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Developments in the Natural Rate of Interest in Japan

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The natural rate of interest is the real interest rate at which economic activity and prices neither accelerate nor decelerate. The basic mechanism of monetary easing -- regardless of whether it is conducted through conventional or unconventional policy means -- consists of driving the real interest rate below the natural rate of interest. Theoretically, therefore, in order to assess the effects of monetary easing, it is necessary to estimate the natural rate of interest, which is by nature unobservable, and determine whether the real interest rate is higher or lower, relative to the estimated natural rate of interest. This paper estimates the natural rate of interest using a number of different approaches. While the estimates differ to some extent depending on the approach taken, the estimation results suggest that it is likely that Japan's natural rate of interest is currently at a low level of around 0 percent.

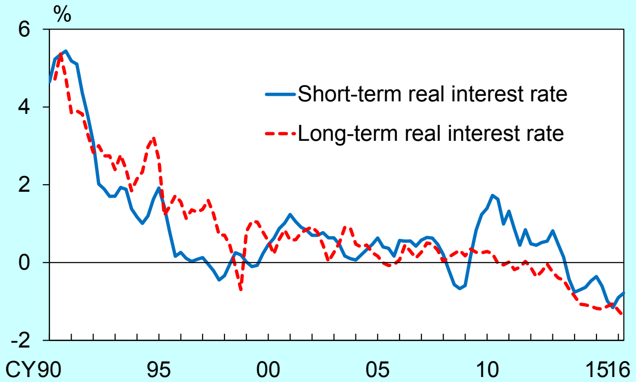
Introduction[1]

The natural rate of interest is the real interest rate at which economic activity and prices neither accelerate nor decelerate. It is also referred to as the equilibrium real interest rate, and, from a theoretical perspective, it can also be defined as the real interest rate which balances savings and investment under full employment. In other words, other things being equal, output falls below the level of full employment, lowering price levels, when the real interest rate exceeds the natural rate of interest (that is, when the real interest rate gap is positive), and output and prices increase when the case is otherwise.

The basic mechanism of monetary easing -- regardless of whether it is conducted through conventional or unconventional policy means -- consists of driving the real interest rate below the natural rate of interest. As shown in Chart 1, real interest rates in Japan have been negative, for both long-term and short-term rates, as nominal interest rates have fallen due to both the downward shift of the entire yield curve and the rise in inflation expectations since the introduction of Quantitative and Qualitative Monetary Easing (QQE).[2] Theoretically, in order to assess the effects of such a decline in real interest rates, it is necessary to estimate the natural rate of interest, which is not observable, and determine whether the real interest rate is higher or lower, relative to the natural rate of interest.

This paper uses a number of different approaches to estimate the natural rate of interest,[3] and provides an assessment of developments in the natural rate of interest following the introduction of QQE.

[Chart 1] Real Interest Rates



Note: The short-term real interest rate is calculated as the uncollateralized overnight call rate minus year-on-year growth rate of the Consumer Price Index (all items, less fresh food and energy, adjusted to exclude the estimated effect of the change in the consumption tax rate). The long-term real interest rate is calculated as 10-year JGB yields minus the forecast of inflation rate for 6 to 10 years taken from "Consensus Forecasts." Semiannual data from the "Consensus Forecasts" up through 2014/Q2 are linearly interpolated.

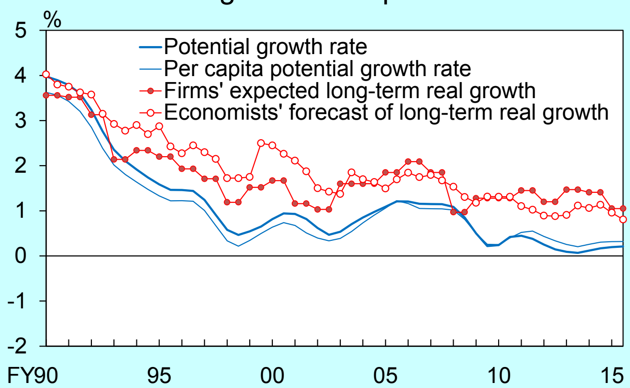
Sources: Consensus Economics Inc., "Consensus Forecasts"; Ministry of Internal Affairs and Communications; Bloomberg; Bank of Japan.

