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An Analysis with a Small-Scale Macroeconomic Model

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Firms' Inflation Expectations under Rational Inattention and Sticky Information: An Analysis with a Small-Scale Macroeconomic Model*

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Abstract

In this paper, we construct a small-scale macroeconomic model that incorporates three hypotheses on the formation of inflation expectations: the full-information rational expectations (FIRE), rational inattention, and sticky information hypotheses. Using data for Japan, including survey data on firms' inflation expectations, we estimate the model to examine the empirical validity of each hypothesis, and analyze how rational inattention and sticky information affect the dynamics of firms' inflation expectations. Our main findings are twofold. First, each one of the three hypotheses has a role to play in explaining the mechanism of the formation of firms' inflation expectations in Japan. In this sense, the manner in which firms form their inflation expectations in Japan is complex. Second, although firms' inflation expectations have been pushed up by the Bank of Japan's introduction of its "price stability target" and the expansion in the output gap amid the Bank's Quantitative and Qualitative Monetary Easing (QQE), the presence of rational inattention and information stickiness has slowed the pace of the rise in firms' inflation expectations.

JEL Classification: D84; E31; E52

Keywords: Survey inflation expectations; Rational inattention; Sticky information

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1 Introduction

Macroeconomics has long recognized inflation expectations as a key determinant of inflation. For example, nearly a hundred years ago, Keynes wrote, "If prices are expected to rise and the business world acts on this expectation, that very fact causes them to rise for a time and, by verifying the expectation, reinforces it; and similarly, if it expects them to fall" (Keynes 1923).

However, macroeconomists have so far failed to reach a consensus on how inflation expectations are actually formed. Since Muth (1961) and Sargent and Wallace (1975), many macroeconomic models, including New Keynesian models, have been built on the assumption that economic agents form their inflation expectations based on *full-information rational expectations* (FIRE). Nonetheless, there has been skepticism on the empirical plausibility of FIRE (e.g. Friedman 1979, Tobin 1980). Indeed, an increasing number of empirical studies on surveys of inflation expectations report the limited explanatory power of FIRE (Coibion, Gorodnichenko, and Kamdar 2018).

In particular, little is known about how firms form their inflation expectations. The number of empirical studies on firms' inflation expectations has been limited because not many countries have conducted surveys on inflation expectations for a wide range of firms. However, in theory, changes in firms' inflation expectations have a crucial effect on inflation dynamics because firms *are* the price-setters as long as they have certain market power and incur some costs (of any form) in changing their prices, as assumed in the New Keynesian Phillips curve. Therefore, uncovering the mechanism of firms' inflation expectations formation is one of the most important topics in current macroeconomics.

In this paper, we construct a small-scale macroeconomic model that incorporates three hypotheses on the formation of inflation expectations. One is FIRE, and the other two are hypotheses based on imperfect information.[1] The first hypothesis based on imperfect information is the *rational inattention hypothesis*, which was proposed by Sims (2003) and

[1] There are also other hypotheses on expectations formation, e.g., the *bounded rationality hypothesis* (Sargent 1993, Gabaix 2014) and the *adaptive learning hypothesis* (Evans and Honkapohja 1999, 2001). Under these hypotheses, while agents have full information on macroeconomic variables, they have only imperfect information on the structure of the economy, and as a result their formation of inflation expectations deviates from FIRE. However, as documented in the comprehensive survey of the literature by Coibion, Gorodnichenko, and Kamdar (2018), most of the recent work on the formation of inflation expectations focuses on hypotheses that relax the assumption of full information on macroeconomic variables.

Maćkowiak and Wiederholt (2009).[2] This hypothesis proposes economic agents' limited capacity to process information as the source of information rigidity. Given their capacity to process information, it is rational for economic agents to allocate that capacity mainly to information that has high relevance to their decision making. As a result, their expectations do not fully reflect information which they think less relevant to them. The second hypothesis based on imperfect information is the *sticky information hypothesis*, which was proposed by Mankiw and Reis (2002). This presupposes the presence of costs in acquiring new information. Due to these costs, agents do not always update their information sets. Consequently, it takes some time for new information to get fully woven into their inflation expectations.

We then estimate the model using Japan's data. As observations of inflation expectations, we basically use survey data from the "outlook for general prices (1-, 3-, and 5-year ahead)" in the *Short-Term Economic Survey of Enterprises in Japan* (*Tankan*). The *Tankan* is a statistical survey conducted by the Bank of Japan covering nearly ten thousand Japanese firms. The data of this survey are therefore those of firms' inflation expectations. However, the *Tankan* only started collecting the data on inflation expectations relatively recently, in 2014. To alleviate this short-sample problem, for our observations of inflation expectations up to 2013Q4, we use the "outlook for price levels (1- and 5-year ahead)" in the *Opinion Survey*, which is a survey of households conducted by the Bank of Japan. In doing so, we put much more weight on the firms' survey data than on the other, by using only the *changes* in the households' data while using the raw *levels* of the firms' data. In addition, we conduct a robustness check of the estimation results to verify that the use of the households' survey data does not drive the main estimation results.

Our main findings are twofold. First, we estimate that the proportions of firms that form expectations with FIRE (*FIRE firms*) and those that are rationally inattentive are both roughly 50%, and that both types of firms update their information sets with a probability of around 60% due to the presence of information stickiness. Therefore, each of the FIRE, rational inattention, and sticky information hypotheses has a role to play

[2] The *noisy information hypothesis*, proposed by Phelps (1970) and Lucas (1972) and extended by Woodford (2003), is a well-known hypothesis based on imperfect information. The rational inattention hypothesis is a variant of the noisy information hypothesis, and it theorizes the formation of expectations based on optimization behavior subject to limited information processing capacity under noisy information.

in explaining the mechanism of the formation of firms' inflation expectations in Japan. Second, although the firms' inflation expectations have been pushed up by the Bank of Japan's introduction of its "price stability target" of 2% inflation and the expansion in the output gap amid the Bank's Quantitative and Qualitative Monetary Easing (QQE), the presence of rational inattention and information stickiness has slowed the pace of the rise in firms' inflation expectations. Meanwhile, the term structure of inflation expectations, or *inflation expectations curve*, in the corporate sector is upward-sloping, reflecting the FIRE firms' expectations that the inflation rate will eventually rise from the current low level to 2%.

A growing number of empirical studies on the mechanism of the formation of inflation expectations have recently used forecaster-level or firm-level micro survey data. Most of them support the empirical validity of the rational inattention and sticky information hypotheses. For instance, using the ECB's survey of professional forecasters, Andrade and Le Bihan (2013) report that forecasters do not revise their forecasts every round of the survey, and that there is a large disagreement among individual forecasts. They argue that these observations are consistent with the rational inattention and sticky information hypotheses. Coibion, Gorodnichenko, and Kumar (2018) and Kumar et al. (2015) examine firm-level survey data of New Zealand firms and point out that, as the rational inattention hypothesis implies, most of the firms do not pay attention to aggregate inflation. Uno, Naganuma, and Hara (2018 a, b) analyze patterns of revisions in inflation expectations using firm-level data from the *Tankan*, and report results consistent with the sticky information hypothesis. In a more recent paper, Inatsugu, Kitamura, and Matsuda (2019) estimate a dynamic panel model using firm-level data from the *Tankan* and report empirical evidence that the noisy information hypothesis (in particular the rational inattention variant) and the sticky information hypothesis are consistent with the formation of firms' inflation expectations in Japan.

The existing studies using macroeconomic models that incorporate imperfect information into the formation of inflation expectations focus mainly on the sticky information hypothesis. For example, using U.S. data, Dupor, Kitamura, and Tsuruga (2010) and Knotek (2010) estimate models where firms face information stickiness as well as price stickiness, and report the probability of no information update (which captures the degree of information stickiness) is 58% and 70%, respectively. Also using U.S. data, Coibion and Gorodnichenko (2011) estimate a DSGE model where a fraction of firms are subject to information stickiness, and report that the fraction of such firms is roughly 20%.[3]

However, almost none of the existing empirical studies uses a macroeconomic model incorporating both rational inattention and sticky information. The only exception we are aware of is Pfajfar and Roberts (2018). In their model, where inflation and inflation expectations variables are defined in deviations from exogenous trend inflation, both rational inattention and sticky information are incorporated into the mechanism of the formation of the inflation expectations. They estimate the model using U.S. data including survey data of households' 1-year-ahead inflation expectations, and report that both the rational inattention and sticky information hypotheses are supported by the data in a recent sample period since the late 1990's.[4]

We build our framework by extending that of Pfajfar and Roberts (2018) in terms of model specifications and estimation methodology in a number of ways. As for specifications, while Pfajfar and Roberts (2018) focus only on 1-year-ahead inflation expectations, we explicitly model the entire term structure of inflation expectations to enable us to fit the model to multiple time series of inflation expectations for different horizons. Additionally, in our model, trend inflation and inflation expectations of rationally inattentive firms are endogenously determined and affected by developments in the actual inflation rate, while they are exogenous in the model of Pfajfar and Roberts (2018). As for estimation, we use survey data of firms' inflation expectations as observations of inflation expectations, while most of the existing studies, including Pfajfar and Roberts (2018), use only survey data of households' or professional forecasters' inflation expectations.

