/(1+\bar{r})$
Returns to scale in production	α	0.70	Ehrlich and Montes (2024)
Hiring cost constant	c^v	0.25	Match unemployment rate in Japan
Posting vacancy parameter	χ	3.45	Bilal et al. (2022)
Average separation rate	$\bar{\delta}$	0.15	Separation rate in Japan excl. retirement
Persistence of individual prod. shock	ρ^u	0.388	Ehrlich and Montes (2024)
S.D. of individual prod. shock	σ^u	0.419	Ehrlich and Montes (2024)
Elasticity of matches w.r.t. job search	μ	0.5	Petrongolo and Pissarides (2001)
Workers, Wholesale firms, and Final goods producing firms			
Relative risk aversion	σ	1	Christiano et al. (2005)
Rotemberg adjustment cost	φ	10	≈ 13.85 in Fueki et al. (2018)
Retail price elasticity	ϵ^p	6.0	Aoki et al. (2023), $\epsilon^p/(\epsilon^p - 1) = 1.2$
Monetary policy			
Taylor rule coefficient	ϕ^π	2.0	standard
Persistence of monetary shock	ρ^{mp}	0.65	0.9^4

returns to scale. The assumption of decreasing returns ensures an interior labor demand solution and stabilizes the firm's dynamic wage and employment problem, as in Ehrlich and Montes (2024).

Labor market The hiring cost constant, c^v, is calibrated so that the steady-state unemployment rate aligns with the actual unemployment rate observed in Japan. The degree of convexity of vacancy-posting cost, χ, adopts the estimated value from Bilal et al. (2022). The average separation rate is set to be consistent with the corresponding data for Japan. The elasticity of matching is set at the standard value of 0.5, as commonly used in the literature (Petrongolo and Pissarides (2001)).

4.3.2 Calibration on λ and ϵ^δ

Under the baseline calibration, the model jointly matches key labor market moments, including the distributions of wage changes, separation rates, and hiring rates. In particular, the parameters governing DNWR, λ, play a central role in shaping the composition of employment adjustments across wages and separations. The quantitative exercises that follow use this calibrated model to evaluate how changes in wage rigidity affect firm-level employment dynamics and aggregate labor market outcomes.

Downward nominal wage rigidity, λ DNWR is introduced through an asymmetric wage adjustment cost. Firms incur a cost only when cutting nominal wages, and the cost is proportional to the size of the nominal wage cut. The parameter governing this cost is calibrated using the distribution

Table 3: *Calibrated* parameters

Parameter		Value
Degree of DNWR	λ	0.9
Separation elasticity in 1997	$\epsilon^{\delta,1997}$	1.6
Separation elasticity in 2003	$\epsilon^{\delta,2003}$	1.4
Separation elasticity in 2024	$\epsilon^{\delta,2024}$	1.4

of wage changes.

Specifically, we simulate wage changes from the model and apply the same kernel-density-based procedure used in the data to measure the fraction of nominal wage cuts that are prevented. The DNWR parameters are chosen such that the model-implied measure of prevented wage cuts matches its empirical counterpart. This indirect calibration strategy ensures that the model reproduces the observed asymmetry in the wage change distribution.

Separation elasticity, ϵ^{δ} Worker separations arise endogenously through a wage-dependent separation rate. The scale parameter of the separation function is calibrated such that the steady-state separation rate matches the average annual separation rate observed in the data. The elasticity of separations with respect to wages, ϵ^{δ}, governs the degree of monopsony power in the labor market. A higher value of ϵ^{δ} implies that worker flows are more sensitive to wages and that firms face a more competitive labor market.

Calibration strategy Choose λ and ϵ^{δ} to match the distribution of nominal wage growth between the data and the model.

We perform a grid search over 31 grid points for parameter λ (ranging from 0.0 to 3.0) and 40 grid points for parameter $\epsilon^{\delta,T}$ (ranging from 0.1 to 4.0), and select the parameter set that minimizes:

$$\min_{\lambda,\{\epsilon^{\delta,T}\}} \left[\frac{\Sigma_T \left\| \mathcal{F}(0 \widehat{< x <} 0.01) - \mathcal{F}(0 < x < 0.01) \right\|}{\text{number of T}} + \Sigma_T \left\| \mathcal{F}\widehat{(x > 0)} - \mathcal{F}(x > 0) \right\| \right] \tag{19}$$

where $\mathcal{F}(\cdot)$ denotes the CDF of nominal wage growth.

Calibration results Figure 9 presents comparisons between the distributions of nominal wage growth as predicted by the model and observed in the data. Table 3 shows the calibrated parameters for λ and ϵ^{δ} for the selected time periods. Our calibrated values for ϵ^{δ} are consistent with the estimates reported by Ehrlich and Montes (2024), which are based on micro-level evidence in Germany regarding wage-separation elasticity. Moreover, we observe that the model-implied aggregate gap between MRPL and wages has widened over time, which aligns with the findings of Aoki et al. (2023).

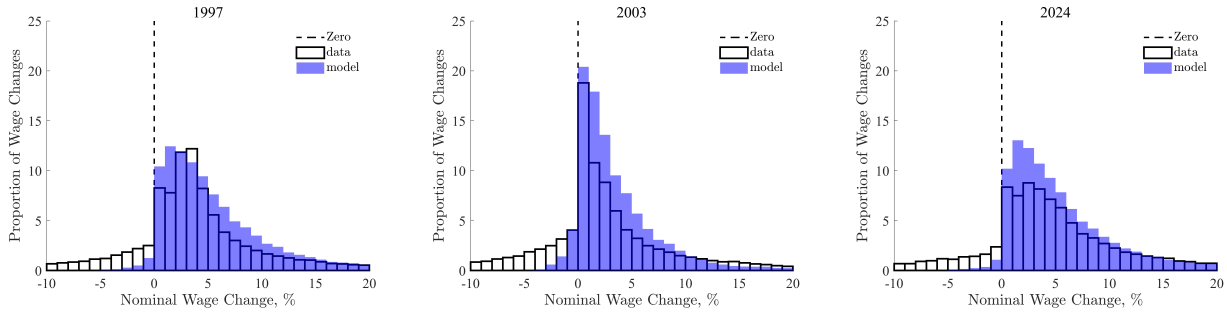

Figure 9: Comparison of nominal wage growth distributions between observed data and the model

5 Wage Markdown and Allocative Efficiency

In this section, we analyze how an increase in trend inflation and an increase in the elasticity of separations affect aggregate wages and productivity. First, in order to gain insight into the effects of π and ϵ^{δ}, we consider another one-period partial equilibrium model in which the firm's problem is partially simplified from the full model presented in the previous section. Next, we present the results based on the full general equilibrium model.

5.1 Analysis Using a One-period Model with Two Types of Firms

Model environment. Here, we assume that firms are divided into two types: high-productivity firms and low-productivity firms. Since firms survive for only one period, we do not consider future firm value and instead study a one-period profit maximization problem.

All parameters other than trend inflation and the elasticity of separations are set as follows. The production function exhibits constant returns to scale ($\alpha = 1$). The scale parameter of the vacancy-posting cost function is set to $c^v = 0.25$; the parameter of the matching function is set to $\mu = 0.5$. The unemployment rate u is assumed to be exogenously given ($u = 0.03$). The price of intermediate goods is normalized to one.

Regarding the wage adjustment costs faced by firms, it is important to note that these costs in the full general equilibrium model are non-linear and kinked. Specifically, firms incur adjustment costs only when they reduce nominal wages. Moreover, the impact of these costs on firms' present and future values depends on the future path of idiosyncratic shocks. For the purpose of one-period theoretical analysis, we approximate the original non-linear function to incorporate the above mechanisms. Specifically, we consider the following nominal wage adjustment cost:

$$\mathcal{D}_j^T\left(w_{j,t}\right) = \lambda_1 \eta \log\left(1 + \exp\left(\frac{w_j / w_{j,t} - (1+\pi)}{\eta}\right)\right) + \lambda_{2,j}, \tag{20}$$

where w_j denotes the steady state level of the real wage at firm j, $\eta > 0$ governs the degree of smoothness, and $\lambda_{2,j}$ is a firm-specific constant chosen to satisfy the normalization:

$$\mathcal{D}_j^T(w_j) = \frac{\lambda_j}{1+\pi}. \tag{21}$$

This adjustment cost indicates that the magnitude of the wage adjustment cost is governed by λ_j, and it diminishes as trend inflation increases. This approximation allows us to reflect the forward-looking nature of wage setting. Firms anticipate the possibility that future wage reductions may become binding. Even when the constraint is not currently active, the option value of avoiding future costly adjustments generates a strictly positive adjustment cost. Thus, the smooth specification can be interpreted as incorporating expected future wage adjustment costs into current decisions. This approximation ensures that the model's outcome remains consistent with the results produced by the full quantitative general equilibrium model, as discussed in a later section.[8]

5.1.1 Allocation of labor across firms and wage markdown

Let employment at firm j be denoted by n_j and its wage by w_j. Define the relative employment and relative wage of the high-productivity firm (denoted by H) with respect to the low-productivity firm (denoted by L) as follows:

$$r \equiv \frac{n_H}{n_L}, \quad q \equiv \frac{w_H}{w_L}. \tag{22}$$

Under these definitions, the following relationship holds between relative employment and relative wages:[9]

$$q = r^{\frac{1+\chi}{1+2\epsilon^\delta(1+\chi)}}. \tag{23}$$

An increase in the relative wage of the high-productivity firm implies an increase in its relative employment. Moreover, when the elasticity of separations rises, relative employment at the high-productivity firm becomes more responsive.

Under an approximation that $\frac{1+\epsilon^\delta \delta_L(r)}{1+\epsilon^\delta \delta_H(r)} \approx 1$, relative employment and relative wages satisfy:

$$r \approx \left(\frac{a_H - \frac{\lambda_H}{1+\pi}}{a_L - \frac{\lambda_L}{1+\pi}} \right)^{\kappa_r(\epsilon^\delta,\chi)}, \qquad \kappa_r(\epsilon^\delta,\chi) \equiv \frac{1+2\epsilon^\delta(1+\chi)}{(1+\epsilon^\delta)(1+\chi)}, \tag{24}$$

$$q = r^{\frac{1+\chi}{1+2\epsilon^\delta(1+\chi)}} \approx \left(\frac{a_H - \frac{\lambda_H}{1+\pi}}{a_L - \frac{\lambda_L}{1+\pi}} \right)^{\kappa_q(\epsilon^\delta)}, \qquad \kappa_q(\epsilon^\delta) \equiv \frac{1}{1+\epsilon^\delta}. \tag{25}$$

Given this relationship, the following proposition holds.

[8]These approximated functions are close to the ones provided by Fahr and Smets (2010) and Iwasaki et al. (2021). See Appendix B.2 for a more detailed discussion.

[9]For the detailed derivation, see Appendix C.2.

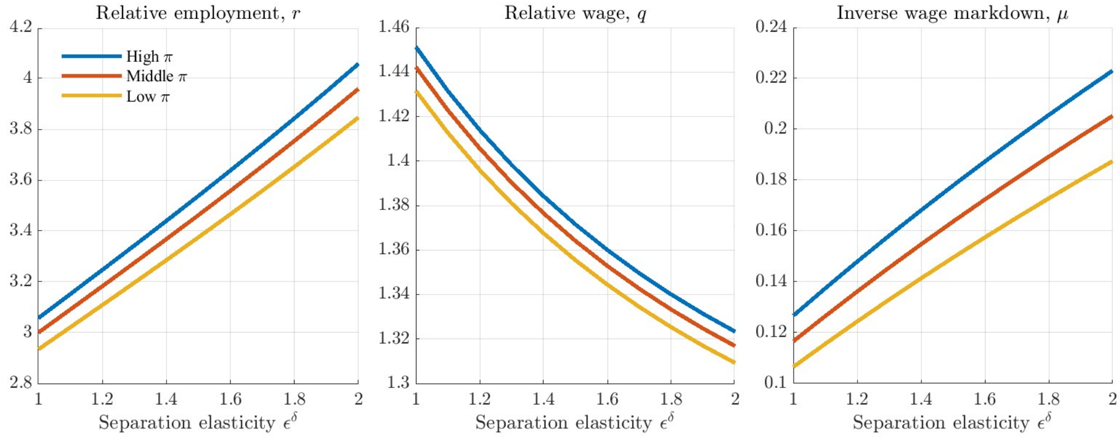

Figure 10: Relative employment, relative wage, and inverse wage markdown

NOTE: In this figure, we change the values for both inflation at the steady state and separation elasticity. We assume firm-specific nominal wage rigidity such that $\lambda_H/a_H > \lambda_L/a_L$ holds.

Proposition 1. *If* $a_H - \frac{\lambda_H}{1+\pi} > a_L - \frac{\lambda_L}{1+\pi}$*, then an increase in separation elasticity* ϵ^δ *raises* r *but lowers* q*. Moreover, if* $\frac{\lambda_H}{a_H} > \frac{\lambda_L}{a_L}$*, then an increase in trend inflation* π *raises both* r *and* q*.*

Proof. See Appendix C.2. ■

The assumption that high-productivity firms face larger wage adjustment costs corresponds, in the full model where firms are subject to idiosyncratic shocks, to the situation in which a high-productivity firm anticipates the possibility of a future decline in productivity and therefore takes into account the constraint that it may be unable to reduce wages.

Figure 10 illustrates the partial equilibrium outcomes of firms under different levels of trend inflation and separation elasticity. As shown in the proposition, when the wage adjustment cost faced by high-productivity firms is large, higher trend inflation and a higher separation elasticity both increase relative employment. An increase in trend inflation reduces effective wage adjustment costs, thereby improving the allocation of labor across firms and moving the economy closer to the frontier. An increase in separation elasticity makes workers more sensitive to their own wage level. The higher wage sensitivity induces low-productivity firms to raise wages more. However, workers still separate from low-wage firms and move toward higher-wage firms.

Relative wages increase as trend inflation rises. However, when separation elasticity increases, relative wages decline. There are two reasons for this. First, as separation elasticity rises, workers become more sensitive to relative wages. Therefore, high-productivity firms can secure employment without setting wages as high as before. Second, low-productivity firms need to set higher wages to prevent separations. Consequently, the dispersion of wage levels across firms declines.

Inverse wage markdown (Wage-to-MRPL ratio). We define the inverse wage markdown for firm type j as the ratio of the wage to the marginal revenue product of labor (MRPL):[10]

$$\mu_j \equiv \frac{w_j}{P^w \text{MRPL}_j}, \qquad \text{MRPL}_j \equiv \alpha a_j n_j^{\alpha-1}. \tag{26}$$

The condition $\mu_j < 1$ indicates that wages lie below marginal revenue product, reflecting monopsony power. Under the parameter setting that $P^w = 1$ and $\alpha = 1$, from the wage first-order condition, we can obtain:

$$\mu_j = \frac{1 - \frac{1}{a_j}\frac{\lambda_j}{1+\pi}}{1 + \frac{1}{\epsilon^\delta \delta_j}}. \tag{27}$$

Given this relationship, we claim the following proposition.