Natural Interest Rate Proxies from a Long-Term Perspective

Under certain conditions, the natural rate of interest coincides in the long run with the potential growth rate. The text book growth theory of the long-run relationship between macroeconomic variables predicts that the natural rate of interest is approximately equal to the potential growth rate or per capita potential growth rate, when an economy is on a balanced growth path where output, consumption, and capital stock grow at a constant rate.[4,5] In the practice of economic analysis, therefore, the potential growth rate is often used as a measure of the natural rate of interest.

Based on the Bank of Japan's estimates, as shown in Chart 2, the potential growth rate has been declining since the 1990s, and has been in the range of 0.0-0.5 percent in recent years. A similar pattern is observed for the potential growth rate per capita, a series for which effects due to population growth rate are excluded. Meanwhile, other indicators related to the potential growth rate, including the long-term forecast of firms and economists on real economic growth, are all currently approximately 1 percent, a rate slightly higher than the potential growth rate.

[Chart 2] Natural Interest Rate Proxies from a Long-Term Perspective



Note: Firms' expected long-term real growth is firms' forecast of real economic growth rates for the next 5 years taken from the "Annual Survey of Corporate Behavior." Economists' forecast of long-term real growth is forecast of real GDP growth rate for 6 to 10 years ahead taken from "Consensus Forecasts."

Sources: Consensus Economics Inc., "Consensus Forecasts"; Ministry of Internal Affairs and Communications; Cabinet Office, etc.

Estimating the Natural Rate of Interest

In the short run, the natural rate of interest deviates from the potential growth rate as it is influenced by factors such as business cycles. Consider a state of the economy where the natural rate of interest coincides with the real interest rate as well as the potential growth rate, the output gap is zero, and the inflation rate is equal to the price stability goal set by the central bank. Suppose further that a negative demand shock occurs in this economy. The output gap becomes negative and the inflation rate falls, if the real interest rate is unchanged from the level of the natural rate of interest before the shock. The real interest rate needs to be cut to maintain full employment. This, by definition, indicates that the natural rate of interest falls due to a negative demand shock and therefore deviates from the potential growth rate in the short-run so long as the potential growth rate itself is unaffected by the shock.

In order to estimate the time path of the natural rate of interest, it is therefore desirable not only to track the secular movements of the potential growth rate, an approximate value in the long-run, but also to incorporate the effects of short-run fluctuations including business cycles into the estimation. There are already a good number of studies about how the natural rate of interest should be estimated, and various approaches have been proposed. In what follows, an estimation of the natural rate of interest in Japan is calculated based on three approaches proposed in the existing studies.

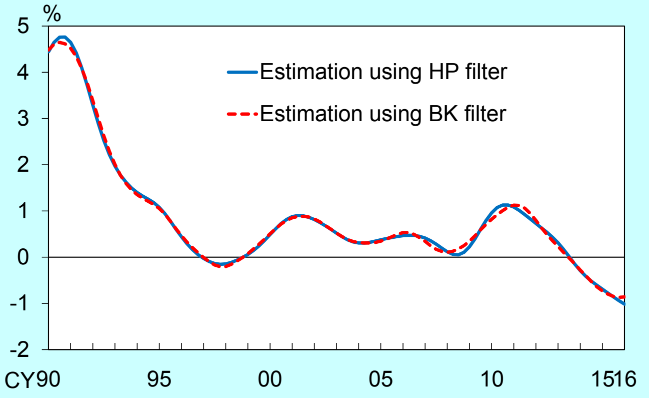
The trend component of the short-term real interest rate

The simplest way to estimate the natural rate of interest is to extract the trend component of the realized short-term real interest rate. The realized short-term real interest should evolve closely around the time path of the natural rate of interest when the central bank follows a variant of the Taylor rule (raising the short-term interest rate in response to an inflationary pressure and excessive demand, and cutting the rate if otherwise), and when such a policy implementation is quickly translated into adjustments in economic activities and prices.[6] When this is the case, the estimated trend component of the short-term real interest rate may be treated as an estimate of the natural rate of interest.