Our paper is also related to the literature on inflation expectations at aggregate level in Japan. Maruyama and Suganuma (2019) estimate the term structure of inflation expectations—the inflation expectations curve—in Japan, combining forecast data of various economic agents. They find that the inflation expectations curve is upward-sloping

[3] As with this paper, Dupor, Kitamura, and Tsuruga (2010) and Knotek (2010) assume that all firms in the model face the sticky information constraint. By contrast, Coibion and Gorodnichenko (2011) assume that only a fraction of firms face the sticky information constraint. Therefore, the estimate of the degree of information stickiness in Coibion and Gorodnichenko (2011) is not comparable with those in Dupor, Kitamura, and Tsuruga (2010), Knotek (2010), and this paper.

[4] They also report that in the sample period up to the late 1990's, information stickiness is present, but the rational inattention hypothesis is not supported by the data.

throughout their sample period. Nishino et al. (2016) and the Bank of Japan (2016) point out that the formation of inflation expectations in Japan is more adaptive than in the U.S., the euro area, and the U.K. While these studies focus on uncovering the characteristics of inflation expectations of various economic agents in Japan, we empirically test the validity of the aforementioned three hypotheses for the formation of firms' inflation expectations in Japan by estimating a macroeconomic model incorporating the hypotheses.

This paper consists of five sections. Section 2 describes our small-scale macroeconomic model. Section 3 documents the data and the methodology we use for estimation. Section 4 shows the estimation results, the impulse responses of the estimated model, and the historical decompositions of recent inflation expectations in Japan based on the estimated model. This section also conducts some robustness checks of our estimation results. Section 5 concludes.

2 Model

This section describes the small-scale macroeconomic model we estimate. We build our model by extending that of Pfajfar and Roberts (2018) in several ways, especially around the specification of the formation of inflation expectations.

2.1 Supply and Demand Sides of the Economy

Supply side of the economy

We model the supply side of the economy with a version of the hybrid New Keynesian Phillips curve,

$$\pi_t = \gamma \pi_{t-1} + (1-\gamma)\, \pi^e_{t,t+1} + \kappa y_t + \eta \pi^m_t + \varepsilon_{\pi,t}, \tag{1}$$

where π_t denotes the inflation rate, $\pi^e_{t,t+s}$ the aggregate expectation for the s-period ahead inflation in the corporate sector at time t, y_t the output gap, and π^m_t the changes in import prices. $\varepsilon_{\pi,t}$ is an *i.i.d.* cost-push shock which is normally distributed with mean zero and variance σ^2_π. γ ($\gamma \in [0,1)$) is the persistence of inflation, κ is the slope of the Phillips curve, and η is the sensitivity of inflation to changes in import prices.

The specification of the Phillips curve is changed from that of Pfajfar and Roberts

(2018) in three ways. First, unlike Pfajfar and Roberts (2018), we impose a restriction that the sum of the coefficients of the lagged inflation and the inflation expectations must be unity, in order to allow the steady-state inflation rate to change as the central bank's inflation target changes.[5] Second, the inflation and the inflation expectations variables are defined in levels, while in Pfajfar and Roberts (2018) they are defined in deviations from exogenous trend inflation. These two changes, in addition to endogenizing trend inflation, which will be discussed later, enable us to analyze how changes in the inflation target affect inflation expectations by changing trend inflation. Third, we include in the specification a term for import price changes, in order to control the effect of commodity prices and exchange rates on domestic inflation rates.

The changes in import prices follow an autoregressive process,

$$\pi_t^m = \rho \pi_{t-1}^m + \varepsilon_{m,t}, \tag{2}$$

where $\varepsilon_{m,t}$ is an *i.i.d.* import price shock which is normally distributed with mean zero and variance σ_m^2. The coefficient ρ ($\rho \in [0,1)$) captures the persistence of import price shocks.

Demand side of the economy

Following Pfajfar and Roberts (2018), we model the demand side of the economy with a reduced-form output gap equation,

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 \left(\pi_{t-1} - \bar{\pi}_{t-1}\right) + \phi_4 \left(\pi_{t-2} - \bar{\pi}_{t-2}\right) + \varepsilon_{y,t}, \tag{3}$$

where $\bar{\pi}_t$ denotes trend inflation whose definition will be provided in subsection 2.3. $\varepsilon_{y,t}$ is an *i.i.d.* output gap shock which is normally distributed with mean zero and variance σ_y^2. ϕ_i ($i = 1, 2, 3, 4$) are the sensitivities of the output gap to the lagged values of output gap and inflation gap ($\pi_t - \bar{\pi}_t$).

In the New Keynesian models widely used in the recent literature on monetary policy, the demand side of the economy is described by a dynamic IS curve (Euler equation) that captures the intertemporal relationship of output gaps and a monetary policy rule

[5] This restriction amounts to assuming that inflation has a unit root. Empirical studies on the New Keynesian Phillips curve commonly make this assumption (Mavroeidis, Plagborg-Møller, and Stock 2014).

that determines nominal interest rates. However, macroeconomists have not yet reached a consensus on how to take into account the presence of an effective lower bound of nominal interest rates and unconventional monetary policy tools in formulating a monetary policy rule. Thus, following Pfajfar and Roberts (2018), in order to focus on the formation of inflation expectations, we abstract the monetary policy rule from the model and specify the demand side as a reduced-form output gap equation. Note that in this setup monetary policy shocks are supposed to be contained in the output gap shocks in equation (3).

2.2 Formations of Inflation Expectations

We consider three hypotheses on the formation of inflation expectations: the FIRE, rational inattention, and sticky information hypotheses. We incorporate them into the model in the following way. First, we assume that there are two types of firms. One is *FIRE firms* who form model-consistent expectations based on FIRE, and the other is *rationally inattentive firms*.[6] Second, we assume that all firms are subject to the sticky information constraint à la Mankiw and Reis (2002). Each firm updates their information sets only with a probability. When they cannot, they do not change their inflation expectations.

In this subsection, we describe the expectation formations of FIRE and rationally inattentive firms in (a), and the details of the sticky information constraint in (b). In (c), we derive an expression for the corporate sector's aggregate inflation expectations.

(a) Two types of firms: FIRE and rationally inattentive firms

FIRE firms

A fraction $1-\mu$ ($\mu \in [0,1]$) of the firms are FIRE firms. These firms fully understand the structure of the economy. They form model-consistent expectations for future inflation using all the available information, including the central bank's inflation target.

Specifically, letting $\pi^{e,FIRE}_{t,t+s}$ denote the s-period-ahead inflation expectations of FIRE firms that can update their information sets at period t, we assume that

$$\pi^{e,FIRE}_{t,t+s} = \mathrm{E}_t \left\{ \pi_{t+s} \right\} \quad \text{for} \quad s = 1, 2, \cdots, \tag{4}$$

[6] This categorization is for simplicity. We acknowledge that firms in the real world cannot be clearly categorized into the two types. For example, the degree of rational inattention of a firm depends on the firm's own characteristics; e.g. its capacity for information processing or its business environment. Therefore, in reality, there can be hybrids of FIRE and rationally inattentive firms.

where E_t is the mathematical expectation operator conditional on the period-t information set. We also assume that FIRE firms believe that inflation will converge to the inflation target in the infinite future. That is,

$$\lim_{s \to \infty} \pi_{t,t+s}^{e,FIRE} = \pi_t^*, \tag{5}$$

where π_t^* denotes the inflation target of the central bank.[7]

The inflation target of the central bank follows a random-walk process,

$$\pi_t^* = \pi_{t-1}^* + \varepsilon_{\pi^*,t}, \tag{6}$$

where $\varepsilon_{\pi^*,t}$ is an *i.i.d.* inflation target shock which is normally distributed with mean zero and variance $\sigma_{\pi^*}^2$.

Rationally inattentive firms

The remaining fraction μ of the firms are rationally inattentive firms. As a result of their rational decision under their limited capacity to store and process information, they do not pay attention to macroeconomic variables such as inflation or the central bank's inflation target.

Pfajfar and Roberts (2018) implicitly assume that the inflation expectations of such economic agents are set at the current level of exogenous trend inflation. Making this assumption amounts to presupposing that rationally inattentive firms know the current level of trend inflation despite the fact that they are assumed to pay only partial attention to other macroeconomic variables. This might be a rather strong assumption.