Proposition 2. *Holding δ_j fixed, higher separation elasticity and higher trend inflation raise μ_j, meaning wages move closer to marginal product.*

Proof. See Appendix C.3. ■

The rightmost panel of Figure 10 shows the relationship between the aggregate inverse wage markdown and both separation elasticity and trend inflation. When separation elasticity increases, the inverse wage markdown also rises. In other words, the gap between MRPL and wages narrows. An increase in separation elasticity implies that workers become more sensitive to relative wages and, consequently, that workers' bargaining power increases. The rise in workers' bargaining power weakens firms' monopsony power, resulting in an increase in the inverse wage markdown.

An increase in the trend inflation rate raises the inverse wage markdown. Under rising inflation, firms can let inflation erode the real value of wages instead of having to lower nominal wages directly. As a result, an increase in trend inflation reduces firms' wage adjustment costs, thereby exerting upward pressure on real wages. The rise in wages reduces the gap between MRPL and wages, leading to an increase in the inverse wage markdown. Conversely, when inflation is low, firms reflect wage adjustment costs in wages, causing the inverse wage markdown to decrease.

Lastly, we consider relative inverse wage markdowns. Under $\alpha = 1$, $\text{MRPL}_j = a_j$, relative inverse wage markdowns satisfy:

$$\frac{\mu_H}{\mu_L} = \frac{w_H/w_L}{\text{MRPL}_H/\text{MRPL}_L} = \frac{q}{a_H/a_L}. \tag{28}$$

[10] $1/\mu_j$ denotes the wage markdown in this paper. We follow Aoki et al. (2023) with respect to the definition of wage markdown.

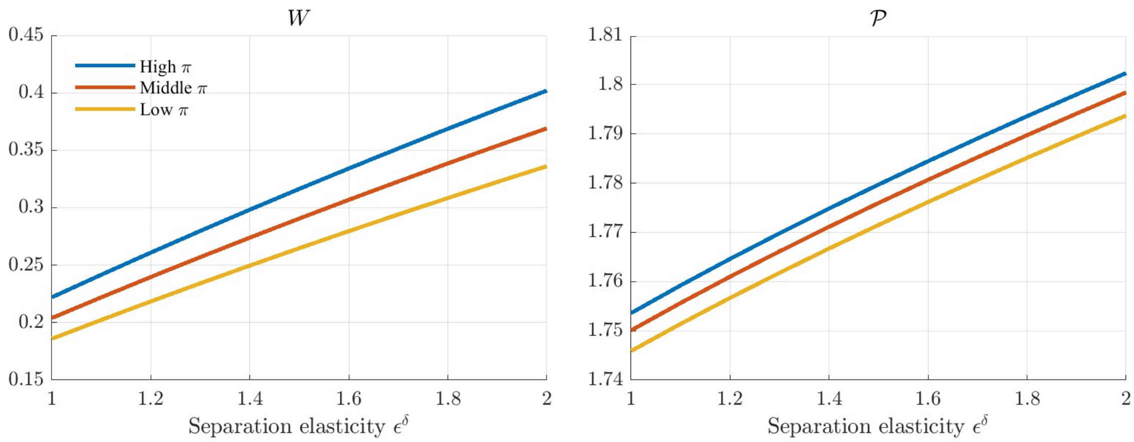

Figure 11: Macro wages and macro productivity

Thus, any force that increases the relative wage q increases the relative inverse markdown proportionally. In the limiting case with no wage markdown—that is, when wages equal MRPL at both firms—the relative wage equals relative productivity.

5.1.2 Macro wages and macro productivity

Macro wages. Below, based on the preceding discussion of firm-level employment and wage setting, we examine macro wages and productivity. Define the aggregate wage as the employment-weighted mean:

$$W \equiv \int w_j s_j \, dj, \qquad s_j \equiv \frac{n_i}{\int n_j dj}. \tag{29}$$

Figure 11 shows macro wages and macro productivity under different levels of trend inflation and separation elasticity. An increase in trend inflation or in separation elasticity raises the macro wage, but through different wage-setting channels. Higher trend inflation raises the relative wage of high-productivity firms, whereas higher separation elasticity lowers that relative wage by inducing low-productivity firms to raise wages more strongly. Nevertheless, both changes raise relative employment at high-productivity firms. Therefore, labor resources shift toward high-productivity firms.

Macro productivity. Define aggregate labor productivity (output per worker) as:

$$\mathcal{P} \equiv \frac{Y}{N} = \frac{\int y_j \, dj}{\int n_j \, dj}. \tag{30}$$

Under the two-type firm setting and the maintained assumption $\alpha = 1$, macro productivity can be written as:

$$\mathcal{P} = a_H s_H + a_L s_L. \tag{31}$$

The same labor-allocation logic used for macro wages applies to macro productivity. An increase in trend inflation or in separation elasticity raises the share of labor allocated to high-productivity firms. As a result, macro productivity increases.

5.2 Analysis Using the General Equilibrium Model

In this section, we examine how changes in trend inflation and separation elasticity affect macro wages and productivity by comparing steady states in a general equilibrium model. We show that the results obtained in the simplified static model are also obtained in the general equilibrium framework.

Here, we take as the benchmark a steady-state inflation rate of 0% and a higher preference for job security. We compare this benchmark with two alternative cases: first, when steady-state inflation rises to 2%; second, when it declines to -1%, corresponding to a deflationary environment. We also consider the case in which separation elasticity increases, corresponding to a lower preference for job security. Regarding separation elasticity, we assume that the elasticity, which declined in Japan during the 2000s, returns to its level observed in the 1990s. Finally, we also compare the benchmark with the case in which both steady-state inflation and separation elasticity increase simultaneously.

The results are shown in Figure 12. First, when steady-state inflation increases, both macro wages and macro productivity rise. The increase in macro wages arises because higher inflation reduces wage adjustment costs, allowing firms to set wages that are more closely aligned with productivity. In addition, the increase in macro productivity suggests that labor is being reallocated toward high-productivity firms, which in turn supports the rise in macro wages. The occurrence of labor reallocation following an increase in inflation is consistent with the results obtained in the static model discussed in the previous section. This also suggests that firms experiencing a positive productivity shock today restrain the wage they set in the current period, since lowering wages in the future would be difficult if they were subsequently hit by a negative productivity shock.[11]

When examining the impact of an increase in separation elasticity on real wages and macroeconomic productivity, it is observed that both improve as a result. In particular, the effect on real wages is substantial. This is because a higher separation elasticity strengthens the effect in which low-productivity firms are compelled to raise wages significantly to retain workers. During this process, the wage gap between high-productivity and low-productivity firms narrows. However, while an increase in separation elasticity does improve macro-level productivity, its impact is relatively small compared to the rise in trend inflation rates. This suggests that the allocation effects on labor caused by changes in separation elasticity are not particularly significant.

[11] The positive effects of higher trend inflation should not be interpreted as monotone. While higher trend inflation relaxes downward nominal wage rigidity by allowing real wages to adjust without nominal wage cuts, sufficiently high trend inflation may also raise the importance of firms' price adjustment costs. Depending on the calibration of price adjustment costs, these distortions may reduce aggregate output and productivity and offset the allocative gains from weaker nominal wage rigidity.

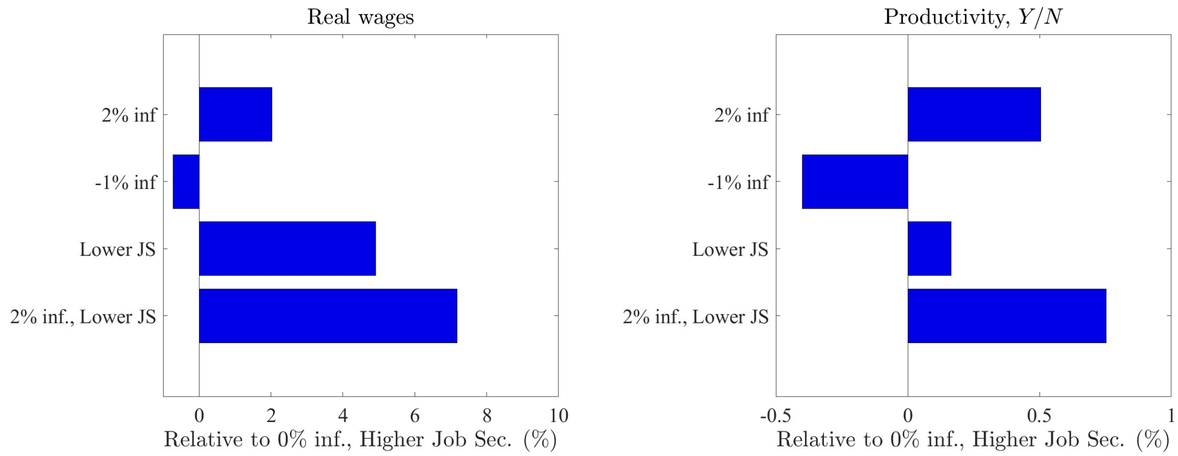

Figure 12: Steady-state comparison with the GE model

NOTE: We set 0% inflation and a higher preference for job security ($\epsilon^{\delta} = 1.4$) as the benchmark. A lower preference for job security corresponds to $\epsilon^{\delta} = 1.6$.

When increases in the inflation rate and separation elasticity occur simultaneously, both macro-level wages and productivity rise accordingly. The magnitude of these increases is approximately equal to the sum of the effects that would occur if each factor were to arise independently.

6 Role of Inflation and Job Security under Productivity Shock

In this section, we examine how changes in trend inflation and separation elasticity affect the transmission mechanism of productivity shocks to macro wages and macro productivity. As in the previous section, we first use a simplified one-period model to explain the key mechanisms, and then present dynamic results based on the full model.

6.1 Analysis Using a One-Period Model

As in the previous section, we abstract from considerations of firm future value and simplify the parameterization. Here, we consider the case in which all firms experience the same positive productivity shock:

$$d \log a_i = d \log z. \tag{32}$$

We first study the pass-through of the productivity shock to macro wages and then examine its pass-through to endogenous macro productivity.

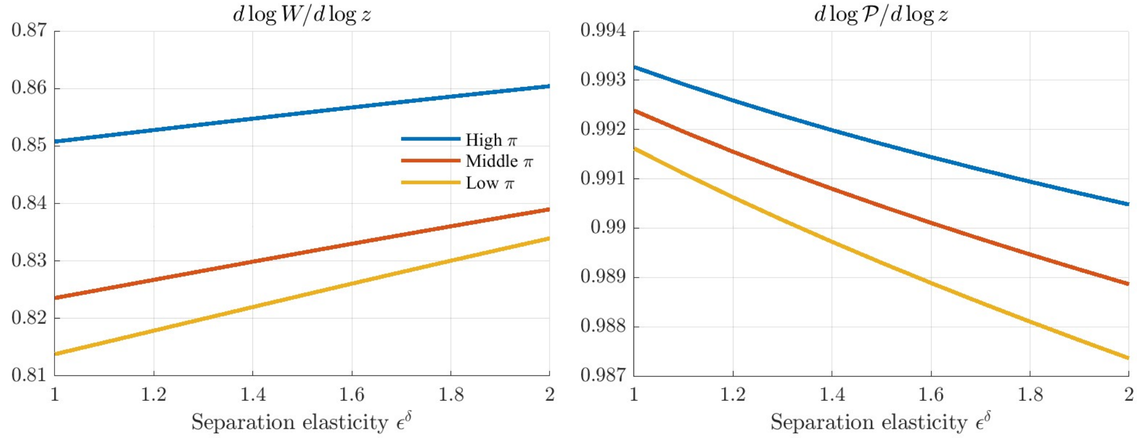

Figure 13: Productivity shock pass-through to macro wages and macro endogenous productivity

6.1.1 Macro wages

Under this setup, the pass-through of a productivity shock to the macro wage can be expressed as follows:[12]

$$\begin{aligned}\frac{d\log W}{d\log z} &= \frac{\int s_j \widetilde{w}_j \theta^w_{z_j} dj + \int s_j \left(\widetilde{w}_j - 1\right)\theta^n_{z_j} dj}{1 - \left[\int s_j \widetilde{w}_j \theta^w_{W_j} dj + \int s_j \left(\widetilde{w}_j - 1\right)\theta^n_{W_j} dj\right]} \\ &= \frac{\mathbb{E}_s[\widetilde{w}\theta^w_z] + \mathbb{C}ov_s\left(\widetilde{w}, \theta^n_z\right)}{1 - \left(\mathbb{E}_s[\widetilde{w}\theta^w_W] + \mathbb{C}ov_s\left(\widetilde{w}, \theta^n_W\right)\right)},\end{aligned} \tag{33}$$

where $\widetilde{w}_j \equiv \frac{w_j}{W}$. $\theta^w_{z_j}$ denotes the pass-through of a productivity shock to firm j's wage, and $\theta^n_{z_j}$ denotes the pass-through of a productivity shock to firm j's employment. Similarly, $\theta^w_{W_j}$ and $\theta^n_{W_j}$ denote the pass-through of macro wage increases to firm-level wages and employment, respectively.

The left panel of Figure 13 shows how the pass-through of a productivity shock to the macro wage changes with trend inflation and separation elasticity. An increase in trend inflation raises the pass-through of a productivity shock to the macro wage. Two main mechanisms operate here. The rise in $\mathbb{E}_s[\widetilde{w}\theta^w_z]$ is attributed to the fact that an increase in the trend inflation rate lowers the wage adjustment costs for firms, making it easier for firms to raise wages in response to productivity shocks. Similarly, the increase in $\mathbb{E}_s[\widetilde{w}\theta^w_W]$ suggests that, under reduced wage adjustment costs, firms find it easier to raise their own wages to keep up with rising wages at other firms in order to secure employment.

The increase in separation elasticity also raises the pass-through rate of macro-level wages to productivity shocks, albeit moderately. Two opposing forces are at play. First, θ^w_W increases because an increase in separation elasticity raises the proportion of workers leaving when macro wages rise.

[12] Each component of pass-through, such as $\theta^w_{z_j}$, is a function of trend inflation and separation elasticity. As the concrete expressions for each pass-through are complex, we discuss the main mechanism in the main text and leave the detailed expressions to Appendix B.4.

As the need to secure employment grows, firms respond by increasing their pass-through rate to macro wages. Conversely, separation elasticity ϵ^{δ} works in the opposite direction. Specifically, θ_z^w decreases. A higher separation elasticity increases the sensitivity of separations to relative wages. When separations become more responsive to wage changes, firms can secure or retain employees through this channel, without needing to raise wages as aggressively. Consequently, wage responses to productivity shocks are dampened. As separation elasticity rises, wage adjustments become more constrained across firms, which in turn reduces the pass-through of productivity to wages.