Chart 3 shows the time path of the trend component of the real interest rate in Japan as estimated by two filters widely used in empirical analysis in economics: the Hodrick–Prescott (HP) filter, and the Baxter-King (BK) filter.[7,8] The trend components of the real interest rates calculated using the HP and BK filters both stood at more than 1 percent around 2010, but subsequently declined sharply and recently reached around minus 1 percent.

These results appear to reflect closely the fact that the estimated series are essentially the moving average of actual real interest rates that have substantially declined since the introduction of QQE.

[Chart 3] Trend Component of Short-Term Real Interest Rate



The Laubach-Williams model

Computing the trend components of short-term real interest rates is relatively easy and simple to implement. It comes at a cost, however, as this approach does not have a strong theoretical foundation. Because it uses the short-term real interest rate as its only input in the estimation process, there is no guarantee, even from a theoretical point of view, that the estimated trend component of the real interest rate is associated with full employment, being contrary to the definition of the natural rate of interest. Several studies therefore propose an alternative approach that makes explicit use of structural relationships between short-term real interest rates, inflation rate, and output gap, and uses these variables as inputs in its estimation. The best-known example of these approaches is the one proposed by Laubach and Williams, economists at the FRB.[9]

The key structural relationships that are considered in the Laubach-Williams model are the three equations shown in Chart 4 (See the Box below for details of the model). The first is the IS curve and shows that the output gap widens positively as the real interest rate gap, defined as the short-term real interest rate minus the natural rate of interest, widens negatively. If the lagged effects of the past values of output gaps are absent, the state where the short-term real interest rate gap is zero is equivalent to the state where the current output gap is zero and no accelerating or deceleration pressures are at work in the economy. The second equation is the Phillips curve, and this shows that inflation increases as the output gap widens positively. The third equation shows the relationship between the natural rate of interest, the potential growth rate, and demand shocks. The natural rate of interest is higher when the potential output grows faster and a positive demand shock is larger.[10] The two unobservable variables, the natural rate of interest and the potential growth rate, are estimated using the Kalman filter, with the observed time series of short-term real interest rates, output, and inflation rates as inputs, based on these structural equations.

[Chart 4] The Laubach-Williams Model

(i) IS curve

Current output gap = *f* (Past output gap (+), Short-term real interest rate gap (−))

Effects on the left hand side of the equation

(ii) Phillips curve

Current inflation rate = *g* (Past inflation rates (+), Past output gap (+))

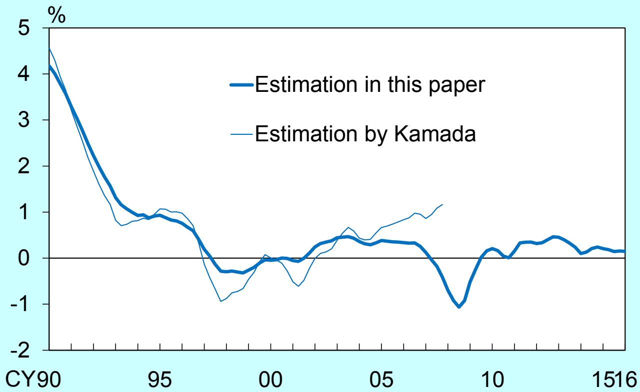
(iii) Natural rate of interest

Current natural rate of interest = *h* (Potential growth rate (+), Demand shock etc (+))

⇒Estimating the natural rate of interest and the potential output based on these equations