We instead assume that rationally inattentive firms have their own "inflation norm" in mind, and that they expect future inflation to accord with their inflation norm without fully acknowledging current inflation and trend inflation. Specifically, their expectations at period t for the s-period ahead inflation, $\pi_{t,t+s}^{e,RI}$, are determined as follows.

$$\pi_{t,t+s}^{e,RI} = \tau_{t-1} \quad \text{for} \quad s = 1, 2, \cdots, \tag{7}$$

[7] As will be discussed later, in this model the steady-state inflation rate conditional on the current inflation target corresponds to the central bank's inflation target. In other words, in this model, the actual inflation rate eventually converges to the inflation target. Thus, the expectation of FIRE firms for infinite-future inflation, as defined in equation (5), is model-consistent.

where τ_{t-1} denotes the inflation norm formed at period $t-1$. In particular, their expectation for the infinite future inflation is

$$\lim_{s\to\infty} \pi^{e,RI}_{t,t+s} = \tau_{t-1}. \tag{8}$$

Thus, rationally inattentive firms do not necessarily believe that inflation will converge to the central bank's target level.

What we call an "inflation norm" here is the inflation rate that is built in social systems and habits, as discussed by Okun (1981). Okun (1981) argues that prices and wages are set based on an inflation norm rather than current inflation. This paper assumes that rationally inattentive firms form their expectation on future inflation only by referring to their inflation norm, without using information about macroeconomic structures and variables.

But how is the inflation norm formed, then? As far as we know, no existing studies model the dynamics of inflation norms, and therefore there are no generally accepted ways of doing so. But the existing studies discussing inflation norms claim that they are formed adaptively in response to the actual inflation rate and that they do not change rapidly, e.g. over the business cycle frequency (Perry 1980, Okun 1981, Schultze 1981). Developing this account, we assume that the inflation norm follows the process described by the equation below.

$$\tau_t = (1-\alpha)\,\tau_{t-1} + \alpha\pi_t + \varepsilon_{\tau,t}, \tag{9}$$

where $\varepsilon_{\tau,t}$ is an *i.i.d.* innovation which is normally distributed with mean zero and variance σ^2_τ. α ($\alpha \in [0,1)$) is the sensitivity of the inflation norm to the actual inflation rate.

The specifications described above imply that, in our model, the range of information to which rationally inattentive firms do not pay attention is wider than the range assumed in the model of Pfajfar and Roberts (2018). As mentioned above, in the model of Pfajfar and Roberts (2018), inflation expectations of rationally inattentive firms are set equal to trend inflation. This presupposes that rationally inattentive firms always pay full attention to factors that affect trend inflation, e.g. the inflation target, despite the fact that they are assumed to pay only partial attention to factors that affect current inflation. In contrast, in our model, rationally inattentive firms do not pay attention to either the inflation target or factors that affect current inflation.[8]

(b) Sticky information

In each period, firms of either type can update their information sets only with a probability $1-\lambda$ ($\lambda \in [0,1)$). Thus, in each period, a fraction λ of all firms cannot update their information sets. Those firms that cannot update their information sets are chosen independently of their history of past information updates.

The firms that fail to obtain new information do not change their expectations from those they had in the previous period. Then, the history-independent nature of information update opportunities mentioned above implies that the average level of their expectations at period t for the inflation at period $t+s$, $\pi^{e,SI}_{t,t+s}$, is equal to the previous period's average expectation in the whole corporate sector, $\pi^{e}_{t-1,t+s}$. That is,

$$\pi^{e,SI}_{t,t+s} = \pi^{e}_{t-1,t+s} \quad \text{for} \quad s = 1, 2, \cdots . \tag{10}$$

The remaining fraction $1-\lambda$ of the firms update their expectations in the ways we described in (a).

(c) Aggregate inflation expectations

Given the setting described above, at each period, the corporate sector consists of three groups of firms: (i) FIRE firms who can update their information set, (ii) rationally inattentive firms who can update their information set, (iii) firms who cannot acquire new information. The fractions of the three groups are $(1-\lambda)(1-\mu)$, $(1-\lambda)\mu$, and λ, respectively. The aggregate inflation expectations of the whole corporate sector are calculated as weighted averages of the expectations of the three groups. That is, the aggregate expectations at time t for the s-period ahead inflation are

$$\pi^{e}_{t,t+s} = (1-\lambda)\left[(1-\mu)\,\pi^{e,FIRE}_{t,t+s} + \mu\pi^{e,RI}_{t,t+s}\right] + \lambda\pi^{e,SI}_{t,t+s} \quad \text{for} \quad s = 1, 2, \cdots , \tag{11}$$

[8] One reason that some firms do not take the central bank's inflation target into account in forming their inflation expectations is that they do not think it has a large effect on their own business conditions. Another reason may be that, in spite of their full awareness of the target, they do not believe that the inflation target will be achieved. Takahashi (2016), who estimates trend inflation in Japan, calls a parameter that corresponds to the proportion of FIRE firms in our model $(1-\mu)$ "the credibility of the inflation target."

Substituting equation (4), (7), and (10) into (11) yields,

$$\pi^e_{t,t+s} = (1-\lambda)\left[(1-\mu)\,\mathrm{E}_t\left\{\pi_{t+s}\right\} + \mu\tau_{t-1}\right] + \lambda\pi^e_{t-1,t+s} \quad \text{for} \quad s = 1, 2, \cdots . \tag{12}$$

Equation (12) holds for any expectation horizon s. By calculating equation (12) for all horizons, we can obtain the term structure of inflation expectations, or the *inflation expectations curve* in the terminology of Maruyama and Suganuma (2019). This enables us to estimate the model using multiple inflation expectations data for different horizons. This cannot be done with the model of Pfajfar and Roberts (2018) because their model includes inflation expectations only for a single horizon.

It should be noted that in the model of Pfajfar and Roberts (2018) the equation corresponding to our equation (12) includes an expectation-specific shock. Introducing such shocks into our model, which includes inflation expectations for all horizons, requires assuming expectation-specific shocks for infinitely many horizons. However, doing so would also require us to make various additional assumptions, e.g. assumptions for the correlations between such shocks for different horizons. There are also technical difficulties in estimating a model that includes a huge number of exogenous shocks. Furthermore, introducing such exogenous expectation-specific shocks into the model may end up making them account for most of the fluctuations in the expectations and, as a consequence, producing biased estimates of the parameters. For these reasons, in this paper we specify equation (12) without assuming any expectation-specific shocks.

2.3 Trend Inflation

In standard New Keynesian models, where all firms form expectations based on FIRE, the expectation for the infinite-future inflation ($\lim_{s\to\infty} \pi^e_{t,t+s}$) corresponds to the steady-state inflation rate ($\lim_{s\to\infty} \mathrm{E}_t\left\{\pi_{t+s}\right\}$). This inflation rate is also defined as trend inflation (Mavroeidis, Plagborg-Møller, and Stock 2014). In such models, trend inflation also coincides with the current inflation target of the central bank (Ascari and Sbordone 2014).

In our model, however, the expectation for the infinite-future inflation does not necessarily correspond to the steady-state inflation rate (conditional on the current inflation target of the central bank) for the following reason.[9] Rationally inattentive firms form their expectations without paying attention to the inflation target, and also, when the inflation target changes, a fraction of firms do not notice the change due to the sticky information constraint. Thus, the expectations of these firms do not necessarily coincide with the inflation target. Nonetheless, as discussed later, in our model it still holds that the steady-state inflation rate is equal to the inflation target. Therefore, the infinite-future inflation expectation does not necessarily correspond to the steady-state inflation rate.

In this paper, we define trend inflation $\bar{\pi}_t$ as the aggregate level of the expectations for infinite-future inflation rather than the steady-state inflation rate. That is,

$$\bar{\pi}_t \equiv \lim_{s\to\infty} \pi^e_{t,t+s}, \tag{13}$$

Substituting equation (10) into equation (11) taking the limit ($s \to \infty$), and then using equation (5), (8), and (13), we obtain another representation of trend inflation,

$$\bar{\pi}_t = (1-\lambda)\left[(1-\mu)\,\pi^*_t + \mu\tau_{t-1}\right] + \lambda\bar{\pi}_{t-1}. \tag{14}$$

Equation (14) indicates that as inflation expectations are revised and the effects of the sticky information constraint fade, trend inflation converges to a weighted average of the inflation norm and the inflation target ($(1-\mu)\,\pi^*_t + \mu\tau_{t-1}$). The inflation norm, however, also gradually converges to the central bank's inflation target in the end because the forward-looking price-settings of FIRE firms, who expect that inflation will converge to the target level, make the actual inflation rate move towards the inflation target. As a result, trend inflation will eventually converge to the inflation target ($\lim_{s\to\infty} \mathrm{E}_t\left\{\bar{\pi}_{t+s}\right\} = \pi^*_t$). This implies that the actual inflation rate also converges to the inflation target ($\lim_{s\to\infty} \mathrm{E}_t\left\{\pi_{t+s}\right\} = \pi^*_t$), that is, the steady-state inflation rate (conditional on the current level of the inflation target) corresponds to the (current level of the) inflation target.[10]

[9] In our model, because the inflation target follows a random walk process, the inflation rate and the inflation expectations are not stationary. Therefore, the "steady-state" inflation rate can only be defined conditional on the current inflation target. In what follows, we use the term with this definition.