6.1.2 Macro productivity

Under the maintained assumption $\alpha = 1$, the change in macro productivity can be expressed as follows:

$$\begin{aligned} d\log\mathcal{P} &= \left[1+\int(\omega_{j,t}-s_{j,t})\theta^n_{z_j}\,dj\right]d\log z+\left[\int(\omega_{j,t}-s_{j,t})\theta^n_{W_j}\,dj\right]d\log W \\ &= \left[1+\mathbb{C}ov_s(\omega/s,\theta^n_z)\right]d\log z+\mathbb{C}ov_s(\omega/s,\theta^n_W)d\log W. \end{aligned} \tag{34}$$

Here, ω represents the share of individual firms in total output. $\theta^n_{z_j}$ denotes the pass-through of productivity shocks to firm-level employment, and $\theta^n_{W_j}$ denotes the pass-through of macro wage increases to firm-level employment. The two covariance terms indicate whether labor resources are reallocated toward firms with high output shares relative to their employment shares when productivity shocks or macro wages increase.

The right panel of Figure 13 shows how the pass-through of productivity shocks to macro productivity changes with trend inflation and separation elasticity. An increase in the trend inflation rate enhances the pass-through rate of productivity shocks to macro productivity. This occurs primarily because the trend inflation rate boosts the pass-through rate of macro wages to productivity shocks, as reflected by the covariance term $\mathbb{C}ov_s(\omega/s,\theta^n_W)$. When macro-level wages rise while individual firms' wages remain constant, a higher rate of separations tends to occur at low-productivity firms compared to high-productivity firms. As a result, labor shifts from low-productivity firms to high-productivity firms, driving a reallocation of resources. This reallocation effect further increases the pass-through rate of macro productivity.

Next, an increase in separation elasticity lowers the pass-through of productivity shocks to macro productivity. The mechanism is as follows. A higher separation elasticity strengthens the responsiveness of separations to relative wages. When separations are highly wage-sensitive, firms can secure or retain employment through the separation channel without raising wages as aggressively. Consequently, wage responses to productivity shocks are attenuated. This mechanism is stronger for firms that rely heavily on the employment margin; wage differentials compress following a productivity shock. As a result, employment increases more at relatively low-wage, low-productivity firms, and labor resources shift toward low-productivity firms.

6.2 IRFs with the General Equilibrium Model

In the previous section, to make the main reallocation mechanism transparent, we approximated the firm distribution by two types—high- and low-productivity firms—and derived a closed-form system for the pass-through of macro productivity shocks to macro wages and productivity. In this section, we investigate whether the main mechanism operates effectively within a full heterogeneous-firm model, which requires numerical methods (i.e., the sequence-space Jacobian in Auclert et al. (2021)).

Figure 14 shows the IRFs of macro real wages and endogenous productivity, which is defined as Y/N. In this exercise, we assume that the economy experiences a 1% permanent increase in productivity. The results are consistent with the aforementioned theory, even under conditions in which firms consider their future value as well as the endogenous forces arising from the dynamics of unemployment. A higher steady-state level of inflation increases the pass-through of exogenous productivity shocks to macro wages and endogenous productivity. Moreover, higher separation elasticity enhances the pass-through of exogenous productivity shocks to macro wages, but has the opposite effect on the pass-through to endogenous productivity, leading to a decline.

The mechanism is as follows. Productivity improvements raise firm-level marginal products, but downward nominal wage rigidity limits the pass-through of these gains into wages, especially at firms that would later like to partially reverse wage increases. Since workers' separation decisions depend on relative wages, this muted wage response also weakens labor reallocation toward more productive firms.

When trend inflation is higher, the expected future cost of wage adjustment falls because firms can reduce real wages through inflation rather than through nominal wage cuts. This makes firms more willing to increase wages in response to positive productivity innovations. Stronger wage responses reduce separations at productive firms and increase their employment shares, thereby generating an endogenous rise in aggregate labor productivity in addition to the direct effect of the aggregate productivity shock itself. The model therefore predicts that higher inflation raises both aggregate real wages and aggregate labor productivity.

When separation elasticity is also higher, the pass-through to the macro productivity is weakened. Wage responses to productivity shocks are attenuated, because firms can secure or retain employment through the separation channel without raising wages as aggressively. This mechanism is stronger for higher productivity firms; thus, wage differentials compress following a productivity shock. Hence, the reallocation effect is weakened when separation elasticity becomes larger.

6.3 Counterfactual Simulations and Discussion of Japan's Experience

In this section, we quantify how Japan's aggregate real wage path would have evolved under alternative inflation and labor-market environments over the past three decades. Our counterfactual exercise is motivated by the mechanism developed in the previous sections: when firms face down-

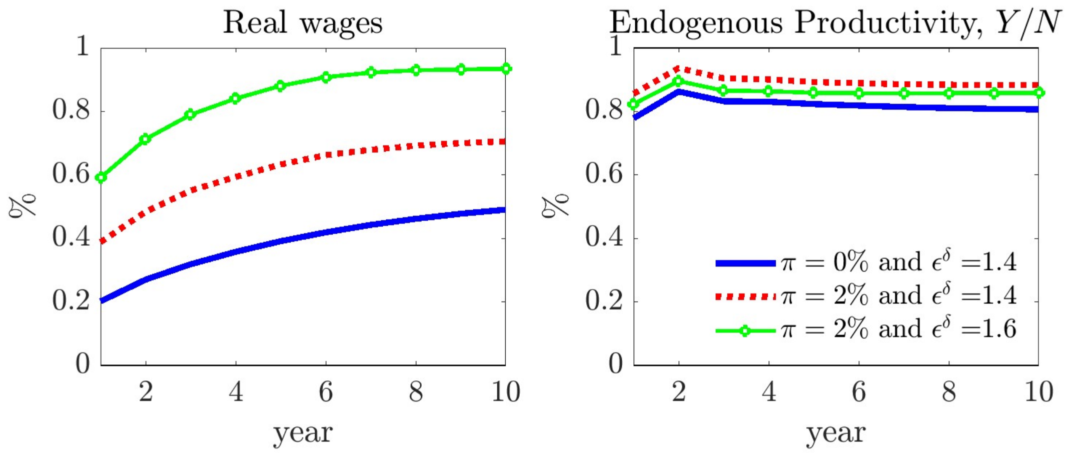

Figure 14: IRFs to an exogeneous 1% productivity shock with persistence ≈ 1

ward nominal wage rigidity, trend inflation relaxes the effective lower bound on real wage adjustment, while a higher separation elasticity makes worker flows more responsive to relative wages and therefore affects wage pass-through and labor reallocation.

The goal of this section is not to explain the entire historical evolution of Japanese real wages with the model alone. Rather, we use the model as a disciplined laboratory to ask the following question: holding fixed the sequence of underlying aggregate productivity disturbances that hit the Japanese economy, how much higher would real wages have been if the economy had operated under a higher inflation environment and, further, under a more fluid labor market? This approach allows us to isolate the propagation mechanism emphasized by the model from other forces that may also have affected actual wages.

We proceed as follows. First, we recover the sequence of aggregate productivity innovations that exactly reproduces the historical Japanese labor productivity path. Next, we simulate the counterfactual scenarios, 2% inflation and 2% inflation/high-separation-elasticity scenarios, using the identical innovation sequence. Finally, we construct the counterfactual real wage series by adding the model-implied deviations from the benchmark to the observed Japanese real wage data.

Recovery of aggregate productivity shocks. We begin by constructing a time series for the innovation ε_t^z in aggregate productivity in Japan over the past 30 years. Let $\mathcal{P}_t^{data}$ denote the observed aggregate labor productivity series in the data, measured as real output per worker. We interpret this historical path as the outcome of a sequence of persistent aggregate productivity shocks. In the model, the aggregate productivity follows an autoregressive process (6). We recover the sequence $\{\varepsilon_t^z\}_{t=1}^T$ recursively so that the model-implied aggregate labor productivity path matches the historical Japanese productivity path under the benchmark scenario: $\mathcal{P}_t^{model} = \mathcal{P}_t^{data}$ for all $t = 1, \ldots, T$.[13]

[13]Operationally, this inversion can be implemented period by period. Starting from the initial state, we choose the innovation ε_t^z in each period so that the model reproduces the observed productivity realization. By construction, the

We consider three scenarios. The first scenario is the benchmark economy, which uses the baseline calibrated parameters from the previous sections. In particular, it adopts the benchmark trend inflation rate, $\pi = 0\%$, and the benchmark separation elasticity, $\epsilon^{\delta} = 1.4$. The second scenario is an economy that steadily achieves the 2% inflation target. In the third scenario, trend inflation remains at 2% and the separation elasticity is further increased to $\epsilon^{\delta} = 1.6$. Let $s \in \{B, S1, S2\}$ index these three scenarios: 0% trend inflation with $\epsilon^{\delta} = 1.4$ (B), 2% trend inflation with $\epsilon^{\delta} = 1.4$ ($S1$), and 2% trend inflation with $\epsilon^{\delta} = 1.6$ ($S2$).

Simulation strategy. After recovering the common productivity shock sequence $\{\varepsilon_t^z\}_{t=1}^T$ from the benchmark economy, we feed that same sequence into each scenario s. For each scenario, we solve the model forward and compute the endogenous paths of aggregate labor productivity and the aggregate real wage:

$$\left\{W_t^{model,s}, \mathcal{P}_t^{model,s}\right\}_{t=1}^T. \tag{35}$$

Although the benchmark model is constructed to replicate the observed path of aggregate labor productivity, it is not designed to match the actual level of Japan's real wage series exactly in every period. Actual real wages are influenced by many forces outside the model, including changes in institutions, taxes, bargaining structures, and measurement issues. For this reason, we do not compare the model-generated wage level mechanically with the historical data. Instead, we use the model to identify scenario-specific deviations from the benchmark and then add those deviations to the observed real wage series.

Let $X \in \{W, \mathcal{P}\}$ denote the endogeneous variables of interest in Japan. We define the counterfactual path under scenario s as:

$$X_t^{cf,s} \equiv X_t^{data} + \left(X_t^{model,s} - X_t^{model,B}\right). \tag{36}$$

Results and discussion. Figure 15 shows the results for this exercise. The model-generated gaps are quantitatively large for real wages. As a result of this, the wage–productivity gaps—real wages minus productivity—also narrow in the two counterfactual scenarios. The results imply that Japan's weak real wage growth over the past three decades was not solely the result of weak underlying productivity growth: it also reflected a propagation mechanism in which low inflation and higher preference for job security reduced the pass-through of productivity gains to wages and slowed the reallocation of labor toward more productive firms. Conversely, an environment with somewhat higher trend inflation and more wage-sensitive worker flows would have allowed a larger fraction of the same productivity gains to be translated into higher real wages.

To conclude, we present the following observations to connect our results with Japan's expe-

recovered shock sequence is the sequence of persistent aggregate productivity innovations ($\rho^z \approx 1$) that rationalizes the observed Japanese productivity path in the economy.

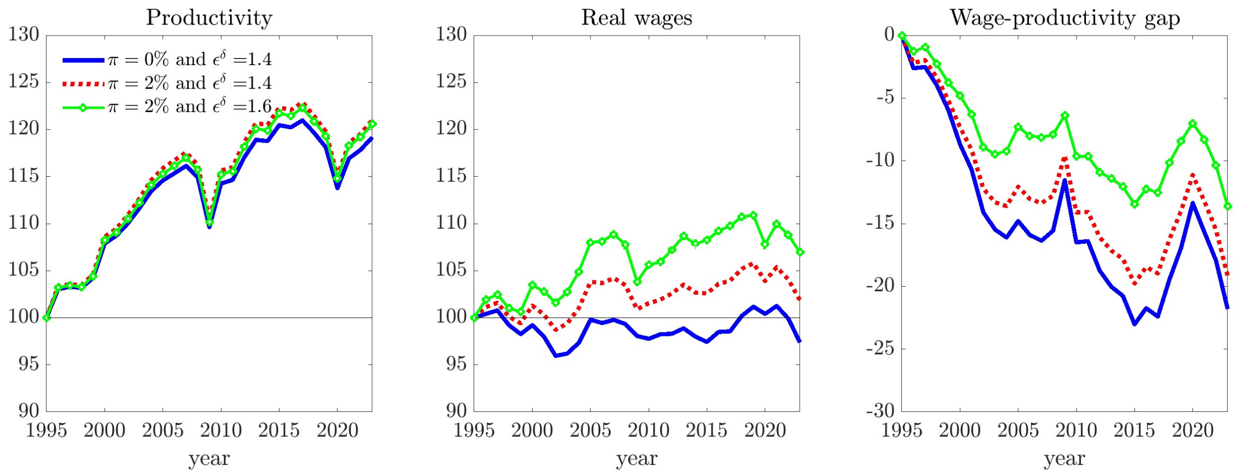

Figure 15: Counterfactural productivity, real wages, and wage–productivity gap over the last three decades

rience. First, Japan has experienced prolonged deflation. As discussed in Section 3, during periods without deflation, a "wage freeze"—where wages neither increased nor decreased—was observed. This suggests that there were costs associated with wage adjustments when firms were setting wages. The deflation that occurred under these costly wage adjustment conditions likely hindered the mechanism through which productivity-enhancing external shocks could translate into increases in real wages. These mechanisms may have also reduced the macroeconomic reallocation effects.

Second, since the 1990s, workers in Japan have placed less emphasis on wage increases in order to maintain their employment relationships. This change in behavior may have further weakened the transmission of productivity shocks to real wages. If an environment were created where workers could move in pursuit of higher wages, their bargaining power would improve, potentially enabling the achievement of higher real wages at the macroeconomic level.

7 Empirical Analysis of Reallocation Effects

In the theoretical analysis, we demonstrated that reducing wage-setting rigidity enhances worker mobility across firms by enabling wages to serve as a signaling mechanism, which, in turn, improves macro-level productivity. In this section, we examine whether the implications of the theoretical model are also supported empirically. Specifically, using firm-level microdata, we investigate whether wage-setting rigidity hinders the efficient allocation of labor in line with firms' productivity.

The effect whereby labor allocation across firms improves as workers move according to firm productivity, thereby increasing macro-level productivity, is generally referred to as the *reallocation effect*. Previous studies that examine the reallocation effect using firm-level microdata mainly employ two approaches. The first approach decomposes macroeconomic productivity growth into (i) productivity gains within individual firms (internal effects) and (ii) changes in the market share of more productive firms, which captures the reallocation component (Foster et al., 2001). The second approach estimates the reallocation effect by regressing the growth rate of individual firms' employment on firm productivity (Liu, 2017; Decker et al., 2020). Within this framework, if the parameter associated with productivity is significantly positive, it suggests that firms with higher productivity experience greater increases in employment, indicating that the reallocation effect contributes positively to macroeconomic productivity.

In this section, we adopt the approach of Decker et al. (2020) to analyze the relationship between individual firms' employment growth rates and their productivity, with the aim of examining whether the mechanism of reallocation effects driven by worker mobility operates in Japan. Furthermore, we explore how this relationship depends on wage-setting rigidity.

7.1 Data

The key variables used in this section are the employment growth rate, productivity, and a firm-level indicator of wage-setting rigidity. Both the employment growth rate and the wage-setting rigidity indicator can be constructed from the panel data of the Basic Survey on Wage Structure (BSWS), which is also used in Section 3. Specifically, the employment growth rate is calculated as the year-over-year change in a firm's total number of employees, while the wage-setting rigidity indicator is defined as the proportion of surveyed employees within each firm whose wages remained unchanged.