The chart in the Box shows the model parameters estimated using data up to 2016Q1, a period after the introduction of "QQE with a Negative Interest Rate." The signs of the estimated parameters in the three structural equations are all in accord with the theoretical prediction. Chart 5 shows the time path of the estimated natural rate of interest. It began to decline in the 1990s, and fell below zero during Japan's financial crisis of the late 1990s, and during the global financial crisis in 2008. It has been about 0% since around the year 2010.[11]

[Chart 5] Estimates of the Natural Interest Rate using the Laubach-Williams Model



Source: Kamada, K. [2009] "Japan's equilibrium real interest rate," in K. Fukao (ed.), *Macroeconomy and Industrial Structures*, pp. 387-427, Keio University Press (in Japanese).

The natural yield curve model

These two approaches to estimating the natural rate of interest, extracting the trend component of real interest rates and the Laubach-Williams model, both focus on the short-term interest rate that has been the key policy instrument in conventional monetary policy. In contrast, the monetary easing implemented by the Bank of Japan so far, namely QQE, "QQE with a Negative Interest Rate," and "QQE with Yield Curve Control," have all been exerting downward pressure on the entire yield curve. It is therefore important to extend the idea of the natural rate of interest beyond the short-term interest rate and consider the natural yield curve when assessing the effects of monetary easing that work on the entire yield curve. The analysis below estimates the natural yield curve following the approach proposed by Imakubo, Kojima, and Nakajima, economists at the Bank of Japan, extending their data set to the period of "QQE with a Negative Interest Rate" and beyond.[12]

Similar to the Laubach-Williams model, the natural yield curve is estimated based on the theoretical relationship characterized by the IS curve.[13] Because yield curves evolve in a complicated manner over time, the yield curve gap, which is essentially the real interest rate gap over the entire yield curve, is summarized by three elements of the yield curve gap that are estimated following the Nelson-Siegel model: the level gap, the slope gap, and the curvature gap. The yield curve gap is then described as the relationship between the output gap and these three elements, as shown in Chart 6. Here, the level captures a parallel shift of the entire yield curve, the slope captures a steepening (or flattening) of the slope as a result of a decline (or rise) in short-term yields, and the curvature captures a decline (or rise) in medium-term yields, resulting in a downward (or upward) bending of the curve, respectively.

[Chart 6] Illustration of the Natural Yield Curve

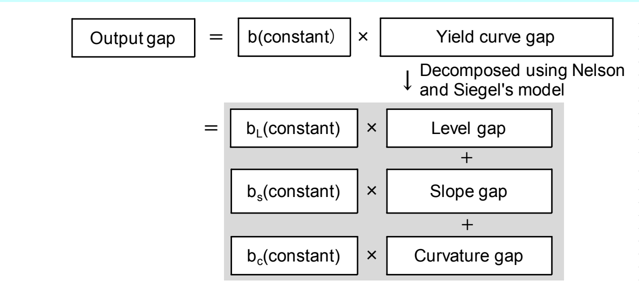


Chart 7 shows some key estimation results from the model, those for the IS curve, based on the sample period that runs up to 2016Q1.[14] The coefficients attached to the level, slope, and curvature gap, are all negative and statistically significant.

[Chart 7] Estimates of Parameters in the IS Curve

Parameter	Estimates (std. err.)
Coefficient on level gap (b_L)	−0.197 (0.047)
Coefficient on slope gap (b_S)	−0.120 (0.015)
Coefficient on curvature gap (b_C)	−0.037 (0.015)

Assuming a specific probability distribution, it is possible to determine the effects of a change in the yield curve gap at different maturities on the output gap. Chart 8 shows the coefficient estimates on the yield curve gap at different maturities, when Beta mixture distribution is assumed. The results indicate that the absolute values of the coefficient estimates are the largest for short maturities, but then gradually diminish as maturities increase,[15] implying that declines in short- and medium-term yields have larger monetary easing effects than declines in long- and super-long-term yields.[16]