[10] This can be verified as follows. Suppose that the output gap (y_t), the changes in the import prices (π^m_t), and the exogenous innovations ($\varepsilon_{\pi,t}$, $\varepsilon_{m,t}$, $\varepsilon_{y,t}$, and $\varepsilon_{\tau,t}$) are zero, and that inflation (π_t), the inflation expectations ($\pi^e_{t,t+s}$), the inflation norm (τ_t), and trend inflation ($\bar{\pi}_t$) are equal to the inflation target at time t (π^*_t). Then all of the equations (1)-(3), (9), (12), and (14) are satisfied.

3 Estimation

We estimate a state-space model whose state equations consist of equations (1)-(3), (6), (9), (12), and (14) with a Bayesian method, following Pfajfar and Roberts (2018).[11] In this section, we document the data, the observation equations, and the estimation procedure.

3.1 Data

We use the time series of 10 observable variables: (1) output gap, (2) inflation, (3)-(7) inflation expectations, (8) inflation norm, (9) inflation target, and (10) changes in import prices. Our sample period ranges from 2004Q2 to 2018Q4, for which the series of inflation expectations are available. The data series used for the estimation are shown in Figure 1.

Data for the variables other than inflation expectations

Series (1) is the output gap estimated by the Bank of Japan.[12] Series (2) is annualized quarter-on-quarter % changes in consumer price index (CPI, less fresh food). The series is adjusted for seasonality and the change in the consumption tax (VAT) rate. Series (8) is the permanent component of series (2) extracted by using the Beveridge-Nelson (BN) decomposition.[13] We set the sample period for this BN decomposition from 1971Q1 to 2018Q4, which is longer than the sample period for the model estimation, in order to take more information into consideration in extracting the permanent component. Regarding series (9), we assume that the inflation target is 1% from 2006Q1 up to 2012Q4, and 2% from 2013Q1 onward. These values are taken from three releases from the Bank of Japan: (1) the median value of the inflation rate Policy Board members considered to be consistent

[11] In the estimation, we replace the equation (12) by the following approximate expression.

$$\pi^e_{t,t+s} = \begin{cases} (1-\lambda)\left[(1-\mu)\,\mathrm{E}_t\{\pi_{t+s}\} + \mu\tau_{t-1}\right] + \lambda\pi^e_{t-1,t+s} & \text{for } s < 200 \\ \bar{\pi}_t & \text{for } s \geq 200 \end{cases} . \qquad (12')$$

Equation (12) holds for any expectation horizon s, and thus it consists of an infinite number of equations. In the computation, however, an infinite number of equations cannot be handled. Therefore, we introduce an additional assumption that the average inflation expectations whose horizons are 200 quarters (50 years) or more are equal to the trend inflation rate. We checked that our estimation results are insensitive to the setting of the maximum horizon around 200 quarters. We also checked that the inflation expectations converge in the trend inflation rate within 200-quarters ahead throughout the sample period in the estimated model.

[12] For details of the estimation of the output gap by the Bank of Japan, see Kawamoto et al. (2017).

[13] For the methodology of the BN decomposition, we follow Takahashi (2016). Takahashi (2016) estimates trend inflation in Japan using the permanent component of CPI as observations of adaptive inflation expectations.

with price stability from a medium- to long-term viewpoint in the "understanding of medium- to long-term price stability" introduced in March 2006 was around 1%, (2) the "price stability goal in the medium to long term" was set to 1% in February 2012, and (3) the "price stability target" introduced in January 2013 was 2%. The observations of the target up to 2005Q4 are treated as missing values. Series (10) is the deviation of quarter-on-quarter % change in the import price index (all commodities, yen basis) from its sample average.

Data for inflation expectations

There are a variety of measures of inflation expectations, ranging from survey data of households, firms, and professional forecasters, to market-based data reflecting expectations of market participants.[14] There is heterogeneity among these measures. Which one is appropriate to use in any particular study should be decided according to the purpose and the context of the study.

In our context, the hybrid New Keynesian Phillips curve (equation 1) is basically derived as a condition for firms' optimization behavior. Therefore, from the viewpoint of economic theory, it is desirable to use survey data of firms as observations of inflation expectations in the Phillips curve.[15]

In this paper, as observations of inflation expectations, we use the average levels of the "outlook for general prices (1-, 3-, 5-year ahead)" (series (3)-(5)) in the *Tankan*, the statistical survey of firms in Japan. However, because the survey on the "outlook for general prices" starts in 2014Q1, the time series of this survey data is not long. To complement this, as observations up to 2013Q4 we use the changes in the average levels of the "outlook for price levels (1- and 5-year ahead)" (series (6) and (7)) in the *Opinion Survey*, a survey of households conducted by the Bank of Japan.[16]

In using the households' survey data in addition to the firms' survey data, we put

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[14] For surveys of the various measures of inflation expectations in Japan, see Maruyama and Suganuma (2019) and Nishiguchi, Nakajima, and Imakubo (2014).

[15] Coibion, Gorodnichenko, Kamdar (2018) claim that when estimating the New Keynesian Phillips curve, historical measures of inflation expectations of price setters, namely firms, should be used if possible. Indeed, Coibion and Gorodnichenko (2015) demonstrate that the explanatory power of the New Keynesian Phillips curve increases when using measures of households' inflation expectations, which are thought of as a relatively better proxy for firms' expectations, instead of professional forecasts.

[16] The *Consumer Confidence Survey*, conducted by the Cabinet Office, also surveys households' inflation outlooks in Japan. However, the *Consumer Confidence Survey* collects inflation expectations only for 1-year ahead. We use the *Opinion Survey* for data of inflation expectations for multiple horizons.

much more weight on the firms' survey data than on the other by using only the *changes* in the households' data while using the raw *levels* of the firms' data. We also take into account potential differences in statistical characteristics between the two, as we will discuss shortly. Moreover, we conduct a robustness check of the estimation results to verify that the use of the households' survey data does not drive the main estimation results.

3.2 Observation Equations

The observation equations are as follows.

- Observed output gap

$$y_t^{obs} = y_t.$$

- Observed inflation rate

$$\pi_t^{obs} = \pi_t.$$

- Observed inflation expectations

$$\begin{aligned}
\pi_{t,1Y}^{Tankan} &= \frac{1-\omega_{1Y}(1-\delta)}{1-\omega_{1Y}} \frac{1}{4} \sum_{s=1}^{4} \pi_{t,t+s}^{e} - \frac{\omega_{1Y}\delta}{1-\omega_{1Y}} \tau_{t-1} + \varepsilon_{\pi_{1Y}^{e},t}, \\
\pi_{t,3Y}^{Tankan} &= \frac{1-\omega_{3Y}(1-\delta)}{1-\omega_{3Y}} \frac{1}{4} \sum_{s=9}^{12} \pi_{t,t+s}^{e} - \frac{\omega_{3Y}\delta}{1-\omega_{3Y}} \tau_{t-1} + \varepsilon_{\pi_{3Y}^{e},t}, \\
\pi_{t,5Y}^{Tankan} &= \frac{1-\omega_{5Y}(1-\delta)}{1-\omega_{5Y}} \frac{1}{4} \sum_{s=17}^{20} \pi_{t,t+s}^{e} - \frac{\omega_{5Y}\delta}{1-\omega_{5Y}} \tau_{t-1} + \varepsilon_{\pi_{5Y}^{e},t}, \\
\pi_{t,1Y}^{OS} - \pi_{t-1,1Y}^{OS} &= \frac{1}{4} \left[\sum_{s=1}^{4} \pi_{t,t+s}^{e} - \sum_{s=1}^{4} \pi_{t-1,t+s-1}^{e} \right] + \varepsilon_{\Delta\pi_{1Y}^{e},t}, \\
\pi_{t,5Y}^{OS} - \pi_{t-1,5Y}^{OS} &= \frac{1}{4} \left[\sum_{s=17}^{20} \pi_{t,t+s}^{e} - \sum_{s=17}^{20} \pi_{t-1,t+s-1}^{e} \right] + \varepsilon_{\Delta\pi_{5Y}^{e},t}.
\end{aligned}$$

- Observed inflation norm

$$\tau_t^{obs} = \tau_t.$$

- Observed inflation target

$$\pi_t^{*,obs} = \pi_t^{*}.$$

- Observed import price factor

$$\pi_t^{m,obs} = \pi_t^m.$$

y_t^{obs}, π_t^{obs}, τ_t^{obs}, $\pi_t^{*,obs}$, and $\pi_t^{m,obs}$ are the observations of the output gap, the inflation, the inflation norm, the inflation target, and the changes in import prices, respectively. $\pi_{t,nY}^{Tankan}$ denotes the n-year ahead inflation expectation in the *Tankan* ($n = 1, 3, 5$), and $\pi_{t,nY}^{OS}$ denotes that in the *Opinion Survey* ($n = 1, 5$). $\varepsilon_{\pi_{1Y}^e,t}$, $\varepsilon_{\pi_{3Y}^e,t}$, $\varepsilon_{\pi_{5Y}^e,t}$, $\varepsilon_{\Delta\pi_{1Y}^e,t}$, and $\varepsilon_{\Delta\pi_{5Y}^e,t}$ are measurement errors that are normally distributed with means zero and variances $\sigma^2_{\pi_{1Y}^e}$, $\sigma^2_{\pi_{3Y}^e}$, $\sigma^2_{\pi_{5Y}^e}$, $\sigma^2_{\Delta\pi_{1Y}^e}$, and $\sigma^2_{\Delta\pi_{5Y}^e}$, respectively.