Since the BSWS does not contain information on firm productivity, we extend the dataset by integrating it with data from the Basic Survey of Japanese Business Structure and Activities (BSJBSA). From 2013 onward, the BSWS and the BSJBSA share a common establishment code, and matching is conducted using this code. For data prior to 2012, only those observations that can be linked using post-2013 establishment codes are traced back using establishment identifiers unique to each survey. However, because there is a discontinuity in the BSWS establishment identifiers between 1997 and 1998, the analysis is restricted to data from 1998 onward. Productivity is measured as real value added per total labor hours at the firm level.[14, 15]

The resulting dataset is a firm-level panel covering the period from 1998 to 2024 and includes a total of 12,665 firm-year observations. With respect to sample coverage, it should be noted that the analysis is restricted to firms that are observed for at least three consecutive periods. Consequently, the dataset does not capture the effects of firm entry and exit on macro-level productivity.

[14]The sum of operating profit, labor costs, taxes and public dues, interest expenses, and fixed asset depreciation is defined as nominal value added. Real value added is obtained by dividing nominal value added by the industry-specific value-added deflator.

[15]Following prior research such as Haba et al. (2024), the "labor costs" paid by each firm are divided by the "hourly wages by industry and firm size" calculated from the Monthly Labor Survey in order to estimate total labor hours in a simplified manner.

7.2 Empirical Framework

In this section, we extend the empirical specification of Decker et al. (2020) by incorporating wage-setting rigidity into the analysis of the relationship between firm-level employment growth and productivity. The baseline estimation equation is as follows:

$$g_{j,t+1} = \beta_0 + \beta_1 \mathbf{1}\{a_{j,t} > \bar{a}_{k,t}\} + \beta_2 \mathbf{1}\{r_{j,t} > \bar{r}_t\} + \beta_3 \left(\mathbf{1}\{a_{j,t} > \bar{a}_{k,t}\} \times \mathbf{1}\{r_{j,t} > \bar{r}_t\}\right) + X'_{j,t}\theta + \epsilon_{j,t+1}. \tag{37}$$

Here, $g_{j,t+1}$ denotes the employment growth rate (log difference) of firm j from period t to $t+1$; $a_{j,t}$ denotes productivity per hour of firm j in period t; and $r_{j,t}$ denotes the proportion of surveyed employees in firm j whose wages remained unchanged (hereafter, the proportion of wage freeze). $\bar{a}_{k,t}$ is the average hourly productivity in industry k, to which firm j belongs, in period t, and $\bar{r}_t$ is the average proportion of wage freeze in period t. The indicator functions $\mathbf{1}\{a_{j,t} > \bar{a}_{k,t}\}$ and $\mathbf{1}\{r_{j,t} > \bar{r}_t\}$ take the value one if, respectively, firm j's hourly productivity and wage-freeze share exceed their corresponding averages, and zero otherwise. $X_{j,t}$ is a vector of control variables, including the number of employees in firm j in period t, the year-over-year change in the unemployment rate in the prefecture where firm j is located, and fixed effects.

The combination of the estimated coefficients β_0, β_1, β_2, and β_3 can be interpreted as representing the average employment growth rates of four subgroups classified by the levels of hourly productivity and the proportion of wage freeze. In particular, the difference in employment growth rates between firms with above-average and below-average productivity is given by β_1 when the proportion of wage freeze is below average, and by $\beta_1 + \beta_3$ when the proportion of wage freeze is above average. If these estimated values are significantly positive, this implies that the higher-productivity group exhibits a higher employment growth rate than the lower-productivity group, suggesting that productivity-driven labor reallocation is functioning. Furthermore, if β_3 is significantly different from zero, the relationship between productivity and employment growth differs depending on whether the proportion of wage freeze is above or below average.

Based on (37), we conduct several estimations that differ in the definition of wage freeze and in the set of fixed effects. First, regarding fixed effects, each specification includes year fixed effects, industry fixed effects, and firm-size fixed effects. In addition, we also consider specifications that include prefecture fixed effects and interaction terms between year and industry fixed effects.

Regarding the definition of wage freeze, we first define it as a case in which the current year's wage is exactly identical to that of the previous year, and construct the proportion of wage freeze $r_{j,t}$ for each firm accordingly. However, even when the current year's wage does not exactly coincide with that of the previous year, wage rigidity may still be present, limiting the extent of wage adjustment. Indeed, as shown by the distribution of wage growth rates using the panel data in Section 3, during periods of deflation, wage growth rates tended to cluster not only exactly at zero but also around values close to zero. Therefore, as a robustness check, we adopt an expanded definition of wage freeze that includes wage growth rates between -0.1% and $+0.1\%$. Under this alternative definition, the proportion of wage freeze $\tilde{r}_{j,t}$ is recalculated for each firm.

7.3 Results

The estimation results are reported in Table 4. Across all specifications, β_1 is estimated to be significantly positive, while β_3 is estimated to be significantly negative. First, the significantly positive estimate of β_1 suggests that when the proportion of wage freeze is low — that is, when wage-setting rigidity is weak — the group of firms with higher productivity exhibits a higher employment growth rate. This result indicates that the mechanism of productivity-driven labor reallocation operates effectively under conditions of lower wage-setting rigidity.

On the other hand, the significantly negative estimate of β_3 suggests that when the proportion of wage freeze is high — that is, when wage-setting is more rigid — the reallocation of workers in response to productivity differences is relatively restrained compared with cases in which wage-setting rigidity is lower.

Taken together, these results suggest that while the reallocation mechanism — whereby workers move in response to firm-level productivity differences — is functioning, such productivity-driven worker mobility is impeded when firms exhibit stronger wage-setting rigidity. This empirical finding is consistent with the analytical results derived from the theoretical model in Section 4. Specifically, under DNWR combined with a low inflation rate, wage-setting becomes more rigid, weakening the signaling role of wages. As a consequence, workers find it more difficult to move in response to firm productivity, thereby reducing the efficiency of labor allocation. Conversely, under higher inflation rates, the signaling role of wages becomes more effective, implying greater scope for improvements in labor allocation across firms.

Table 4: Wage setting rigidity and reallocation effects

	(1)	(2)	(3)	(4)	(5)	(6)
Prod: β_1	0.012***	0.012***	0.014***	0.013***	0.013***	0.015***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Rigidity $\left(r_{j,t}\right)$: β_2	-0.001	-0.001	0.005			
	(0.003)	(0.003)	(0.004)			
Prod $\times$ Rigidity $\left(r_{j,t}\right)$: β_3	-0.008*	-0.008**	-0.007*			
	(0.004)	(0.004)	(0.003)			
Rigidity $\left(\tilde{r}_{j,t}\right)$: β_2				0.002	0.002	0.007
				(0.003)	(0.003)	(0.004)
Prod $\times$ Rigidity $\left(\tilde{r}_{j,t}\right)$: β_3				-0.007**	-0.008**	-0.009**
				(0.003)	(0.003)	(0.004)
Observations	12,665	12,665	12,131	12,665	12,665	12,131
Adjusted R^2	0.038	0.040	0.062	0.038	0.040	0.062
Fixed effects						
Year	✓	✓	✓	✓	✓	✓
Industry	✓	✓	✓	✓	✓	✓
Firm size	✓	✓	✓	✓	✓	✓
Prefecture		✓	✓		✓	✓
Year $\times$ Industry			✓			✓

NOTE: ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively. Robust standard errors clustered by industry are reported in parentheses.

8 Conclusion

Motivated by the striking divergence between labor productivity and real wages in Japan over the past three decades, this paper examines why productivity gains do not translate into wage growth as strongly in Japan as in Western European economies. We focus on two forces that are particularly relevant in the Japanese context: prolonged low inflation or deflation, and labor-market practices that place greater weight on employment stability than on wage growth. Our objective is to clarify whether these factors can account for the widening wage–productivity gap and to identify the conditions under which that gap may narrow.

To address this question, we combine empirical and theoretical analyses. First, using long-term worker-level microdata for Japan covering 1990–2024, we jointly estimate upward and downward nominal wage rigidities and document how wage-setting behavior evolves over time. Second, we develop a heterogeneous-firm general equilibrium model with downward nominal wage rigidity and monopsonistic wage setting, in which workers' separations respond to firms' wages relative to the aggregate wage. This framework allows us to analyze how trend inflation and labor-market frictions shape wage setting, worker reallocation, and the transmission of productivity gains to wages. Third, we use the model to conduct counterfactual simulations based on Japan's historical productivity path and to quantify how alternative inflation and labor-market environments change real wage dynamics. Finally, we bring the mechanism back to the data by examining whether wage-setting rigidity weakens worker reallocation toward more productive firms.

Our findings highlight three main points. First, wage setting in Japan exhibits rigidity in both upward and downward directions during the deflationary period, and these wage freezes suppress wage growth and widen the wage–productivity gap. Second, in the model, downward nominal wage rigidity distorts firms' wage-setting incentives by making future wage cuts costly, which induces firms to restrain wage increases and weakens the pass-through of productivity gains to wages. Third, this distortion becomes particularly severe in a low-inflation environment and in a labor market with higher preference for job security, where wage signals are less effective in reallocating workers toward more productive firms. Together, these results imply that both deflation and labor-market frictions are central to understanding Japan's prolonged wage stagnation.

The counterfactual and firm-level evidence together suggest that Japan's prolonged real wage stagnation reflects not only weak productivity growth itself, but also a transmission mechanism in which low inflation, rigid wage setting, and weak worker reallocation dampen the wage response to productivity improvements. More productive firms tend to expand employment faster, but this relationship weakens when wage freezes are more prevalent. These results imply that nominal wage flexibility matters not only for wage growth itself but also for the efficient allocation of labor across firms.

Several issues remain for future research. Most importantly, a fuller understanding of the wage–productivity gap requires going beyond the channels studied here. In particular, it is important to deepen our understanding of how terms-of-trade movements may affect the wage–productivity gap, especially in an open economy such as Japan where import prices, exchange rates, and external cost pressures can drive a wedge between real wages and labor productivity. More generally, other factors not explicitly analyzed in this paper—including changes in industrial composition, non-regular employment, globalization, and broader institutional features of wage bargaining—may also shape the relationship between productivity and wages.

We examine how trend inflation affects the wage-productivity gap in this paper, but trend inflation itself is an endogenous object in the economy. Productivity growth through the reallocation effect, for example, could change the marginal cost at the macro level, which also changes inflation dynamics. Considering the interaction among trend inflation, nominal wage rigidity, and the worker reallocation effect is also an important agenda for future work.

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Appendix

A Data Appendix

A.1 Real Wages and Productivity

The real wages presented in Figure 1 (Figure A.1) are based on real wages per employee, while productivity is measured based on real productivity per employee. Real wages per employee are calculated by dividing the real compensation of employees by the number of employees, adjusted to full-time equivalents. Real productivity per employee is derived by dividing real GDP by the number of employees. The data sources for these measures are from the OECD.

For reference, we also provide a comparison of real wages per hour and productivity per hour (Figure A.2). Real wages per hour are calculated by dividing the real compensation of employees by the total number of hours worked. Productivity per hour is calculated by dividing real GDP by the total number of hours worked. The data source for Japan is the Cabinet Office, while data for other countries is sourced from the OECD.

As noted by Date et al. (2024), the wages of most full-time employees in Japan are determined based on their monthly base salary. Consequently, their hourly wages are subject to considerable fluctuations due to factors such as differences in the number of weekdays in each year. Taking this into account, this study focuses on scheduled wages instead of hourly wages as the primary object of analysis.

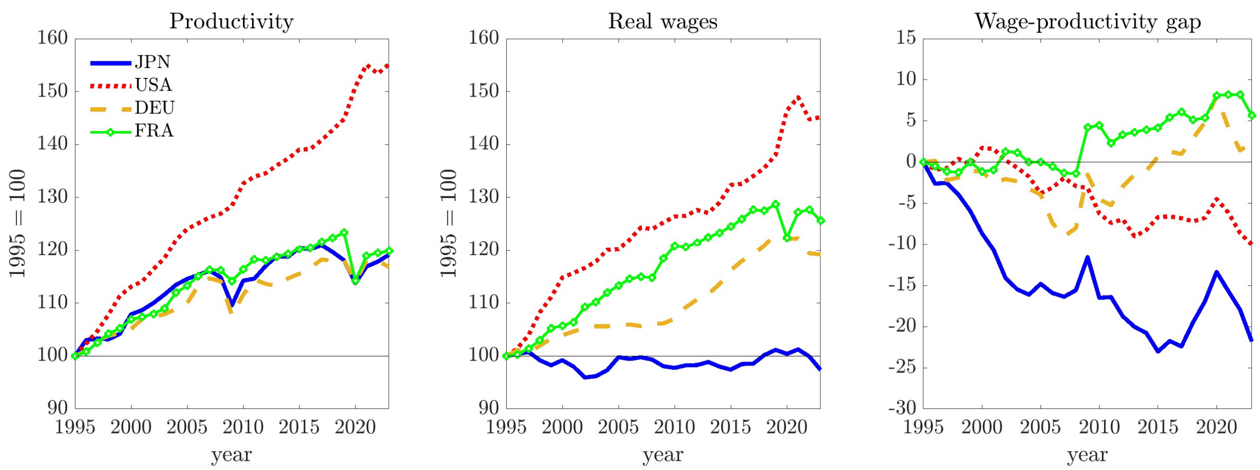

Figure A.1: Productivity and real wages per person, and wage-productivity gap

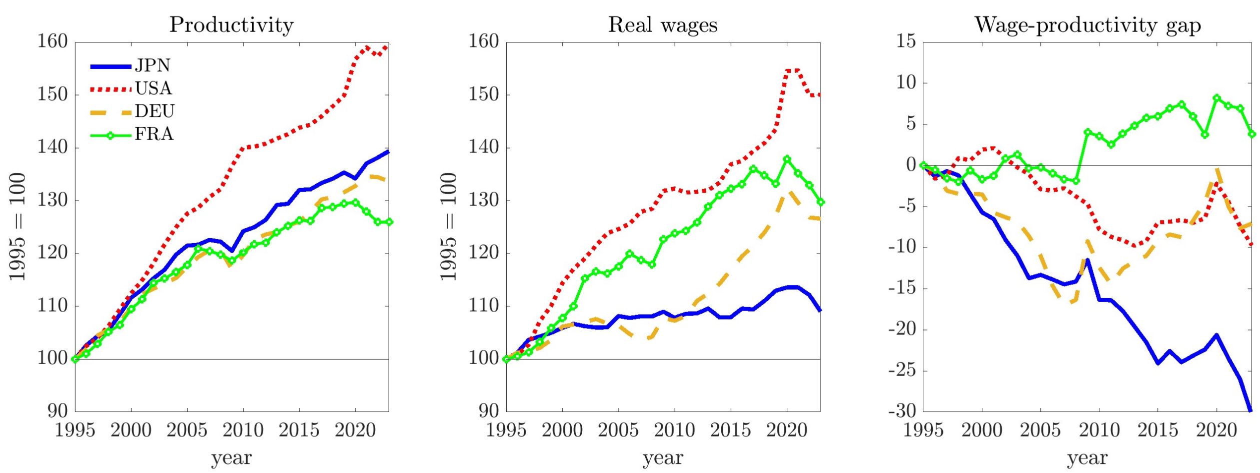

Figure A.2: Productivity and real wages per hour, and wage-productivity gap

B Theoretical Appendix

B.1 Firms' FOCs

B.1.1 Derivation of the wage FOC

As a preliminary, we define the continuation marginal value of wage and employment. Let

$$M^w_{j,t+1} \equiv \mathbb{E}_t\left[\frac{\partial V_{j,t+1}}{\partial w_{j,t}}\right], \qquad M^n_{j,t+1} \equiv \mathbb{E}_t\left[\frac{\partial V_{j,t+1}}{\partial n_{j,t}}\right]. \tag{B.1}$$

This is the marginal value today of having one more worker at the end of period t (i.e., at the start of period $t+1$), evaluated at the optimum.