[Chart 8] Coefficient Estimates on the Yield Curve Gap at Different Maturities

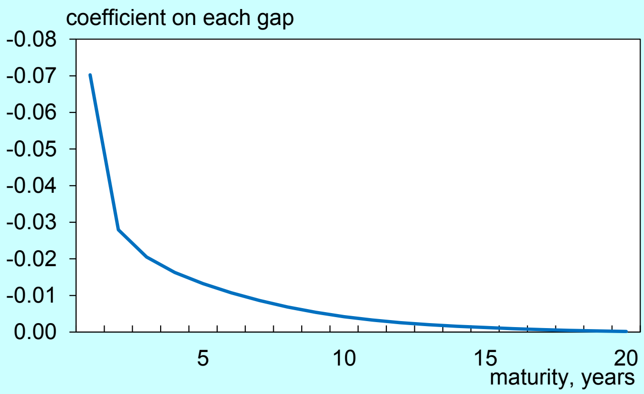
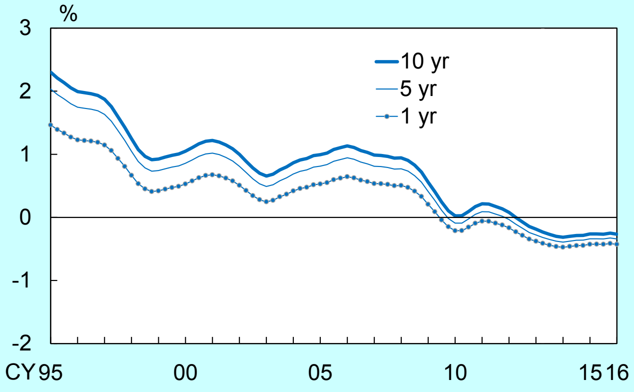


Chart 9 shows the time path of the estimated natural rate of interest in the short-tem (one year), mid-term (five years), and long-term (ten years). For all maturities, interest rates show a declining trend and are about zero or slightly negative since around the year 2010. In addition, the yield curve is flattening, as shown by the shrinkage of spread between 10-year yield and 1-year yield.

[Chart 9] Estimation using the Natural Yield Curve Model



Concluding remarks

This paper uses a number of different approaches to estimate the natural rate of interest in Japan. The results show that the natural rate of interest has followed a downward trend since the 1990s, reflecting the slowdown in the potential growth rate. They also suggest that the natural rate of interest is likely to be around 0% for recent years, while care must be exercised when interpreting these results as estimates of the natural rate of interest differ depending on the method used.

Given that real interest rates have been negative, both short-term and long-term, since the introduction of QQE, it is considered that they are at levels well below the natural rate of interest. This suggests that Japan's financial conditions can be judged to be highly accommodative, even though the natural rate of interest has exhibited a secular decline.

Finally, it is noteworthy that there is non-negligible uncertainty regarding estimates of the natural rate of interest, regardless of which economy is being analyzed. Figures calculated using similar methodologies can differ if there are disagreements in the details of assumptions regarding the economic structure in the model.[17] In addition, there is the real-time estimation problem, in that there is always the possibility of a change in the estimates of past values of the natural rate of interest due to the extension of the data sample or revisions in the data used as inputs. It is thus important to understand these issues and to continue studies of the natural rate of interest based on a broad range of approaches and data.

【BOX】Estimation using the Laubach-Williams model

In Laubach and Williams's approach, the IS curve and the Phillips curve are given by[18]

$$y_t = y_t^* + \alpha_1(y_{t-1} - y_{t-1}^*) + \alpha_2(y_{t-2} - y_{t-2}^*) - \frac{\alpha_3(r_{t-1} - r_{t-1}^* + r_{t-2} - r_{t-2}^*)}{2} + \varepsilon_t^y, \tag{1}$$

$$\pi_t = \beta_1\pi_{t-1} + \beta_2 \sum_{i=2}^{4} \frac{\pi_{t-i}}{3} + (1 - \beta_1 - \beta_2) \sum_{i=5}^{8} \frac{\pi_{t-i}}{4} + \beta_3(y_{t-1} - y_{t-1}^*) + \beta_4(\pi_t^I - \pi_t) + \beta_5(\pi_t^O - \pi_t) + \varepsilon_t^\pi, \tag{2}$$

where y denotes the output, y^* is the potential output, r and r^* are the actual short-term real interest rate and the natural rate of interest, respectively; π is the inflation rate of the general price, π^I is the inflation rate of the import price, and π^O is the inflation rate of oil price; ε_t^y and ε_t^π are error terms.