As for the observation equations of inflation expectations shown above, we carefully adjusted the details of the specifications both to deal with some data issues and to take into account as much information from the firms' survey data as possible (see Appendix for details). First, to deal with potential heterogeneity of the measurement errors between the *Tankan* data and the *Opinion Survey* data, we do not directly connect these time series, but specify a separate observation equation for each time series. Second, to put more weight on the survey data of firms than those of households, we use only the *changes* in the expectations data of the *Opinion Survey* while we use the *levels* of the *Tankan* data. Third, we specify the observation equations for the Tankan data using the share of the respondents answering "don't know" as well. In every round of the *Tankan* survey, non-negligible shares of the firms choose the option of "don't know." For example, about 40% of the firms, on average, answer "don't know" for the 5-year-ahead inflation outlook. If we were to estimate the model without taking into account the firms answering "don't know," we would end up assuming a priori that these firms have the same inflation expectations on average as the firms that answer numerical values, or in other words, that the share of rationally inattentive firms in them is the same as that in the latter firms. However, the very fact that a firm chose the "don't know" option suggests that this particular firm is rationally inattentive to macroeconomic variables. Therefore, the share of rationally inattentive firms in these respondents is likely to be higher than that in the whole corporate sector. In order to take this possibility into account, in our specification of the observation equations for the *Tankan* data, the inflation expectations of the firms answering "don't know" are expressed as weighted averages of the inflation norm and average inflation expectations in the corporate sector. The weight on the inflation norm, δ, is estimated simultaneously with the other parameters.[17] The share of firms answering "don't know" is set to the averages from the March 2014 survey to the December 2018 survey. Specifically, ω_{1Y}, ω_{3Y}, and ω_{5Y} are 14.9%, 29.7%, and 40.4%, respectively.[18]

3.3 Estimation Procedure and Prior Distributions

The likelihood function of the state-space model which consists of the state and observation equations described above can be computed using the Kalman filter. We combine the likelihood function with the prior distributions of the parameters to obtain the posterior distributions numerically. In this process, we use the Markov chain Monte Carlo (MCMC) method with the random walk Metropolis-Hastings algorithm. We create a sample of 100,000 draws in MCMC, discarding the initial 50,000 draws as burn-in.[19]

The types, means, and standard deviations of the prior distributions of each parameter are shown in Table 1. In principle, we set them following Pfajfar and Roberts (2018). The prior distributions of the parameters for which there are no counterparts in the model of Pfajfar and Roberts (2018) are determined as follows. The prior mean of the sensitivity of the inflation norm to the actual inflation rate, α, is obtain by estimating equation (9) with the ordinary least squares method, using series (2) and series (8). For the following three parameters, we assume flat prior distributions to let the observations inform the estimation as much as possible: the sensitivity of inflation to changes in import prices, η; the persistence of the import price shock, ρ; and the weight of the inflation norm in the inflation expectations of the firms answering "don't know" in the *Tankan*, δ. The size of the import price shock is based on the standard deviation of the quarter-on-quarter changes in the import price index in our sample period.

As for the choice of the prior distributions, we conduct a robustness check by re-estimating the model with different prior distributions of three key parameters: the degree of rational inattention (μ), the degree of information stickiness (λ), and the sensitivity of the inflation norm to the actual inflation rate (α).

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[17] Using only the aggregates of the numerical answers and discarding the answers of "don't know" is equivalent to assuming a priori that δ is zero.

[18] As Uno, Naganuma, and Hara (2018b) report, the share of firms who answer "don't know" is stable throughout the sample period. Thus, in this paper we treat ω_{1Y}, ω_{3Y}, and ω_{5Y} as constants.

[19] We checked the convergence of the sampling with the diagnostic test of Brooks and Gelman (1998).

4 Result

In this section we first report the posterior distributions of the parameters and the estimated inflation expectations curve and show the impulse responses of the estimated model. We also show the historical decompositions of recent inflation expectations in Japan based on the estimated model. In addition, we conduct some robustness checks of the parameter estimation results.

4.1 Prior Distribution

The last three columns of Table 1 show the posterior means and the lower and upper bounds of the 90% intervals of the parameters.

The posterior mean of the share of rationally inattentive firms, μ, is 0.51 with a 90% interval (0.35, 0.65). Thus, the posterior mean of the share of FIRE firms, $1-\mu$, is 0.49 with a 90% interval (0.35, 0.65). The posterior mean of the degree of information stickiness, λ, is 0.57 with a 90% interval (0.45, 0.70). These posterior means imply that, at each period, (i) the share of FIRE firms who update information is 21%, (ii) that of rationally inattentive firms who update information is 22%, and (iii) that of firms who cannot obtain new information is 57%.

The estimation result indicates that the FIRE, rational inattention, and sticky information hypotheses are all empirically valid for firms' inflation expectations in Japan. In other words, the manner in which firms in Japan form their inflation expectations is more complex than that assumed in simple FIRE models.

The posterior mean of the sensitivity of the inflation norm to the actual inflation rate, α, is estimated to be a small value, 0.09, even though its 90% interval is located in the positive region. This implies that the inflation norm depends heavily on the historical experience of inflation, and that it changes only gradually in response to the actual inflation rate. This is consistent with the explanations of the inflation norm provided in the literature such as Okun (1981). The slope of the Phillips curve, κ, is 0.14 and the inflation persistence, γ, is 0.31 at their posterior means. Their 90% intervals are estimated to be positive. Our estimate of the inflation persistence is smaller than that reported in existing works that estimate the New Keynesian Phillips curve in Japan.[20] This suggests that most of the persistence of inflation can be accounted for by the persistence of inflation expectations, once the latter is explicitly modeled with rational inattention and sticky information, as is done in this paper.

4.2 Term Structure of Firms' Inflation Expectations

As mentioned above, calculating equation (12) for all the horizons provides us with a term structure of the inflation expectations—the inflation expectations curve—in the corporate sector. Figure 2 shows the estimated inflation expectations curve for Japanese firms. The curve is upward-sloping throughout most of the sample period. This is consistent with the findings of Maruyama and Suganuma (2019), who estimate the inflation expectations curve in Japan using various measures of inflation expectations, including survey data and market data.

The positive slope reflects the expectations of the FIRE firms, who form model-consistent expectations in a forward-looking way. FIRE firms expect that the actual inflation rate will eventually converge to the central bank's inflation target, although it has been below the target level throughout the sample period.

Note that the levels of the inflation expectations just after the introduction of QQE are lower and the falls in the expectations in 2014-15 are smaller in our model than in the *Tankan* data. This is because we estimate the inflation expectations not only using the observation data but also taking into account the historical relationship between the inflation expectations and other economic variables. Compared with our model's estimated inflation expectations, the inflation expectations observed in the *Tankan* rose by a larger amount after the introduction of QQE, and also fell by a larger amount in 2014-15. The gaps between our model's estimated expectations and the observed data are identified as measurement errors.

[20] Blanchard, Cerutti, and Summers (2015) estimate the hybrid-type New Keynesian Phillips curve with time-varying parameters for Japan and report γ is around 0.6. Takahashi (2016) estimates that the parameter that is equivalent to our γ is about 0.8.

4.3 Impulse Responses

Next, using the estimated model, we calculate impulse responses to a temporary output gap shock and those to a permanent inflation target shock in order to uncover the features of our model's dynamics.[21]

Figure 3 shows impulse responses of the inflation expectations and the actual inflation rate to a temporary shock that increases the output gap by 1% point on impact.[22] Figure 4 shows those to a permanent shock which raises the inflation target by 1% point. In each panel, the red bold line represents the mean of the impulse responses in the estimated model, and the shaded area represents 90% intervals of the response. For comparison, we also show the mean responses in a model without rational inattention ($\mu = 0$, depicted by the dotted lines) and those in a model without rational inattention or sticky information ($\mu = \lambda = 0$, depicted by the circled lines).[23] The graphs reveal the following two features of the estimated model's dynamics.

First, the responses of the inflation expectations and the actual inflation rate to the shocks are delayed in the estimated model, where both rational inattention and sticky information are present. For example, the response of the inflation expectation (1-year ahead) to the output gap shock reaches its peak in the third quarter after the shock in the estimated model. By contrast, in the model without rational inattention or sticky information it immediately reaches its peak in the impact period. Similarly, in the estimated model the inflation expectations and the actual inflation rate rise gradually after the inflation target shock happens, while in the model without rational inattention or sticky information the rise in the inflation target feeds rapidly into the inflation expectations and the actual inflation rate.