The FOC for $w_{j,t}$ is

$$\frac{d\Pi_{j,t}}{dw_{j,t}} + \beta\mathbb{E}_t\left[\frac{dV_{j,t+1}}{dw_{j,t}}\right] = 0. \tag{B.2}$$

Apply the chain rule to the continuation term:

$$\mathbb{E}_t\left[\frac{dV_{j,t+1}}{dw_{j,t}}\right] = M^n_{j,t+1}\frac{\partial n_{j,t}}{\partial w_{j,t}} + M^w_{j,t+1} \tag{B.3}$$

Compute $\frac{d\Pi_{j,t}}{dw_{j,t}}$. Differentiate profits:

$$\frac{d\Pi_{j,t}}{dw_{j,t}} = P^w_t\frac{\partial y_{j,t}}{\partial w_{j,t}} - \frac{\partial}{\partial w_{j,t}}(w_{j,t}n_{j,t}) - \frac{\partial}{\partial w_{j,t}}(\mathcal{D}_{j,t}n_{j,t}) - \frac{\partial \mathcal{C}_{j,t}}{\partial w_{j,t}}. \tag{B.4}$$

Since $y_{j,t} = a_{j,t}n^{\alpha}_{j,t}$ and $\mathcal{C}_{j,t}$ does not depend on $w_{j,t}$,

$$\frac{\partial y_{j,t}}{\partial w_{j,t}} = MRPL_{j,t}\frac{\partial n_{j,t}}{\partial w_{j,t}}, \qquad \frac{\partial \mathcal{C}_{j,t}}{\partial w_{j,t}} = 0, \tag{B.5}$$

where $MRPL_{j,t} \equiv \frac{\partial y_{j,t}}{\partial n_{j,t}}$. Then,

$$\begin{aligned}\frac{d\Pi_{j,t}}{dw_{j,t}} &= P^w_t MRPL_{j,t}\frac{\partial n_{j,t}}{\partial w_{j,t}} - \left(n_{j,t} + w_{j,t}\frac{\partial n_{j,t}}{\partial w_{j,t}}\right) - \left(\frac{\partial \mathcal{D}_{j,t}}{\partial w_{j,t}}n_{j,t} + \mathcal{D}_{j,t}\frac{\partial n_{j,t}}{\partial w_{j,t}}\right)\\ &= -n_{j,t}\left(1 + \frac{\partial \mathcal{D}_{j,t}}{\partial w_{j,t}}\right) + \left(P^w_t MRPL_{j,t} - w_{j,t} - \mathcal{D}_{j,t}\right)\frac{\partial n_{j,t}}{\partial w_{j,t}}.\end{aligned} \tag{B.6}$$

Combine pieces to obtain the wage FOC. Plug (B.3) and (B.6) into (B.2):

$$0 = -n_{j,t}\left(1 + \frac{\partial \mathcal{D}_{j,t}}{\partial w_{j,t}}\right) + \left(P^w_t MRPL_{j,t} - w_{j,t} - \mathcal{D}_{j,t} + \beta M^n_{j,t+1}\right)\frac{\partial n_{j,t}}{\partial w_{j,t}} + \beta M^w_{j,t+1}. \tag{B.7}$$

After rearranging, we obtain:

$$\left(P^w_t MRPL_{j,t} - w_{j,t} - \mathcal{D}_{j,t} + \beta M^n_{j,t+1}\right)\frac{\partial n_{j,t}}{\partial w_{j,t}} + \beta M^w_{j,t+1} = n_{j,t}\left(1 + \frac{\partial \mathcal{D}_{j,t}}{\partial w_{j,t}}\right), \tag{B.8}$$

where, holding $(W_t, n_{j,t-1}, v_{j,t})$ fixed when differentiating w.r.t. $w_{j,t}$, we have:

$$\frac{\partial n_{j,t}}{\partial w_{j,t}} = \epsilon^{\delta} \delta \left(\frac{w_{j,t}}{W_t} \right) \frac{n_{j,t-1}}{w_{j,t}}. \tag{B.9}$$

Note that the marginal product of labor at firm j is $MRPL_{j,t} = \alpha a_{j,t} n_{j,t}^{\alpha-1}$.

B.1.2 Derivation of the vacancy FOC

The FOC for $v_{j,t}$ is

$$\frac{d\Pi_{j,t}}{dv_{j,t}} + \beta \mathbb{E}_t \left[\frac{\partial V_{j,t+1}}{\partial v_{j,t}} \right] = 0. \tag{B.10}$$

Since $v_{j,t}$ affects $V_{j,t+1}$ only through the endogenous state $n_{j,t}$,

$$\mathbb{E}_t \left[\frac{\partial V_{j,t+1}}{\partial v_{j,t}} \right] = \mathbb{E}_t \left[\frac{\partial V_{j,t+1}}{\partial n_{j,t}} \right] \frac{\partial n_{j,t}}{\partial v_{j,t}} = M^n_{j,t+1} \frac{\partial n_{j,t}}{\partial v_{j,t}}. \tag{B.11}$$

Differentiate profits:

$$\frac{d\Pi_{j,t}}{dv_{j,t}} = P^w_t \frac{\partial y_{j,t}}{\partial v_{j,t}} - \frac{\partial}{\partial v_{j,t}} (w_{j,t} n_{j,t}) - \frac{\partial}{\partial v_{j,t}} (\mathcal{D}_{j,t} n_{j,t}) - \frac{\partial \mathcal{C}_{j,t}}{\partial v_{j,t}}. \tag{B.12}$$

Because $y_{j,t} = a_{j,t} n_{j,t}^{\alpha}$, $\frac{\partial y_{j,t}}{\partial v_{j,t}} = MRPL_{j,t} \frac{\partial n_{j,t}}{\partial v_{j,t}}$. Also,

$$\frac{\partial}{\partial v_{j,t}} (w_{j,t} n_{j,t}) = w_{j,t} \frac{\partial n_{j,t}}{\partial v_{j,t}}, \qquad \frac{\partial}{\partial v_{j,t}} (\mathcal{D}_{j,t} n_{j,t}) = \mathcal{D}_{j,t} \frac{\partial n_{j,t}}{\partial v_{j,t}}. \tag{B.13}$$

Therefore,

$$\frac{d\Pi_{j,t}}{dv_{j,t}} = \left(P^w_t MRPL_{j,t} - w_{j,t} - \mathcal{D}_{j,t} \right) \frac{\partial n_{j,t}}{\partial v_{j,t}} - \frac{\partial \mathcal{C}_{j,t}}{\partial v_{j,t}}. \tag{B.14}$$

Substituting (B.11) and (B.14) into (B.10),

$$0 = \left(P^w_t MRPL_{j,t} - w_{j,t} - \mathcal{D}_{j,t} \right) \frac{\partial n_{j,t}}{\partial v_{j,t}} - \frac{\partial \mathcal{C}_{j,t}}{\partial v_{j,t}} + \beta M^n_{j,t+1} \frac{\partial n_{j,t}}{\partial v_{j,t}} \tag{B.15}$$

After rearranging, we obtain:

$$\left(P^w_t MRPL_{j,t} - w_{j,t} - \mathcal{D}_{j,t} + \beta M^n_{j,t+1} \right) \frac{\partial n_{j,t}}{\partial v_{j,t}} = \frac{\partial \mathcal{C}_{j,t}}{\partial v_{j,t}}, \tag{B.16}$$

where, holding $(w_{j,t}, W_t, n_{j,t-1})$ fixed when differentiating w.r.t. $v_{j,t}$,

$$\frac{\partial n_{j,t}}{\partial v_{j,t}} = m'(v_{j,t}). \tag{B.17}$$

The marginal posting cost is

$$\frac{\partial \mathcal{C}(v_{j,t}, n_{j,t-1})}{\partial v_{j,t}} = c^v \left(\frac{v_{j,t}}{n_{j,t-1}} \right)^{\chi} \tag{B.18}$$

The left-hand side of (B.16) is the marginal benefit of an additional vacancy via the induced increase in employment, valued at the net marginal surplus $(P^w MRPL - w - \mathcal{D} + \beta M^n)$, while the right-hand side is the marginal posting cost.

B.2 Approximation to the Kinked Wage Adjustment Cost

In the baseline specification, the wage adjustment cost is kinked and given by:

$$\mathcal{D}(x) = \lambda_1 \big(x - (1+\pi)\big)_+, \qquad x \equiv \frac{w_{t-1}}{w_t}, \tag{B.19}$$

where $(z)_+ \equiv \max\{z, 0\}$. The cost is zero for $x \leq 1+\pi$ and increases linearly for $x > 1+\pi$. While this specification captures DNWR in a transparent way, it introduces a non-differentiability at the kink point $x = 1+\pi$, which complicates local analysis and linearization.

To obtain a differentiable representation while preserving the economic structure of the kink, we consider the following smooth approximation:

$$\mathcal{D}^T(x) = \lambda_1 \eta \log\left(1 + \exp\left(\frac{x - (1+\pi)}{\eta}\right)\right) + \lambda_2, \tag{B.20}$$

where $\eta > 0$ governs the degree of smoothness and λ_2 is a constant chosen to satisfy the normalization:

$$\mathcal{D}^T(1) = \frac{\lambda}{1+\pi}. \tag{B.21}$$

For small but positive η, the approximation tracks the original linear branch closely on the right-hand side of the kink and replicates its slope asymptotically. Figure B.1 compares the original DNWR function with the approximated function under an example value for each parameter. We show the case with the following parameters: $\lambda_1 = 10$, $\lambda = 0.02$, $\pi = 0.02$, and $\eta = 0.005$.

Usefulness of the approximation. The smooth formulation retains three key properties that make it a useful approximation. First, it provides a close and controlled approximation to the original kinked function. For $x > 1+\pi$, the derivative satisfies: $\frac{\partial \mathcal{D}^T(x)}{\partial x} = \lambda_1 \frac{1}{1+\exp\left(-\frac{x-(1+\pi)}{\eta}\right)}$, which converges to λ_1 as x moves sufficiently to the right of the kink. Hence, the smooth curve becomes locally parallel to the original linear function, preserving the effective marginal cost of wage cuts. Moreover, for $\pi > 0$, this derivative is close to zero at the steady state, where $x = 1$, provided that η is sufficiently small relative to π (so that π/η is large). This property is useful for analyzing a one-period model and is consistent with the original kinked function.

Second, unlike the original specification, the smooth approximation implies:

$$\mathcal{D}^T(x) > 0 \quad \text{even for } x < 1+\pi. \tag{B.22}$$

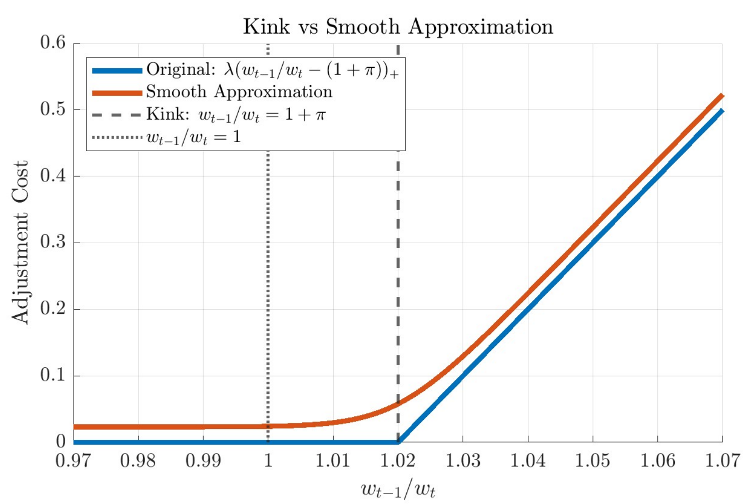

Figure B.1: Approximation of DNWR Function

This feature reflects the forward-looking nature of wage setting. Firms anticipate the possibility that future wage reductions may become binding. Even when the constraint is not currently active, the option value of avoiding future costly adjustments generates a strictly positive adjustment cost. Thus, the smooth specification can be interpreted as incorporating expected future wage adjustment costs into current decisions.

Third, the smooth approximation eliminates the non-differentiability at the kink and delivers a well-defined marginal condition. In particular, the marginal cost around the steady state becomes smooth, and under the chosen normalization the derivative at the steady state can be controlled directly. This property is crucial for analytical tractability, especially when deriving first-order conditions or log-linearized equilibrium systems.

Having stated the above, for the sake of later use, we provide the change in slope of the cost around the stationary point. Define $s_\pi \equiv [1+\exp(\pi/\eta)]^{-1}$ and $s_\lambda \equiv \lambda_1/w$. At the stationary point $w_{t-1} = w_t = w$ (so $x_t = 1$), the derivatives with respect to the current wage are

$$\mathcal{D}^T_{w_t} = -s_\lambda s_\pi, \qquad w\mathcal{D}^T_{w_t w_t} = s_\lambda s_\pi \left[2 + \frac{1-s_\pi}{\eta}\right].$$

Therefore, provided $1 - s_\lambda s_\pi > 0$, the local marginal-wedge elasticity is

$$\kappa_{c,j}(\pi, \lambda_1) \equiv \frac{w\mathcal{D}^T_{w_t w_t}}{1+\mathcal{D}^T_{w_t}} = \frac{s_\lambda s_\pi\,[2+(1-s_\pi)/\eta]}{1-s_\lambda s_\pi} \quad \Longrightarrow \quad d\log(1+\mathcal{D}^T_{w_t}) = \kappa_{c,j}\, d\log w_t.$$

For $\pi \geq 0$,

$$\frac{ds_\pi}{d\pi} = -\frac{s_\pi(1-s_\pi)}{\eta} < 0, \qquad \frac{\partial \kappa_{c,j}}{\partial s_\pi} = \frac{s_\lambda\left[2+(1-2s_\pi+s_\lambda s_\pi^2)/\eta\right]}{(1-s_\lambda s_\pi)^2} > 0,$$
$$\frac{\partial \kappa_{c,j}}{\partial \lambda_1} = \frac{s_\pi\,[2+(1-s_\pi)/\eta]}{w(1-s_\lambda s_\pi)^2} > 0.$$

Hence $\partial\kappa_{c,j}/\partial\pi < 0$: trend inflation weakens local DNWR, whereas a steeper cost schedule strengthens it. In the analysis below, we therefore treat the first derivative of the adjustment cost, $\partial D^T/\partial w_t$, as approximately zero around the steady state, while retaining its second derivative, $\partial^2 D^T/\partial w_t^2$, as non-negligible. The resulting local curvature of the marginal adjustment-cost wedge is summarized by $\kappa_{c,j}$, as defined above. It should be noted that a similar set of derivatives of the nominal wage rigiditiy cost can be derived with the quadratic adjustment cost.