The law of motion of the unobservable variables, y^* and r^*, is specified as follows:

$$y_t^* = y_{t-1}^* + g_{t-1} + \varepsilon_t^{y^*}, \tag{3}$$

$$g_t = g_{t-1} + \varepsilon_t^g, \tag{4}$$

$$r_t^* = cg_t + z_t, \tag{5}$$

$$z_t = z_{t-1} + \varepsilon_t^z, \tag{6}$$

where g denotes the trend of the potential output growth; ε^{y^*} and ε^g are temporary and persistent shock to the potential growth rate, respectively; z represents all other factors, such as changes in the discount rate, that affect the potential growth rate, and ε_t^z denotes a shock that varies z, including a shock to the discount rate.

The estimation procedure is as follows. First, by substituting Equation (5) into (1), a state space representation with five equations is obtained; Here, Equations (1) and (2) are observation equations; Equations (3), (4), and (6) are state equations. Using the Kalman filter, the unobservable variables, y^*, g, and z, and parameters are estimated. Second, the estimate of r^* is obtained by substituting estimates of those variables into Equation (5). The chart below shows the result of the estimation using Japanese data.[19]

【BOX Chart】Estimation Result

Parameters	Estimates (std. err.)	Parameters	Estimates (std. err.)
α_1	0.71 (0.24)	c	0.70 (0.27)
α_2	0.04 (0.14)	std. dev. of ε^y	0.43 (0.19)
α_3	0.06 (0.04)	std. dev. of ε^π	1.43 (0.19)
β_1	0.19 (0.08)	std. dev. of ε^{y^*}	0.85 (0.11)
β_2	0.68 (0.16)	std. dev. of ε^g	0.15 (0.04)
β_3	2.41 (1.52)	std. dev. of ε^z	0.64
β_4	0.01 (0.01)		
β_5	0.01 (0.00)	MUE	$\lambda_z = 0.064$

Note: Existing studies point out that the estimated standard deviation of ε^z may be biased toward 0, owing to the so-called *pile-up* problem, if it is estimated directly using the Kalman filter. In order to avoid this issue, the standard deviation is estimated in the following two steps. First, a value $\lambda_z \equiv (\text{std. dev. of } \varepsilon^z / \text{std. dev. of } \varepsilon^y)\alpha_3/\sqrt{2}$ is estimated using a median-unbiased estimator (MUE). Second, the standard deviation of ε^z is calculated using the estimates of λ_z together with estimates of the standard deviation of ε^y and α_3.[20]

[1] This article is a supplement to *Comprehensive Assessment: Developments in Economic Activity and Prices as well as Policy Effects since the Introduction of Quantitative and Qualitative Monetary Easing (QQE)* released by the Bank of Japan in September 2016.

[2] Figures for real interest rates vary depending on what series is used as expected inflation rates when calculated. In Chart 1, actual inflation rates and economists' forecasts are used as a proxy of the short-term and the long-term expected inflation rates, respectively.

[3] Pioneering studies that estimate the natural rate of interest in Japan include, Oda and Muranaga [2003], and Kamada [2009]. In particular, Kamada [2009] estimates the natural rate of interest based on several different approaches. Some of the approaches described in the current paper are also studied in his paper.

Oda, N. and J. Muranaga [2003] "On the Natural Rate of Interest: Theory and Estimates," Bank of Japan Working Paper Series, No.03-J-5 (in Japanese).