Second, the impulse responses to the output gap shock are smaller in the estimated model than in the model without rational inattention or sticky information. For example,

[21] The impulse responses shown below depict the responses of economic variables to the shocks that occurred during the sample period, based on the average relationship between the variables during the sample period. The responses of the economic variables to similar shocks that occur outside the sample period can differ from these impulse responses, because the relationship between the variables can differ from the one observed during the sample period.

[22] To compute the responses, we obtain 1,000 random draws for the parameter vector using the posterior distributions.

[23] Except for the degrees of rational inattention and sticky information, we use the same random draws for the parameters as the ones we use to compute the responses of the estimated model.

the mean response of the inflation expectation (1-year ahead) to the output gap shock is +0.2% points at its peak in the estimated model. This is almost only a fourth of the response in the model without rational inattention or sticky information (+0.8% points).

These two features originate from the presence of rationally inattentive firms and the sticky information constraint. In our model, rationally inattentive firms expect future inflation to conform to their inflation norm, which depends on their historical experience of inflation. It takes a long time for the improvement in the output gap and the rise in the inflation target to push up the inflation norm of these firms by driving up the actual inflation rate. Also, the presence of the firms who cannot update their information sets and do not change their expectations under the sticky information constraint delays the response of the aggregate inflation expectations. And the maximum responses to temporary shocks are smaller in the estimated model because these shocks disappear before rationally inattentive firms and firms who cannot update their information sets have fully incorporated the effects of the shocks into their expectations.

4.4 Historical Decompositions of Inflation Expecations

Figure 5 shows the historical decomposition of the cumulative changes since 2012Q4 of the average inflation expectations (1-year ahead, year-on-year % changes), where the parameters are set to their posterior means.

Figure 5 reveals that the introduction of the "price stability target" of 2% in January 2013 and the expansion of the output gap amid QQE have been pushing up inflation expectations in the corporate sector. Figure 5 also indicates that rational inattention and sticky information have slowed the rise of inflation expectations towards 2%. Specifically, the contributions of the inflation target shock to the inflation expectations have been expanding only gradually since the Bank of Japan raised the inflation target from 1% to 2% by publishing the "price stability target." In addition, the contribution of the output gap shock lags one year behind developments in the output gap (depicted by the fine line).

As we mentioned in the precious subsection, the slow response of inflation expectations is caused by the presence of rationally inattentive firms and the constraint of sticky information. It takes a long time for the rise in the inflation target and the expansion of the output gap to raise the inflation norm of rationally inattentive firms by pushing up the actual inflation rate. Furthermore, the sticky information constraint makes the lag longer.

4.5 Robustness Checks

Finally, to check the robustness of the parameter estimation result shown above (the baseline result, hereafter), we re-estimate the model (a) without using the households' survey data as observations of inflation expectations, and (b) with different prior distributions of key parameters.

(a) Estimation without using the *Opinion Survey* data

As mentioned in subsection 3-1, in the baseline estimation we used the changes in the households' inflation expectations data of the *Opinion Survey* as observations of inflation expectations for the sample period up to 2013Q4, where the firms' inflation expectation data of the *Tankan* does not exist. As the first robustness check, we examine whether this use of the households' data in addition to the firms' data drives the main estimation result. To this end, we re-estimate the model treating the observations of the inflation expectations from 2004Q2 to 2013Q4 as missing values.

Table 2 shows the estimation result in this case. The posterior distributions of all the parameters, including the degrees of rational inattention and information stickiness, are almost unchanged from those in the baseline result (Table 1). Thus, the baseline result is robust with respect to whether households' inflation expectations data is used.

(b) Estimation with different prior distributions of key parameters

Next we check the robustness of the baseline result with respect to the choice of the prior distributions of three key parameters: the degree of rational inattention (μ), the degree of information stickiness (λ), and the sensitivity of the inflation norm to the actual inflation rate (α). Specifically, we re-estimate the model setting the prior distribution of each of the three parameters to a uniform distribution with the interval of $[0, 1]$. Using uniform distributions as the prior distributions of these parameters amounts to estimating the model without taking into account any prior information on these parameters.

Table 3 shows the estimation result in this case. The posterior distributions of the three key parameters μ, λ, and α remain nearly unchanged from those in the baseline result (Table 1). In addition, the posterior distributions of the other parameters are broadly the same as those in the baseline result. Therefore, we conclude that the baseline result is robust with respect to the choice of the prior distributions of the key parameters.

5 Conclusion

In this paper, we construct a small-scale macroeconomic model that incorporates three hypotheses on the formation of inflation expectations: the FIRE, rational inattention, and sticky information hypotheses. Using data for Japan, including survey data on firms' inflation expectations, we estimate the model to examine the empirical validity of each hypothesis. Our main findings are twofold.

First, each one of the three hypotheses has a role to play in explaining the mechanism of the formation of firms' inflation expectations in Japan. In this sense, the manner in which firms form their inflation expectations in Japan is complex. Second, although firms' inflation expectations have been pushed up by the introduction of the "price stability target" and the expansion in the output gap amid QQE, rational inattention and sticky information have slowed the pace of the rise in firms' inflation expectations. Meanwhile, the term structure of inflation expectations, or *inflation expectations curve*, in the corporate sector is upward-sloping, reflecting the FIRE firms' expectations that the inflation rate will eventually rise from the current low level to 2%.

Note that there is room for explicitly modeling the relationship between inflation expectations and monetary policy. In this paper, we abstract the monetary policy rule from the model and describe the demand side of the economy as a reduced-form output gap equation in order to focus on the mechanism of the formation of inflation expectations. Investigating whether our results still hold in a model incorporating the monetary policy rule may be one of the fruitful ways to pursue future research.

Appendix. Observation Equations of Inflation Expectations

In this appendix, we document the details of the observation equations of the inflation expectations in our model.

As mentioned in section 3, as observations of the inflation expectations, we use the "outlook for general prices (1-, 3-, and 5-year ahead)" in the *Tankan* since 2014Q1, and the "outlook for price levels (1- and 5-year ahead)" in the *Opinion Survey* up to 2013Q4. As for the observation equations of inflation expectations, we carefully adjusted the details of the specifications both to put more weight on the firms' inflation expectations data from the *Tankan* and to take into account as much information from the firms' survey data as possible.

Combining two measures of inflation expectations

One way to use two measures of the inflation expectations at the same time is to directly connect the two sets of the time series data. As discussed later, however, there are differences in levels between the *Tankan* and the *Opinion Survey* data. Thus, it is not appropriate to directly connect these two series. Moreover, connecting the two time series directly assumes that these time series contain a homogeneous measurement error. But the measurement errors of the two series are potentially heterogeneous because these series are collected in different surveys.

Instead of directly connecting the time series, we use the two observed series at the same time by specifying a separate observation equation for each time series. That is to say, we specify three observation equations for the *Tankan* data (outlooks for 1-, 3-, and 5-year ahead) and two observation equations for the *Opinion Survey* data (outlooks for 1- and 5-year ahead). These specifications allow us to take account of the differences in the levels and the potential heterogeneity in the measurement errors.

In the observation equations for firm's inflation expectations, the *Tankan* data up to 2013Q4 are missing. In order to prioritize the data of firms' inflation expectations, we set the observations of the *Opinion Survey* data since 2014Q1 to missing values in the observation equations for households' inflation expectations. Being able to deal with the missing observations is one of the advantages of the state-space approach which we take in this paper (see, for example, Durbin and Koopman (2012)) .

Use of the *Opinion Survey* data

As Figure 1 shows, the levels of the inflation expectations in the *Opinion Survey* are clearly higher than those in the *Tankan* and the actual inflation rate. One of the reasons of the upward-bias is the difference in the respondents of the two surveys: the *Tankan* is a survey of firms, while the *Opinion Survey* is a survey of households. Another reason is that, unlike the *Tankan*, the *Opinion Survey* does not ask respondents to state their inflation expectations in terms of a specific price indicator such as CPI.[24,25]

In order to put more weight on the firms' inflation expectations data of the *Tankan*, we first take the time-series difference in the *Opinion Survey* data to obtain the changes in the inflation expectations from the previous survey, and then link them to the changes in the model's inflation expectations through the observation equations. As for the *Tankan* data, we directly use the raw levels of the data as the observations of inflation expectations.[26]

The choice of "don't know" in the *Tankan*

The options in the questions of the "outlook for general prices" in the *Tankan* include "don't know." If we were to estimate the model without taking into account the firms answering "don't know," we would end up assuming a priori that these firms have the same inflation expectations on average as the firms that answer numerical values, or in other words, that the share of rationally inattentive firms in them is the same as that in the latter firms.