B.3 Partial Equilibrium Analysis Using a One-period Model

B.3.1 Wage FOC

We consider a one-period model, which ignores the one-period-ahead valuation terms, $M^n_{j,t+1}$ and $M^w_{j,t+1}$. We assume that the price of intermediate goods is given as $P^w_t = 1$. Let

$$x_{j,t} \equiv \tfrac{w_{j,t}}{W_t}, \qquad \delta_{j,t} \equiv \delta(x_{j,t}), \qquad f(x_{j,t}) \equiv \epsilon^\delta \delta_{j,t}.$$

Rearranging the wage FOC and defining a new term as follows:

$$R_{j,t} \;\equiv\; \frac{n_{j,t}\left(1+\frac{\partial \mathcal{D}^T(w_{j,t})}{\partial w_{j,t}}\right)}{\epsilon^\delta \delta\left(\frac{w_{j,t}}{W_t}\right)\frac{n_{j,t-1}}{w_{j,t}}} = \frac{n_{j,t} w_{j,t}}{f(x_{j,t})\, n_{j,t-1}}\left(1+\frac{\partial \mathcal{D}^T(w_{j,t})}{\partial w_{j,t}}\right). \tag{B.23}$$

Therefore the wage FOC can be written as:

$$MRPL_{j,t} - w_{j,t} - \mathcal{D}^T(w_{j,t}) = R_{j,t}. \tag{B.24}$$

Steady state. We linearize (B.24) around a steady state satisfying

$$w_{j,t} = w_j, \qquad n_{j,t} = n_j, \qquad x_{j,t} = x_j. \tag{B.25}$$

Note that we assume at this point,

$$\mathcal{D}^T(w_j) = \frac{\lambda_j}{1+\pi}, \qquad \frac{\partial \mathcal{D}^T(w_{j,t})}{\partial w_{j,t}}\Big|_{w_j} \approx 0. \tag{B.26}$$

Total differential of the left-hand side. Differentiating the left-hand side yields the following result.

$$dMRPL_{j,t} = MRPL_{j,t}\left(d\log a_{j,t} + (\alpha-1)\, d\log n_{j,t}\right). \tag{B.27}$$

Using (B.26), the total differential of the left-hand side of (B.24) at the linearization point is

$$\begin{aligned} d\Big(MRPL_{j,t} - w_{j,t} - \mathcal{D}^T(w_{j,t})\Big) &= dMRPL_{j,t} - dw_{j,t} \\ &= MRPL_{j,t}\left(d\log a_{j,t} + (\alpha-1)\, d\log n_{j,t}\right) - w_{j,t}\, d\log w_{j,t}, \end{aligned} \tag{B.28}$$

where we assume $\frac{\partial \mathcal{D}^T(w_{j,t})}{\partial w_{j,t}} \approx 0$ around the steady state.

Total differential of the right-hand side $R_{j,t}$. Start from (B.23). At the linearization point (B.25), the bracketed term equals 1, and its first-order change is

$$d\log\left(1 + \frac{\partial \mathcal{D}^T_{j,t}}{\partial w_{j,t}}\right) = \kappa_{c,j} d\log w_{j,t}. \tag{B.29}$$

Next, use the definition of ϵ^δ as the (positive) elasticity of the separation rate:

$$\epsilon^\delta \equiv -\frac{d\log\delta(x)}{d\log x} \quad\Longrightarrow\quad d\log\delta(x_{j,t}) = -\epsilon^\delta\, d\log x_{j,t}. \tag{B.30}$$

Since $x_{j,t} = w_{j,t}/W_t$,

$$d\log x_{j,t} = d\log w_{j,t} - d\log W_t, \qquad \Rightarrow \qquad -d\log\delta(x_{j,t}) = \epsilon^\delta\left(d\log w_{j,t} - d\log W_t\right). \tag{B.31}$$

Now take logs of (B.23) and totally differentiate:

$$d\log R_{j,t} = d\log n_{j,t} + d\log w_{j,t} - d\log f(x_{j,t}) - d\log n_{j,t-1} + d\log\left(1 + \frac{\partial \mathcal{D}^T_{j,t}}{\partial w_{j,t}}\right). \tag{B.32}$$

Because $f(x_{j,t}) = \epsilon^\delta \delta(x_{j,t})$ and ϵ^δ is constant, $d\log f(x_{j,t}) = d\log\delta(x_{j,t})$. We consider the deviation from the steady state, then using (B.29), and (B.31), we get

$$\begin{aligned} d\log R_{j,t} &= d\log n_{j,t} + d\log w_{j,t} - \ d\log\delta(x_{j,t}) + \kappa_{c,j} d\log w_{j,t} \\ &= d\log n_{j,t} + \left(1 + \epsilon^\delta + \kappa_{c,j}\right) d\log w_{j,t} - \epsilon^\delta\, d\log W_t. \end{aligned} \tag{B.33}$$

Converting back to levels (first order): $dR_{j,t} = R_{j,t}\, d\log R_{j,t}$, hence

$$dR_{j,t} = R_{j,t}\left[d\log n_{j,t} + \left(1 + \epsilon^\delta + \kappa_{c,j}\right) d\log w_{j,t} - \epsilon^\delta\, d\log W_t\right]. \tag{B.34}$$

First-order (total-differential) version of the FOC. Taking the total differential of (B.24) and using (B.28)–(B.34):

$$\begin{aligned}
0 &= dMRPL_{j,t} - dw_{j,t} - dR_{j,t} \\
&= MRPL_{j,t}\left(d\log a_{j,t} + (\alpha-1)\, d\log n_{j,t}\right) - w_{j,t}\, d\log w_{j,t} \\
&\quad - R_{j,t}\left[d\log n_{j,t} + \left(1+\epsilon^{\delta}+\kappa_{c,j}\right) d\log w_{j,t} - \epsilon^{\delta}\, d\log W_t\right].
\end{aligned} \tag{B.35}$$

Collect terms in $(d\log w_{j,t}, d\log a_{j,t}, d\log n_{j,t}, d\log W_t)$:

$$\begin{aligned}
\left[w_{j,t} + R_{j,t}\left(1+\epsilon^{\delta}+\kappa_{c,j}\right)\right] d\log w_{j,t} &= MRPL_{j,t}\, d\log a_{j,t} \\
&\quad - \left[R_{j,t} + (1-\alpha) MRPL_{j,t}\right] d\log n_{j,t} + R_{j,t}\epsilon^{\delta}\, d\log W_t.
\end{aligned} \tag{B.36}$$

Express everything in terms of $f(x_j)$ and define $g(x_j)$. At the linearization point (B.25), (B.23) implies

$$R_j \equiv \frac{n_j w_j}{f(x_j)\, n_j} = \frac{w_j}{f(x_j)}. \tag{B.37}$$

Also, the steady-state level FOC (B.24) becomes

$$MRPL_j - w_j - \frac{\lambda_j}{1+\pi} = R_j. \tag{B.38}$$

Substitute (B.37) into (B.38) to eliminate w_j:

$$MRPL_j - \frac{\lambda_j}{1+\pi} = w_j\left(1+\frac{1}{f(x_j)}\right) = w_j\frac{1+f(x_j)}{f(x_j)} \implies \frac{MRPL_j}{R_j} = \frac{MRPL_j}{MRPL_j - \frac{\lambda_j}{1+\pi}}(1+f(x_j)). \tag{B.39}$$

Define

$$g(x_j) \;\equiv\; \frac{MRPL_j}{R_j}. \tag{B.40}$$

Now divide both sides of (B.36) by R_j and use $w_j/R_j = f(x_j)$ from (B.37). The coefficient on $d\log w_{j,t}$ becomes

$$\frac{w_j}{R_j} + \left(1+\epsilon^{\delta}+\kappa_{c,j}\right) = f(x_j) + \left(1+\epsilon^{\delta}+\kappa_{c,j}\right).$$

Also, $MRPL_j/R_j = g(x_j)$, and

$$\frac{R_j + (1-\alpha)MRPL_j}{R_j} = 1 + (1-\alpha)\frac{MRPL_j}{R_j} = 1 + (1-\alpha)g(x_j).$$

Therefore (B.36) becomes

$$\left[(1+\epsilon^{\delta}+\kappa_{c,j}) + f(x_j)\right] d\log w_{j,t} = g(x_j)\, d\log a_{j,t} - \left[1+(1-\alpha)g(x_j)\right] d\log n_{j,t} + \epsilon^{\delta}\, d\log W_t. \tag{B.41}$$

Finally, divide (B.41) by $(1+\epsilon^{\delta}+\kappa_{c,j}) + f(x_j)$:

$$d\log w_{j,t} = \frac{1}{(\epsilon^{\delta}+\kappa_{c,j}+1) + f\,(x_j)}\left[g\,(x_j)\;\; d\log a_{j,t} - \{1+(1-\alpha)g\,(x_j)\}\;\; d\log n_{j,t} + \epsilon^{\delta}\, d\log W_t\right]. \tag{B.42}$$

B.3.2 FOC for Vacancy Posting

This section derives the steady-state characterization and the first-order (local) approximation of the vacancy-posting condition around the stationary equilibrium. Consider the vacancy-posting first-order condition written as

$$MRPL_{j,t} - w_{j,t} - \mathcal{D}^T(w_{j,t}) = \frac{c^v \left(\frac{v_{j,t}}{n_{j,t-1}}\right)^\chi}{(1-\mu)\, v_{j,t}^{-\mu}\, u_{t-1}^{\mu}}. \tag{B.43}$$

Define the right-hand side as

$$H_{j,t} \equiv \frac{c^v \left(\frac{v_{j,t}}{n_{j,t-1}}\right)^\chi}{(1-\mu)\, v_{j,t}^{-\mu}\, u_{t-1}^{\mu}} = \frac{c^v}{1-\mu}\, v_{j,t}^{\mu+\chi}\, n_{j,t-1}^{-\chi}\, u_{t-1}^{-\mu}. \tag{B.44}$$

Then (B.43) is equivalently[16]

$$MRPL_{j,t} - w_{j,t} - \mathcal{D}^T(w_{j,t}) = H_{j,t}. \tag{B.45}$$

Steady state. We linearize around a stationary point satisfying

$$w_{j,t} = w_j, \qquad n_{j,t} = n_{j,t-1} \equiv n_j, \qquad v_{j,t} \equiv v_j, \qquad a_{j,t} \equiv a_j, \qquad u_{t-1} \equiv u. \tag{B.46}$$

Evaluating (B.45) at (B.46) gives the steady-state condition

$$MRPL_j - w_j - \frac{\lambda_j}{1+\pi} = H_j, \tag{B.47}$$

where $MRPL_j \equiv \alpha a_j n_j^{\alpha-1}$ and $H_j \equiv \frac{c^v}{1-\mu}\, v_j^{\mu+\chi}\, n_j^{-\chi}\, u^{-\mu}$.

Local approximation. We now derive the first-order relationship among $(v_{j,t}, a_{j,t}, n_{j,t}, w_{j,t})$ around the steady state. Consistent with the firm's static vacancy choice, the predetermined stock $n_{j,t-1}$ is taken as fixed in period t, and we impose

$$d\log n_{j,t-1} = 0. \tag{B.48}$$

Moreover, in the local approximation used in the paper we treat aggregate unemployment at the beginning of the period as fixed,

$$d\log u_{t-1} = 0. \tag{B.49}$$

Taking the total differential of (B.45) at the steady state yields

$$dMRPL_{j,t} - dw_{j,t} - d\mathcal{D}^T(w_{j,t}) = dH_{j,t}. \tag{B.50}$$

Since we have $d\mathcal{D}^T(w_{j,t}) = 0$ at first order, so (B.50) becomes

$$dMRPL_{j,t} - dw_{j,t} = dH_{j,t}. \tag{B.51}$$

With respect to $MRPL_{j,t}$ we have,

$$dMRPL_{j,t} = MRPL_{j,t}\left(d\log a_{j,t} + (\alpha-1)d\log n_{j,t}\right), \tag{B.52}$$

and $dw_{j,t} = w_{j,t}\, d\log w_{j,t}$. Evaluating at the steady state,

$$dMRPL_{j,t} - dw_{j,t} = MRPL_j\left(d\log a_{j,t} + (\alpha-1)d\log n_{j,t}\right) - w_j\, d\log w_{j,t}. \tag{B.53}$$

Next, from (B.44),

$$d\log H_{j,t} = (\mu+\chi)d\log v_{j,t} - \chi d\log n_{j,t-1} - \mu d\log u_{t-1}. \tag{B.54}$$

[16] Note that $H_{j,t} = R_{j,t}$ from the equilibrium conditions.

Using (B.48) and (B.49), we obtain

$$d \log H_{j,t} = (\mu + \chi) d \log v_{j,t}. \tag{B.55}$$

Converting back to levels at first order gives $dH_{j,t} = H_j \, d \log H_{j,t}$, hence

$$dH_{j,t} = H_j (\mu + \chi) d \log v_{j,t}. \tag{B.56}$$

Substituting (B.53) and (B.56) into (B.51) yields

$$MRPL_j \left(d \log a_{j,t} + (\alpha - 1) d \log n_{j,t} \right) - w_j \, d \log w_{j,t} = H_j (\mu + \chi) d \log v_{j,t}. \tag{B.57}$$

Finally, divide both sides of (B.57) by H_j and define

$$g(x_j) \equiv \frac{MRPL_j}{H_j}, \qquad f(x_j) \equiv \frac{w_j}{H_j}. \tag{B.58}$$

Then (B.57) becomes

$$(\mu + \chi) d \log v_{j,t} = g(x_j) \, d \log a_{j,t} - (1 - \alpha) g(x_j) \, d \log n_{j,t} - f(x_j) \, d \log w_{j,t}. \tag{B.59}$$

Dividing by $(\mu + \chi)$ delivers the desired local approximation:

$$d \log v_{j,t} = \frac{1}{\mu + \chi} \left[g\left(x_j\right) d \log a_{j,t} - (1 - \alpha) g\left(x_j\right) d \log n_{j,t} - f\left(x_j\right) d \log w_{j,t} \right]. \tag{B.60}$$

B.3.3 Law of motion of employment

This section derives the local relationship between firm-level employment, wages, and vacancies around the steady state.