Kamada, K. [2009] "Japan's Equilibrium Real Interest Rate," in K. Fukao (ed.), *Macroeconomy and Industrial Structures*, pp. 387-427, Keio University Press (in Japanese).

[4] In general, the natural rate of interest coincides with the potential growth rate when the following two conditions are met. First, the intertemporal elasticity of substitution of households in the economy is unity, which implies that, other things being equal, a 1% point change in the real interest rate is translated into a 1% point change in real consumption growth. Second, the discount rate is zero, which implies that households receive the same utility flow from consumption in the current and next period. See Oda and Muranaga [2003] for details of the related discussion.

[5] In a representative agent model, an arbitrage condition, the so-called Euler equation, holds for the relationship between the per capita consumption growth and real interest rate, as a consequence of a household's optimization behavior. Using per capita potential growth rates as a proxy for the natural rate of interest is to some extent consistent with this theoretical prediction. In a heterogeneous agent model, such as an overlapping generations model, however, the natural rate of interest may be affected by changes in demographics of the economy including the relative size of youth and elderly population, making the relationship between the natural rate of interest and demographics more complex. See the following for the relationship between the natural rate of interest and demographics.

Hirata, W. [2012] "Population Growth and Economic Growth: Lessons Learned from a Growth Theory," *Kinyu- Kenkyu*, 31(2), pp.121-162 (in Japanese).

[6] The Taylor rule is a policy rule for nominal interest rate adjustments. Under some assumptions, however, it can be interpreted as a rule for real interest rate adjustments that sets real interest rates to a value higher than the natural rate of interest in response to a price increase and excessive demand, and sets real interest rates to a value below the natural rate of interest if the case is otherwise.

[7] The HP filter and BK filter are proposed by Hodrick and Prescott [1997] and Baxter and King [1999], respectively. See also Kamada [2009] for the similarities and differences between the two filters.

Hodrick, R. J. and E. C. Prescott [1997] "Post-War U.S. Business Cycles: An Empirical Investigation," *Journal of Money, Credit and Banking*, 29(1), pp.1-16.

Baxter, M. and R. G. King [1999] "Measuring Business Cycles: Approximate Band-Pass Filters for Economic Time Series," *Review of Economics and Statistics*, 81(4), pp.575-593.

[8] In estimating the trend components, the uncollateralized overnight call rate (figures for 1985Q2 and before are estimated using the collateralized overnight call rate) is used as a measure of the short-term interest rate and year-on-year growth rate of the Consumer Price Index (all items, less fresh good and energy, adjusted to exclude seasonality and the estimated effect of changes in the consumption tax rate) is used as a measure of the expected inflation rate. The value of the smoothing parameter in the HP filter and the length of filters and the ranges of the frequency in the BK filter are set following Kamada [2009]. The sample period runs from 1981Q1 to 2016 Q1.

[9] See Laubach and Williams [2003]. Other approaches that explicitly use the structural relationship between variables include NAILO （non-accelerating inflation level of output） proposed by Kamada and Hirose [2001], and NAIRI （non-accelerating inflation rate of interest） proposed by Kamada [2009].

Laubach, T. and J. C. Williams [2003] "Measuring the Natural Rate of Interest," *Review of Economics and Statistics*, 85(4), pp.1063-1070.

Hirose, Y. and K. Kamada [2001] "A New Technique for Simultaneous Estimation of the Output Gap and Phillips Curve," Bank of Japan Working Paper Series, 01-7.

[10] The two conditions listed in the footnote 4 above are not imposed when estimating the system described in Chart 4.

[11] It is pointed out that estimates of the natural rate of interest using the Laubach-Williams model may be affected by the estimation strategy, including the initial values used for the Kalman filter. The estimates reported in the current paper are calculated following closely the strategy adopted in Kamada [2009]. As a result, the time path of the estimated natural rate of interest tracks closely that of the natural rate of interest reported in his paper.

[12] See Imakubo, Kojima, and Nakajima [2015a, b].