[24] In the *Tankan*, the prices that respondents should have in mind in answering their inflation expectations are explicitly defined as "general prices (as measured by the consumer price index)." On the other hand, in the *Opinion Survey*, the definition of prices is phrased as "overall prices of goods and services you purchase." Therefore, the respondents in the *Opinion Survey* do not necessarily state their inflation expectations in terms of CPI. In this regard, Nishiguchi, Nakajima, and Imakubo (2014) who investigate the respondent-level data of the *Opinion Survey* find that, while households raise their inflation expectations in response to a rise in prices of frequently purchased items (food and energy), only a part of the households revise their inflation expectations in response to a rise in prices of infrequently purchased items (goods and services other than food and energy). They claim that this observation implies that not all the respondents in the *Opinion Survey* state their inflation outlooks with reference to CPI.

[25] Furthermore, Kamada (2013) points out the downward rigidity of the inflation outlooks in the *Opinion Survey*, that is, the scarcity of respondents giving negative values for their inflation outlooks. Kamada, Nakajima, and Nishiguchi (2015) claim that this downward rigidity may have become stronger after the survey method was changed from a direct-visit method to a mail survey method in 2006.

[26] The fact that the answers in the *Opinion Survey* are not necessarily in terms of CPI inflation might affect not only the average levels but also the changes in inflation expectations. In our model, such potential biases of the observed changes in the expectations are captured by the measurement errors.

However, the very fact that a firm chose the "don't know" option suggests that this particular firm is rationally inattentive to macroeconomic variables. Therefore, the share of rationally inattentive firms in these respondents is likely to be higher than that in the whole corporate sector.

We take this possibility into account in our specification of the observation equations for the *Tankan* data. First, we assume that the inflation expectations of the firms that answer with numerical values is the average outlook released in the *Tankan*, minus a measurement error. We then assume that the inflation expectations of the firms that answer "don't know" are weighted averages of inflation norm and average inflation expectations in the corporate sector. The weight on the inflation norm is denoted by δ. That is,

$$\frac{1}{4}\sum_{s=1}^{4}\pi_{t,t+s}^{e} = (1-\omega_{1Y})\left[\pi_{t,1Y}^{Tankan} - \hat{\varepsilon}_{\pi_{1Y}^{e},t}\right] + \omega_{1Y}\left[\delta\tau_{t-1} + (1-\delta)\frac{1}{4}\sum_{s=1}^{4}\pi_{t,t+s}^{e}\right], \tag{A1}$$

$$\frac{1}{4}\sum_{s=9}^{12}\pi_{t,t+s}^{e} = (1-\omega_{3Y})\left[\pi_{t,3Y}^{Tankan} - \hat{\varepsilon}_{\pi_{3Y}^{e},t}\right] + \omega_{3Y}\left[\delta\tau_{t-1} + (1-\delta)\frac{1}{4}\sum_{s=9}^{12}\pi_{t,t+s}^{e}\right], \tag{A2}$$

$$\frac{1}{4}\sum_{s=17}^{20}\pi_{t,t+s}^{e} = (1-\omega_{5Y})\left[\pi_{t,5Y}^{Tankan} - \hat{\varepsilon}_{\pi_{5Y}^{e},t}\right] + \omega_{5Y}\left[\delta\tau_{t-1} + (1-\delta)\frac{1}{4}\sum_{s=17}^{20}\pi_{t,t+s}^{e}\right], \tag{A3}$$

where $\hat{\varepsilon}_{\pi_{1Y}^{e},t}$, $\hat{\varepsilon}_{\pi_{3Y}^{e},t}$, and $\hat{\varepsilon}_{\pi_{5Y}^{e},t}$ are measurement errors. The share of the firms choosing "don't know" for 1-, 3-, and 5-year ahead inflation is set to their averages from the March 2014 survey to the December 2018 survey. Specifically, ω_{1Y}, ω_{3Y}, and ω_{5Y} are 14.9%, 29.7%, and 40.4%, respectively. δ is estimated simultaneously with the other parameters.

Observation equations for inflation expectations

In short, the observation equations for inflation expectations in our model consist of the following five equations.

$$\pi_{t,1Y}^{Tankan} = \frac{1-\omega_{1Y}(1-\delta)}{1-\omega_{1Y}}\frac{1}{4}\sum_{s=1}^{4}\pi_{t,t+s}^{e} - \frac{\omega_{1Y}\delta}{1-\omega_{1Y}}\tau_{t-1} + \varepsilon_{\pi_{1Y}^{e},t}, \tag{A4}$$

$$\pi_{t,3Y}^{Tankan} = \frac{1-\omega_{3Y}(1-\delta)}{1-\omega_{3Y}}\frac{1}{4}\sum_{s=9}^{12}\pi_{t,t+s}^{e} - \frac{\omega_{3Y}\delta}{1-\omega_{3Y}}\tau_{t-1} + \varepsilon_{\pi_{3Y}^{e},t}, \tag{A5}$$

$$\pi_{t,5Y}^{Tankan} = \frac{1-\omega_{5Y}(1-\delta)}{1-\omega_{5Y}}\frac{1}{4}\sum_{s=17}^{20}\pi_{t,t+s}^{e} - \frac{\omega_{5Y}\delta}{1-\omega_{5Y}}\tau_{t-1} + \varepsilon_{\pi_{5Y}^{e},t}, \tag{A6}$$

$$\pi_{t,1Y}^{OS} - \pi_{t-1,1Y}^{OS} = \frac{1}{4}\left[\sum_{s=1}^{4}\pi_{t,t+s}^{e} - \sum_{s=1}^{4}\pi_{t-1,t+s-1}^{e}\right] + \varepsilon_{\Delta\pi_{1Y}^{e},t}, \tag{A7}$$

$$\pi_{t,5Y}^{OS} - \pi_{t-1,5Y}^{OS} = \frac{1}{4}\left[\sum_{s=17}^{20}\pi_{t,t+s}^{e} - \sum_{s=17}^{20}\pi_{t-1,t+s-1}^{e}\right] + \varepsilon_{\Delta\pi_{5Y}^{e},t}. \tag{A8}$$

Equations (A4)-(A6) are obtained by solving equations (A1)-(A3) for the observations. In this process, the measurement errors are normalized ($\varepsilon_{\pi_{1Y}^{e},t}$, $\varepsilon_{\pi_{3Y}^{e},t}$, and $\varepsilon_{\pi_{5Y}^{e},t}$ denote the normalized measurement errors). $\varepsilon_{\pi_{1Y}^{e},t}$, $\varepsilon_{\pi_{3Y}^{e},t}$, $\varepsilon_{\pi_{5Y}^{e},t}$, $\varepsilon_{\Delta\pi_{1Y}^{e},t}$, and $\varepsilon_{\Delta\pi_{5Y}^{e},t}$ are measurement errors that are normally distributed with means zero and variances $\sigma_{\pi_{1Y}^{e}}^{2}$, $\sigma_{\pi_{3Y}^{e}}^{2}$, $\sigma_{\pi_{5Y}^{e}}^{2}$, $\sigma_{\Delta\pi_{1Y}^{e}}^{2}$, and $\sigma_{\Delta\pi_{5Y}^{e}}^{2}$, respectively.

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Table 1. Estimated Parameters

		Prior			Posterior		
		Type	Mean	S.D.	Mean	5%	95%
New Keynesian Phillips Curve							
γ	Inflation persistence	B	0.50	(0.15)	**0.31**	0.20	0.41
κ	Slope	N	0.10	(0.10)	**0.14**	0.08	0.19
η	Sensitivity to import prices	N	0.00	(1.00)	**0.11**	0.08	0.14
ρ	Persistence of import price shock	B	0.50	(0.20)	**0.35**	0.18	0.52
Formation of Inflation Expectations							
μ	Degree of rational inattention	B	0.50	(0.20)	**0.51**	0.35	0.65
λ	Degree of information stickiness	B	0.75	(0.15)	**0.57**	0.45	0.70
δ	Weight used in calculating the inflation expectations of the firms answering "don't know"	B	0.50	(0.20)	**0.73**	0.52	0.96
Reduced-Form Output Gap Equation							
ϕ_1	Output gap (1-quarter lagged)	N	1.30	(0.50)	**1.51**	1.31	1.72
ϕ_2	Output gap (2-quarter lagged)	N	-0.50	(0.50)	**-0.54**	-0.71	-0.37
ϕ_3	Inflation gap (1-quarter lagged)	N	0.10	(1.00)	**0.07**	-0.06	0.20
ϕ_4	Inflation gap (2-quarter lagged)	N	0.10	(1.00)	**-0.39**	-0.53	-0.25
Formation of Inflation Norm							
α	Sensitivity to actual inflation	B	0.09	(0.01)	**0.09**	0.07	0.10
Size of Shocks							
σ_π	Cost-push shock	invG	1.00	(Inf)	**0.68**	0.57	0.79
σ_y	Output gap shock	invG	1.00	(Inf)	**0.61**	0.52	0.71
σ_τ	Inflation norm shock	invG	1.00	(Inf)	**0.14**	0.12	0.16
σ_{π^*}	Inflation target shock	invG	1.00	(Inf)	**0.18**	0.15	0.21
σ_m	Import price shock	invG	5.00	(Inf)	**4.84**	4.11	5.56
$\sigma_{\pi^e_{1Y}}$	Measurement error (*Tankan* , 1-year ahead)	invG	1.00	(Inf)	**0.54**	0.40	0.68
$\sigma_{\pi^e_{3Y}}$	Measurement error (*Tankan* , 3-year ahead)	invG	1.00	(Inf)	**0.33**	0.23	0.43
$\sigma_{\pi^e_{5Y}}$	Measurement error (*Tankan* , 5-year ahead)	invG	1.00	(Inf)	**0.36**	0.26	0.47
$\sigma_{\Delta\pi^e_{1Y}}$	Measurement error (*Opinion Survey* , 1-year ahead)	invG	1.00	(Inf)	**1.04**	0.85	1.22
$\sigma_{\Delta\pi^e_{5Y}}$	Measurement error (*Opinion Survey* , 5-year ahead)	invG	1.00	(Inf)	**0.57**	0.46	0.67

Note: "N" stands for normal distribution, "B" for beta distributution, "invG" for inverse gamma distribution.