Environment. Firm-level employment evolves according to

$$n_{j,t} = \left(1 - \delta_{j,t}\right) n_{j,t-1} + m_{j,t}, \tag{B.61}$$

where $\delta_{j,t}$ is the separation rate and $m_{j,t}$ denotes new hires. Hiring is produced by vacancies and unemployment,

$$m_{j,t} = v_{j,t}^{1-\mu} u_{t-1}^{\mu}, \tag{B.62}$$

and separations depend on relative wages,

$$\delta_{j,t} = \bar{\delta} \left(\frac{w_{j,t}}{W_t} \right)^{-\epsilon^{\delta}}. \tag{B.63}$$

Steady state. In the steady state, employment is constant, $n_{j,t} = n_{j,t-1} \equiv n_j$, hence (B.61) implies

$$m_j = \delta_j n_j. \tag{B.64}$$

Local approximation. We consider a first-order approximation around the steady state and impose

$$d \log u_{t-1} = 0, \qquad d \log n_{j,t-1} = 0, \tag{B.65}$$

reflecting that unemployment and lagged employment are predetermined in period t. Taking a total differential of (B.61) gives

$$dn_{j,t} = -n_{j,t-1} \, d\delta_{j,t} + dm_{j,t}. \tag{B.66}$$

Evaluating at the steady state ($n_{j,t-1} = n_j$) and dividing by n_j yields

$$d \log n_{j,t} = -d\delta_{j,t} + \frac{dm_{j,t}}{n_j}. \tag{B.67}$$

From (B.63), taking logs and differentiating,

$$\log \delta_{j,t} = \log \bar{\delta} - \epsilon^{\delta}(\log w_{j,t} - \log W_t),$$

so

$$d\delta_{j,t} = \delta_j \, d \log \delta_{j,t} = -\delta_j \epsilon^{\delta} \left(d \log w_{j,t} - d \log W_t \right). \tag{B.68}$$

Hence,

$$-\, d\delta_{j,t} = \delta_j \epsilon^{\delta} \left(d \log w_{j,t} - d \log W_t \right). \tag{B.69}$$

Next, from (B.62),

$$d \log m_{j,t} = (1-\mu) d \log v_{j,t} + \mu d \log u_{t-1}. \tag{B.70}$$

Using $d \log u_{t-1} = 0$ from (B.65),

$$d \log m_{j,t} = (1-\mu) d \log v_{j,t}. \tag{B.71}$$

Since $m_j = \delta_j n_j$ in (B.64), we have $\frac{m_j}{n_j} = \delta_j$, and thus

$$\frac{dm_{j,t}}{n_j} = \frac{m_j}{n_j} d \log m_{j,t} = \delta_j (1-\mu) \, d \log v_{j,t}. \tag{B.72}$$

Substituting (B.69) and (B.72) into (B.67) yields

$$d \log n_{j,t} = \delta_j \epsilon^{\delta} \left(d \log w_{j,t} - d \log W_t \right) + \delta_j (1-\mu) \, d \log v_{j,t}. \tag{B.73}$$

B.4 Solving for $(d \log w_{j,t}, d \log n_{j,t})$

This section derives closed-form expressions for the local responses of firm-level wages and employment to firm productivity and the aggregate wage index. The derivation starts from the three local equations for wage setting, vacancy posting, and the law of motion of employment, and then eliminates vacancies to obtain a two-equation system in $(d \log w_{j,t}, d \log n_{j,t})$.

B.4.1 Solving for $(d \log w_{j,t}, d \log n_{j,t})$

Summary of steady-state objects. Let relative wages be $x_{j,t} \equiv w_{j,t}/W_t$ and the separation function be

$$\delta(x_j) = \bar{\delta} x_j^{-\epsilon^{\delta}}, \qquad f(x_j) = \epsilon^{\delta} \delta(x_j). \tag{B.74}$$

Define

$$g(x_j) = \frac{MRPL_j}{MRPL_j - \frac{\lambda_j}{1+\pi}} (1 + f(x_j)), \tag{B.75}$$

and the composite term

$$\Theta_j \equiv 1 + (1-\mu) \delta\left(x_j\right) \frac{(1-\alpha) g\left(x_j\right)}{\mu + \chi}. \tag{B.76}$$

The three local equations. We start from the log-linear wage equation

$$d \log w_{j,t} = \frac{1}{\left(\epsilon^{\delta} + \kappa_{c,j} + 1\right) + f\left(x_j\right)} \left[g\left(x_j\right) d \log a_{j,t} - \left\{ 1 + (1-\alpha) g\left(x_j\right) \right\} d \log n_{j,t} + \epsilon^{\delta} d \log W_t \right]. \tag{B.77}$$

Define for later use

$$D_j \equiv \left(\epsilon^{\delta} + \kappa_{c,j} + 1 \right) + f\left(x_j\right), \qquad E_j \equiv 1 + (1-\alpha) g\left(x_j\right), \tag{B.78}$$

so that (B.77) can be written compactly as

$$d\log w_{j,t} = \frac{g(x_j)}{D_j} d\log a_{j,t} - \frac{E_j}{D_j} d\log n_{j,t} + \frac{\epsilon^{\delta}}{D_j} d\log W_t. \tag{B.79}$$

Next, the vacancy-posting condition implies the local vacancy response

$$d\log v_{j,t} = \frac{1}{\mu+\chi}\left[g\left(x_j\right) d\log a_{j,t} - (1-\alpha) g\left(x_j\right) d\log n_{j,t} - f\left(x_j\right) d\log w_{j,t}\right]. \tag{B.80}$$

Finally, the local approximation of the law of motion of employment is

$$d\log n_{j,t} = f\left(x_j\right)\left(d\log w_{j,t} - d\log W_t\right) + (1-\mu)\delta_j\, d\log v_{j,t}, \qquad \delta_j \equiv \delta(x_j). \tag{B.81}$$

Substitute out vacancies in the employment equation. Substitute (B.80) into (B.81):

$$\begin{aligned} d\log n_{j,t} &= f(x_j)\left(d\log w_{j,t} - d\log W_t\right) \\ &\quad + \frac{(1-\mu)\delta_j}{\mu+\chi}\left[g(x_j)\, d\log a_{j,t} - (1-\alpha) g(x_j)\, d\log n_{j,t} - f(x_j)\, d\log w_{j,t}\right]. \end{aligned} \tag{B.82}$$

Collect terms in $d\log n_{j,t}$ on the left:

$$\begin{aligned} \left[1 + \frac{(1-\mu)\delta_j}{\mu+\chi}(1-\alpha) g(x_j)\right] d\log n_{j,t} &= \frac{(1-\mu)\delta_j}{\mu+\chi} g(x_j)\, d\log a_{j,t} \\ &\quad + \left[f(x_j) - \frac{(1-\mu)\delta_j}{\mu+\chi} f(x_j)\right] d\log w_{j,t} - f(x_j)\, d\log W_t. \end{aligned} \tag{B.83}$$

Using the definition of Θ_j in (B.76), divide both sides of (B.83) by Θ_j to obtain

$$d\log n_{j,t} = \frac{(1-\mu)\delta\left(x_j\right) g\left(x_j\right)}{(\mu+\chi)\Theta_j} d\log a_{j,t} + \frac{f\left(x_j\right)}{\Theta_j}\left[1 - \frac{(1-\mu)\delta\left(x_j\right)}{\mu+\chi}\right] d\log w_{j,t} - \frac{f\left(x_j\right)}{\Theta_j} d\log W_t. \tag{B.84}$$

Define coefficients (A_j, B_j, C_j). Define

$$A_j = \frac{(1-\mu)\delta\left(x_j\right) g\left(x_j\right)}{(\mu+\chi)\Theta_j}, \qquad B_j = \frac{f\left(x_j\right)}{\Theta_j}\left[1 - \frac{(1-\mu)\delta\left(x_j\right)}{\mu+\chi}\right], \qquad C_j = \frac{f\left(x_j\right)}{\Theta_j}. \tag{B.85}$$

Then (B.84) becomes

$$d\log n_{j,t} = A_j\, d\log a_{j,t} + B_j\, d\log w_{j,t} - C_j\, d\log W_t. \tag{B.86}$$

Solve the two-equation system in $(d\log w_{j,t}, d\log n_{j,t})$. Combine the wage equation (B.79) with (B.86). Substitute (B.86) into (B.79):

$$\begin{aligned} D_j\, d\log w_{j,t} &= g(x_j)\, d\log a_{j,t} - E_j\Big(A_j\, d\log a_{j,t} + B_j\, d\log w_{j,t} - C_j\, d\log W_t\Big) + \epsilon^{\delta}\, d\log W_t \\ &= \left(g(x_j) - E_j A_j\right) d\log a_{j,t} - E_j B_j\, d\log w_{j,t} + \left(\epsilon^{\delta} + E_j C_j\right) d\log W_t. \end{aligned} \tag{B.87}$$

Therefore,

$$d\log w_{j,t} = \frac{g\left(x_j\right) - E_j A_j}{D_j + E_j B_j} d\log a_{j,t} + \frac{\epsilon^{\delta} + E_j C_j}{D_j + E_j B_j} d\log W_t. \tag{B.88}$$

Next, substitute (B.88) into (B.86):

$$d\log n_{j,t} = A_j\, d\log a_{j,t} + B_j \left(\frac{g(x_j) - E_jA_j}{D_j + E_jB_j} d\log a_{j,t} + \frac{\epsilon^\delta + E_jC_j}{D_j + E_jB_j} d\log W_t \right) - C_j\, d\log W_t. \tag{B.89}$$

Collect coefficients on $d\log a_{j,t}$ and $d\log W_t$:

$$d\log n_{j,t} = \left[A_j + \frac{B_j(g(x_j) - E_jA_j)}{D_j + E_jB_j} \right] d\log a_{j,t} + \left[\frac{B_j(\epsilon^\delta + E_jC_j)}{D_j + E_jB_j} - C_j \right] d\log W_t. \tag{B.90}$$

Therefore,

$$d\log n_{j,t} = \frac{A_jD_j + B_jg\left(x_j\right)}{D_j + E_jB_j} d\log a_{j,t} + \frac{B_j\epsilon^\delta - C_jD_j}{D_j + E_jB_j} d\log W_t. \tag{B.91}$$

Mapping firm productivity to aggregate productivity. Suppose firm productivity can be written as $a_{j,t} = a_j z_t$, so that $d\log a_{j,t} = d\log z_t$ for all j. Then one may write the reduced forms

$$d\log w_{j,t} = \theta^w_{z,j}\, d\log z_t + \theta^w_{W,j}\, d\log W_t, \qquad d\log n_{j,t} = \theta^n_{z,j}\, d\log z_t + \theta^n_{W,j}\, d\log W_t, \tag{B.92}$$

with coefficients identified from (B.88)–(B.91) as

$$\theta^w_{z,j} \equiv \frac{g(x_j) - E_jA_j}{D_j + E_jB_j}, \qquad \theta^w_{W,j} \equiv \frac{\epsilon^\delta + E_jC_j}{D_j + E_jB_j}, \tag{B.93}$$

and

$$\theta^n_{z,j} \equiv \frac{A_jD_j + B_jg(x_j)}{D_j + E_jB_j}, \qquad \theta^n_{W,j} \equiv \frac{B_j\epsilon^\delta - C_jD_j}{D_j + E_jB_j}. \tag{B.94}$$

Equations (B.88)–(B.94) provide the desired closed-form characterization of local wage and employment responses.

B.4.2 Understanding each pass-through expression[17]

Productivity pass-through to firm wage setting, $\theta^w_{z,j}$. The coefficient $\theta^w_{z,j}$ measures the local elasticity of a firm's wage with respect to an aggregate productivity shock, holding the aggregate wage index constant. It captures how much of a macro productivity improvement is transmitted into firm-level wage adjustment through the wage-vacancy-employment block.

When aggregate productivity increases, the marginal revenue product of labor rises. Absent frictions, firm wages would adjust proportionally. However, wage adjustment is dampened by two mechanisms embedded in the denominator $D_j + E_jB_j$: local nominal wage rigidity, summarized by $\kappa_{c,j}$, and the employment feedback effect arising from vacancy and separation responses. The level DNWR wedge, $\lambda/(1+\pi)$, enters $g(x_j)$ through $MRPL_j - \lambda/(1+\pi)$. The numerator $g(x_j) - E_jA_j$ captures the direct marginal-productivity effect net of the employment-adjustment channel.

High-productivity firms (high x_j) exhibit a lower pass-through than low-productivity firms. The reason is that their employment margin is more valuable: because their marginal product is high, preserving employment is crucial for production. As a result, the employment feedback term E_jA_j is relatively strong, dampening wage adjustment. Low-productivity firms, by contrast, face weaker marginal productivity incentives and therefore adjust wages more in response to aggregate productivity movements.

[17] The comparative-static interpretations in this subsection should be understood as local results evaluated at the calibrated steady state. In particular, the signs and cross-firm rankings discussed hold under the parameter values and parameter ranges used in the quantitative exercises; they are not claimed to be global analytical properties of the model and may change outside the calibrated region.

Trend inflation affects $\theta^w_{z,j}$ through both the level DNWR wedge, $\lambda/(1+\pi)$, and the local marginal-wedge elasticity, $\kappa_{c,j}(\pi, \lambda_1)$. Higher trend inflation lowers the former and, under the smooth cost specification, also lowers $\kappa_{c,j}$. These changes affect $g(x_j)$ and reduce the contribution of local nominal rigidity to D_j. Under the calibrated parameter values, the net effect is to increase pass-through, with a disproportionately large increase at high-productivity firms. Thus, inflation raises overall pass-through and narrows the gap between high- and low-productivity firms in the calibrated exercises.

Separation elasticity ϵ^δ operates in the opposite direction. A higher separation elasticity strengthens the responsiveness of separations to relative wages. When separations are highly wage-sensitive, firms can secure or retain employment through the separation channel without raising wages as aggressively. This amplifies the employment feedback embedded in B_j and effectively increases the denominator $D_j + E_j B_j$. Consequently, wage responses to productivity shocks are attenuated. As separation elasticity rises, wage adjustments become more limited across firms, and the productivity pass-through to wages declines. Because this mechanism is stronger for firms that rely heavily on the employment margin, wage differentials compress following a productivity shock.

In summary, productivity pass-through to firm wages is increasing in trend inflation and decreasing in separation elasticity. Higher inflation relaxes effective nominal rigidity and strengthens wage responsiveness, particularly for high-productivity firms. Higher separation elasticity enhances employment adjustment and reduces the need for wage adjustment, dampening pass-through and compressing cross-firm wage responses to productivity shocks.

Macro wage pass-through to firm wage setting, $\theta^w_{W,j}$. $\theta^w_{W,j}$ denotes the pass-through of the macro wage to an individual firm's wage. This pass-through is positive: when the aggregate wage increases, each firm raises its own wage in order to limit separations and secure employment. In the calibrated exercises, the pass-through is smaller for high-productivity firms than for low-productivity firms. It increases with trend inflation because weaker nominal rigidity makes it easier to adjust wages in response to the aggregate wage. It also increases with separation elasticity because firms' separations become more sensitive to their wage relative to the aggregate wage.

Productivity pass-through to firm employment, $\theta^n_{z,j}$. $\theta^n_{z,j}$ denotes the pass-through of a macro productivity shock to firm employment. When a positive productivity shock occurs, firms increase employment. The effect is smaller for high-productivity (high-wage) firms and larger for low-productivity (low-wage) firms. This is because wage increases at high-productivity firms are more moderate than at low-productivity firms. When separation elasticity increases, the difference in this pass-through across firms becomes larger. This reflects the suppression of wage increases at high-productivity firms and the stronger incentive for low-productivity firms to raise wages, which reduces wage differentials across firms. The effect of changes in trend inflation is not large.