Imakubo, K., H. Kojima, and J. Nakajima [2015a] "The Natural Yield Curve: Its Concept and Measurement," Bank of Japan Working Paper Series, No.15-E-5.

Imakubo, K., H. Kojima, and J. Nakajima[2015b] "The Natural Yield Curve: Its Concept and Developments in Japan," Bank of Japan Research Laboratory Series, No.15-E-3.

[13] Following the discussion in Clark and Kozicki [2005], when estimating the model of the natural yield curve, the output gap and potential growth rate are treated as the observable variables, and the theoretical relationship characterized by the Phillips curve is not imposed.

Clark, T. E. and S. Kozicki [2005] "Estimating Equilibrium Real Interest Rates in Real Time," *North American Journal of Economics and Finance*, 16(3), pp.395-413.

[14] The model and data set used in the current estimation are the same as those used in Imakubo, Kojima, and Nakajima [2015a, b]. For the data, the government bond yields with maturity of 1, 2, 3, 7, 10, and 20 years, inflation expectations by maturity obtained from the Consensus Forecasts, and estimates by the Bank of Japan are used for the output gap and the potential growth rate, respectively. Only the estimates of the IS curve are reported in the Box since the estimates are little changed from those reported in Imakubo, Kojima, and Nakajima [2015a, b], in terms of the sizes of parameters and their statistical significance, though the sample period is extended by five quarters to 2016Q1 from the sample period used in their paper.

[15] A similar pattern regarding differences across maturity is obtained when an alternative distribution, such as step distribution, is assumed instead. This pattern seems to be

related to the fact that in addition to the contribution of variations in the level gap, that of variations in slope gap to variations in output gap is particularly large.

[16] This finding seems to reflect the fact that short- to medium-term funds account for a large part of borrowing by firms and households. It should be noted, however, that there is some evidence for changes in the funding behavior of borrowing firms, including an increase in the issuance of super-long-term corporate bonds, as a result of the unprecedentedly low interest rate environment. The qualitative and quantitative relationship obtained here may vary if such developments result in a substantial change in firms' financial structures.

[17] One other approach that is not discussed in this paper is the methodology that uses the dynamic stochastic general equilibrium (DSGE) model (see for example, Kamada [2009], and Iwata and Samikawa et al. [2016] for studies that estimate the natural rate of interest using DSGE). This approach overcomes the Lucas critique, and is practically useful because it can account for variations in the natural rate of interest by decomposing them into structural shocks. The estimated results sometimes differ across models, however, depending on the model settings.

Iwata, K., I. Samikawa, and Japan Center for Economic Research [2016] "Negative Interest Rate Policy, Effect and Limits of the Monetary Easing in terms of Three Dimensions," Nikkei Publishing Inc. (in Japanese).

[18] Strictly speaking, the model used in the current paper is a version of the model of Laubach and Williams [2003] that is constructed by Kamada [2009].

[19] The data set used in the estimation is essentially the same as that used in Kamada [2009]. It consists of real GDP (the logarithm), year-on-year growth rate of the Consumer Price Index (all items, less fresh food from 1969 and beyond, and all items, less seasonal products, before 1969, adjusted to exclude seasonality and the estimated effect of changes in the consumption tax rate), year-on-year growth rate of the Imported Price Index, and year-on-year growth rate of the crude oil price reported in the Trade Statistics, as a measure of the output, the inflation rate, the growth rate of imported goods prices, and the growth rate of the oil price, respectively. The short-term real interest rate is constructed from the uncollateralized overnight call rate (figures for 1985Q2 and before are estimated using the collateralized overnight call rate) minus the predicted value of the inflation rate estimated by the AR(3) model. The sample period runs from 1968Q4 to 2016Q1.

[20] See Stock and Watson [1998] for details of the median-unbiased estimator.

Stock, J. H. and M. W. Watson [1998] "Median Unbiased Estimation of Coefficient Variance in a Time-Varying Parameter Model," *Journal of the American Statistical Association*, 93(441), pp.349-358.

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