Macro wage pass-through to firm employment, $\theta^n_{W,j}$. $\theta^n_{W,j}$ measures the total response of firm employment to an increase in the aggregate wage index, allowing the firm's own wage to adjust endogenously. In equation (B.86), the direct effect holding the firm's own wage fixed is $-C_j$: a higher aggregate wage lowers the firm's relative wage, raises its separation rate, and reduces employment. The endogenous wage response, $B_j\theta^w_{W,j}$, partially offsets this direct effect. Because unemployment is held fixed in this partial-equilibrium exercise, the response should not be interpreted as operating through a change in aggregate unemployment. Under the calibrated parameter values, the total employment response is more negative for low-wage, low-productivity firms, and a higher separation elasticity widens the cross-firm difference.

C Two-type Firm Analysis

C.1 Derivation of Equilibrium System and Relative Wages

Consider two types of atomistic firms, $j \in \{H, L\}$, with productivity (a_H, a_L). Each type has the same mass, and each individual firm takes the aggregate wage W and unemployment u as given.

The analysis is a one-period steady-state partial equilibrium. We normalize $P^w = 1$ throughout this section. Let employment and wages be (n_j, w_j) and define:

$$q \equiv \frac{w_H}{w_L}, \qquad r \equiv \frac{n_H}{n_L}, \qquad s \equiv \frac{n_H}{n_H + n_L} = \frac{r}{1+r}.$$

Let the aggregate wage be the employment-weighted mean:

$$W \equiv s w_H + (1-s) w_L.$$

Define relative wages:

$$x_j \equiv \frac{w_j}{W}.$$

We impose $\mu = \frac{1}{2}$ and allow $\chi \geq 0$. Let the steady-state DNWR cost-level wedge be $\lambda_j/(1+\pi)$. Let the marginal revenue product of labor be $MRPL_j = \alpha a_j n_j^{\alpha-1}$. Vacancy posting costs are convex:

$$C(v_j, n_j) = \frac{c^v}{\chi+1}\left(\frac{v_j}{n_j}\right)^{\chi} v_j, \qquad \frac{\partial C}{\partial v_j} = c^v \left(\frac{v_j}{n_j}\right)^{\chi}.$$

Steady-state accounting. Employment evolves via retention plus hiring:

$$n_j = \left[1 - \delta(x_j)\right] n_j + m_j \quad \Longrightarrow \quad m_j = \delta_j n_j. \tag{C.1}$$

With $\mu = \frac{1}{2}$, matches are

$$m_j = v_j^{1/2} u^{1/2}. \tag{C.2}$$

Combining yields

$$\delta_j n_j = v_j^{1/2} u^{1/2} \quad \Longrightarrow \quad v_j = \frac{\delta_j^2}{u} n_j^2. \tag{C.3}$$

From the vacancy FOC, we have:

$$\left(\text{MRPL}_j - w_j - \frac{\lambda_j}{1+\pi}\right)\frac{\partial m_j}{\partial v_j} = \frac{\partial C}{\partial v_j}. \tag{C.4}$$

Since $\frac{\partial m_j}{\partial v_j} = \frac{1}{2} v_j^{-1/2} u^{1/2}$, we obtain

$$\left(\text{MRPL}_j - w_j - \frac{\lambda_j}{1+\pi}\right)\frac{1}{2} v_j^{-1/2} u^{1/2} = c^v \left(\frac{v_j}{n_j}\right)^{\chi}.$$

Solving for the wedge and using $v_j = \frac{\delta_j^2}{u} n_j^2$, we obtain:

$$\text{MRPL}_j - w_j - \frac{\lambda_j}{1+\pi} = 2 c^v \delta_j^{2\chi+1} n_j^{\chi+1} u^{-(\chi+1)}. \tag{C.5}$$

Wage FOC and Closed-Form Wage Schedule. The wage FOC implies

$$\text{MRPL}_j - w_j - \frac{\lambda_j}{1+\pi} = \frac{w_j}{\epsilon^{\delta} \delta_j}. \tag{C.6}$$

Equating gives

$$w_j = 2 c^v \epsilon^{\delta} u^{-(\chi+1)} \delta_j^{2\chi+2} n_j^{\chi+1}. \tag{C.7}$$

Equilibrium System. The two steady-state conditions are

$$\alpha a_L n_L^{\alpha-1} - \frac{\lambda_L}{1+\pi} = 2c^v u^{-(\chi+1)} \left(\epsilon^\delta \delta_L(r) + 1\right) \delta_L(r)^{2\chi+1} n_L^{\chi+1}, \tag{C.8}$$

$$\alpha a_H (r n_L)^{\alpha-1} - \frac{\lambda_H}{1+\pi} = 2c^v u^{-(\chi+1)} \left(\epsilon^\delta \delta_H(r) + 1\right) \delta_H(r)^{2\chi+1} (r n_L)^{\chi+1}. \tag{C.9}$$

These determine (r, n_L). Wages follow from the closed-form wage schedule.

Relative wages x_H, x_L in terms of (q, r). Using $W = s w_H + (1-s) w_L$ and $s = r/(1+r)$,

$$\frac{W}{w_L} = s\frac{w_H}{w_L} + (1-s) = sq + (1-s) = \frac{rq+1}{1+r}. \tag{C.10}$$

Hence

$$x_L \equiv \frac{w_L}{W} = \frac{1}{sq+(1-s)} = \frac{1+r}{1+rq}, \qquad x_H \equiv \frac{w_H}{W} = \frac{q}{sq+(1-s)} = \frac{q(1+r)}{1+rq} = q\, x_L. \tag{x(q,r)}$$

C.2 Proof of Proposition 1

Under $\alpha = 1$, the steady-state conditions derived in the text imply, for each $j \in \{H, L\}$,

$$a_j - \frac{\lambda_j}{1+\pi} = 2c^v u^{-(\chi+1)} (1 + \epsilon^\delta \delta_j)\, \delta_j^{2\chi+1}\, n_j^{\chi+1}. \tag{C.11}$$

Taking ratios across types yields the following relationship:

$$\frac{a_H - \frac{\lambda_H}{1+\pi}}{a_L - \frac{\lambda_L}{1+\pi}} = \frac{1+\epsilon^\delta \delta_H}{1+\epsilon^\delta \delta_L} \left(\frac{\delta_H}{\delta_L}\right)^{2\chi+1} r^{\chi+1}, \qquad r \equiv \frac{n_H}{n_L}. \tag{C.12}$$

Next, the closed-form wage schedule obtained by equating the vacancy and wage first-order conditions implies

$$q \equiv \frac{w_H}{w_L} = \left(\frac{\delta_H}{\delta_L}\right)^{2\chi+2} r^{\chi+1}. \tag{C.13}$$

Separations satisfy $\delta(x) = \bar{\delta} x^{-\epsilon^\delta}$. Since the wage aggregator implies $x_H/x_L = q$, we have

$$\frac{\delta_H}{\delta_L} = q^{-\epsilon^\delta}. \tag{C.14}$$

Combining (C.13) and (C.14) yields

$$q^{1+2(\chi+1)\epsilon^\delta} = r^{\chi+1} \quad \Longrightarrow \quad q = r^{\frac{\chi+1}{1+2(\chi+1)\epsilon^\delta}}, \qquad \frac{\delta_H}{\delta_L} = r^{-\epsilon^\delta \frac{\chi+1}{1+2(\chi+1)\epsilon^\delta}}. \tag{C.15}$$

In the following, we suppose $\epsilon^\delta \delta_j$ is small enough that

$$\frac{1+\epsilon^\delta \delta_H}{1+\epsilon^\delta \delta_L} \approx 1. \tag{C.16}$$

Closed forms under (C.16). Using (C.16) in (C.12) and substituting (C.15) gives

$$\begin{aligned} \frac{a_H - \frac{\lambda_H}{1+\pi}}{a_L - \frac{\lambda_L}{1+\pi}} &\approx \left(r^{-\epsilon^\delta \frac{\chi+1}{1+2(\chi+1)\epsilon^\delta}}\right)^{2\chi+1} r^{\chi+1} \\ &= r^{(\chi+1) - \epsilon^\delta \frac{(\chi+1)(2\chi+1)}{1+2(\chi+1)\epsilon^\delta}} = r^{\frac{(\chi+1)(1+\epsilon^\delta)}{1+2(\chi+1)\epsilon^\delta}}. \end{aligned} \tag{C.17}$$

Rearranging yields (24). Combining (24) with the first equality in (C.15) yields (25). In particular,

$$\log q = \frac{1}{1+\epsilon^{\delta}} \left[\log\Big(a_H - \frac{\lambda_H}{1+\pi}\Big) - \log\Big(a_L - \frac{\lambda_L}{1+\pi}\Big) \right]. \tag{C.18}$$

Effect of separation elasticity. Define

$$\Delta(\pi) \equiv \log\Big(a_H - \frac{\lambda_H}{1+\pi}\Big) - \log\Big(a_L - \frac{\lambda_L}{1+\pi}\Big).$$

Then $\log r = \kappa_r(\chi, \epsilon^{\delta})\Delta(\pi)$, and

$$\frac{\partial \kappa_r(\chi, \epsilon^{\delta})}{\partial \epsilon^{\delta}} = \frac{2\chi + 1}{(\chi+1)(1+\epsilon^{\delta})^2} > 0. \tag{C.19}$$

If $a_H - \frac{\lambda_H}{1+\pi} > a_L - \frac{\lambda_L}{1+\pi}$, then $\Delta(\pi) > 0$, so $\partial \log r / \partial \epsilon^{\delta} > 0$. Moreover, differentiating (C.18) yields

$$\frac{\partial \log q}{\partial \epsilon^{\delta}} = -\frac{1}{(1+\epsilon^{\delta})^2}\Delta(\pi) < 0, \tag{C.20}$$

so higher separation elasticity decreases relative wages.

Effect of trend inflation. Using $a_j - \frac{\lambda_j}{1+\pi} > 0$,

$$\frac{\partial}{\partial\,(1+\pi)} \log\Big(a_j - \frac{\lambda_j}{1+\pi}\Big) = \frac{1}{a_j - \frac{\lambda_j}{1+\pi}} \cdot \frac{\lambda_j}{(1+\pi)^2}, \qquad i \in \{H, L\},$$

hence

$$\frac{\partial \Delta}{\partial\,(1+\pi)} = \frac{1}{(1+\pi)^2} \left(\frac{\lambda_H}{a_H - \frac{\lambda_H}{1+\pi}} - \frac{\lambda_L}{a_L - \frac{\lambda_L}{1+\pi}} \right). \tag{C.21}$$

Because denominators are positive, the bracketed term satisfies

$$\begin{aligned} \frac{\lambda_H}{a_H - \frac{\lambda_H}{1+\pi}} > \frac{\lambda_L}{a_L - \frac{\lambda_L}{1+\pi}} \quad &\iff \quad \lambda_H\Big(a_L - \frac{\lambda_L}{1+\pi}\Big) > \lambda_L\Big(a_H - \frac{\lambda_H}{1+\pi}\Big) \\ &\iff \quad \lambda_H a_L > \lambda_L a_H \quad \iff \quad \frac{\lambda_H}{a_H} > \frac{\lambda_L}{a_L}. \end{aligned}$$

This implies $\partial\Delta/\partial\pi > 0$ under the condition $\frac{\lambda_H}{a_H} > \frac{\lambda_L}{a_L}$. Since $\kappa_r(\chi, \epsilon^{\delta}) > 0$ and $\kappa_q(\epsilon^{\delta}) > 0$, it follows from $\log r = \kappa_r(\chi, \epsilon^{\delta})\Delta(\pi)$ and (C.18) that $\partial \log r / \partial\,(1+\pi) > 0$ and $\partial \log q / \partial\,(1+\pi) > 0$. Therefore higher trend inflation raises both relative employment and relative wages under $\frac{\lambda_H}{a_H} > \frac{\lambda_L}{a_L}$.

C.3 Proof of Proposition 2

We define the inverse wage markdown for firm type j as the ratio of the wage to the marginal revenue product of labor:

$$\mu_j \equiv \frac{w_j}{\mathrm{MRPL}_j}, \qquad \mathrm{MRPL}_j \equiv \alpha a_j n_j^{\alpha-1}. \tag{C.22}$$

Note that the condition $\mu_j < 1$ indicates that wages are below the marginal revenue product of labor, reflecting monopsony power.

The wage first-order condition implies:

$$\text{MRPL}_j = w_j + \frac{\lambda_j}{1+\pi} + \frac{w_j}{\epsilon^\delta \delta_j}, \tag{C.23}$$

where $\delta_j = \bar{\delta} x_j^{-\epsilon^\delta}$. Dividing both sides by MRPL_j and using $\mu_j = w_j / \text{MRPL}_j$ yields:

$$1 = \mu_j + \frac{\lambda_j}{(1+\pi)\ \text{MRPL}_j} + \frac{\mu_j}{\epsilon^\delta \delta_j}. \tag{C.24}$$

Solving for μ_j gives

$$\mu_j = \frac{1-\phi_j}{1+\psi_j}, \qquad \phi_j \equiv \frac{\lambda_j}{(1+\pi)\ \text{MRPL}_j}, \qquad \psi_j \equiv \frac{1}{\epsilon^\delta \delta_j}. \tag{C.25}$$

Expression (C.25) decomposes the inverse wage markdown into two wedges: (i) the DNWR wedge ϕ_j and (ii) the monopsony/retention wedge ψ_j.

Effect of separation elasticity. Holding δ_j and MRPL_j fixed,

$$\frac{\partial \psi_j}{\partial \epsilon^\delta} = -\frac{1}{\left(\epsilon^\delta\right)^2 \delta_j} < 0. \tag{C.26}$$

Since

$$\frac{\partial \mu_j}{\partial \psi_j} = -\frac{1-\phi_j}{(1+\psi_j)^2},$$

we obtain

$$\frac{\partial \mu_j}{\partial \epsilon^\delta} = \frac{1-\phi_j}{(1+\psi_j)^2} \cdot \frac{1}{\left(\epsilon^\delta\right)^2 \delta_j} > 0, \tag{C.27}$$

provided $\text{MRPL}_j > \frac{\lambda_j}{1+\pi}$. Thus, higher separation elasticity raises μ_j, meaning wages move closer to the marginal revenue product of labor.

Effect of trend inflation. Holding δ_j and MRPL_j fixed,

$$\frac{\partial \phi_j}{\partial (1+\pi)} = -\frac{\lambda_j}{(1+\pi)^2\ \text{MRPL}_j} < 0. \tag{C.28}$$

Because

$$\frac{\partial \mu_j}{\partial \phi_j} = -\frac{1}{1+\psi_j},$$

we obtain

$$\frac{\partial \mu_j}{\partial (1+\pi)} = \frac{1}{1+\psi_j} \cdot \frac{\lambda_j}{(1+\pi)^2\ \text{MRPL}_j} > 0. \tag{C.29}$$

Therefore, higher trend inflation increases the wage-to-MRPL ratio by compressing the DNWR wedge.