Table 2. Result of the Estimation without the *Opinion Survey* Data

		Prior			Posterior		
		Type	Mean	S.D.	Mean	5%	95%
New Keynesian Phillips Curve							
γ	Inflation persistence	B	0.50	(0.15)	**0.29**	0.19	0.39
κ	Slope	N	0.10	(0.10)	**0.14**	0.09	0.20
η	Sensitivity to import prices	N	0.00	(1.00)	**0.11**	0.08	0.14
ρ	Persistence of import price shock	B	0.50	(0.20)	**0.34**	0.18	0.51
Formation of Inflation Expectations							
μ	Degree of rational inattention	B	0.50	(0.20)	**0.53**	0.39	0.67
λ	Degree of information stickiness	B	0.75	(0.15)	**0.61**	0.50	0.72
δ	Weight used in calculating the inflation expectations of the firms answering "don't know"	B	0.50	(0.20)	**0.73**	0.51	0.95
Reduced-Form Output Gap Equation							
ϕ_1	Output gap (1-quarter lagged)	N	1.30	(0.50)	**1.51**	1.31	1.70
ϕ_2	Output gap (2-quarter lagged)	N	-0.50	(0.50)	**-0.53**	-0.70	-0.36
ϕ_3	Inflation gap (1-quarter lagged)	N	0.10	(1.00)	**0.05**	-0.08	0.19
ϕ_4	Inflation gap (2-quarter lagged)	N	0.10	(1.00)	**-0.40**	-0.55	-0.25
Formation of Inflation Norm							
α	Sensitivity to actual inflation	B	0.09	(0.01)	**0.09**	0.07	0.10
Size of Shocks							
σ_{π}	Cost-push shock	invG	1.00	(Inf)	**0.68**	0.58	0.79
σ_{y}	Output gap shock	invG	1.00	(Inf)	**0.62**	0.51	0.71
σ_{τ}	Inflation norm shock	invG	1.00	(Inf)	**0.14**	0.12	0.17
σ_{π^*}	Inflation target shock	invG	1.00	(Inf)	**0.18**	0.15	0.21
σ_{m}	Import price shock	invG	5.00	(Inf)	**4.83**	4.11	5.57
$\sigma_{\pi^e_{1Y}}$	Measurement error (*Tankan* , 1-year ahead)	invG	1.00	(Inf)	**0.53**	0.39	0.67
$\sigma_{\pi^e_{3Y}}$	Measurement error (*Tankan* , 3-year ahead)	invG	1.00	(Inf)	**0.33**	0.24	0.43
$\sigma_{\pi^e_{5Y}}$	Measurement error (*Tankan* , 5-year ahead)	invG	1.00	(Inf)	**0.37**	0.26	0.47

Note: "N" stands for normal distribution, "B" for beta distributution, "invG" for inverse gamma distribution.

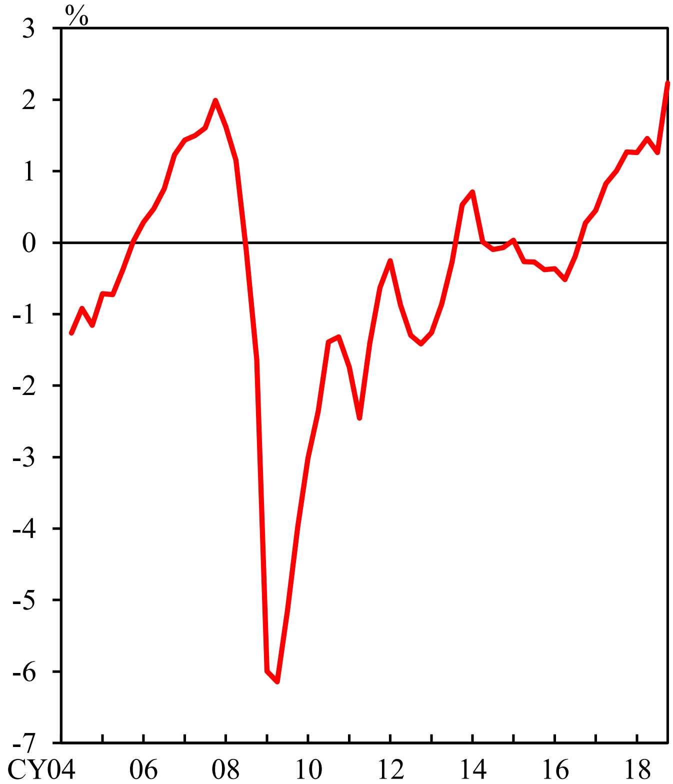
Table 3. Result of the Estimation Using the Different Prior Distribution

		Prior			Posterior		
		Type	Mean	S.D.	Mean	5%	95%
New Keynesian Phillips Curve							
γ	Inflation persistence	B	0.50	(0.15)	**0.30**	0.19	0.40
κ	Slope	N	0.10	(0.10)	**0.13**	0.07	0.18
η	Sensitivity to import prices	N	0.00	(1.00)	**0.10**	0.07	0.13
ρ	Persistence of import price shock	B	0.50	(0.20)	**0.35**	0.19	0.52
Formation of Inflation Expectations							
μ	Degree of rational inattention	uniform distribution with the interval [0, 1]			**0.49**	0.30	0.66
λ	Degree of information stickiness	uniform distribution with the interval [0, 1]			**0.51**	0.32	0.69
δ	Weight used in calculating the inflation expectations of the firms answering "don't know"	B	0.50	(0.20)	**0.74**	0.52	0.96
Reduced-Form Output Gap Equation							
ϕ_1	Output gap (1-quarter lagged)	N	1.30	(0.50)	**1.50**	1.30	1.71
ϕ_2	Output gap (2-quarter lagged)	N	-0.50	(0.50)	**-0.54**	-0.70	-0.36
ϕ_3	Inflation gap (1-quarter lagged)	N	0.10	(1.00)	**0.07**	-0.06	0.22
ϕ_4	Inflation gap (2-quarter lagged)	N	0.10	(1.00)	**-0.39**	-0.53	-0.25
Formation of Inflation Norm							
α	Sensitivity to actual inflation	uniform distribution with the interval [0, 1]			**0.08**	0.04	0.11
Size of Shocks							
σ_π	Cost-push shock	invG	1.00	(Inf)	**0.67**	0.57	0.77
σ_y	Output gap shock	invG	1.00	(Inf)	**0.61**	0.52	0.71
σ_τ	Inflation norm shock	invG	1.00	(Inf)	**0.15**	0.12	0.17
σ_{π^*}	Inflation target shock	invG	1.00	(Inf)	**0.18**	0.15	0.21
σ_m	Import price shock	invG	5.00	(Inf)	**4.81**	4.09	5.50
$\sigma_{\pi^e_{1Y}}$	Measurement error (*Tankan* , 1-year ahead)	invG	1.00	(Inf)	**0.54**	0.40	0.68
$\sigma_{\pi^e_{3Y}}$	Measurement error (*Tankan* , 3-year ahead)	invG	1.00	(Inf)	**0.33**	0.23	0.43
$\sigma_{\pi^e_{5Y}}$	Measurement error (*Tankan* , 5-year ahead)	invG	1.00	(Inf)	**0.37**	0.26	0.47
$\sigma_{\Delta\pi^e_{1Y}}$	Measurement error (*Opinion Survey* , 1-year ahead)	invG	1.00	(Inf)	**1.03**	0.84	1.21
$\sigma_{\Delta\pi^e_{5Y}}$	Measurement error (*Opinion Survey* , 5-year ahead)	invG	1.00	(Inf)	**0.56**	0.46	0.66

Note: "N" stands for normal distribution, "B" for beta distributution, "invG" for inverse gamma distribution.

Figure 1. Data Used for Estimation (1)

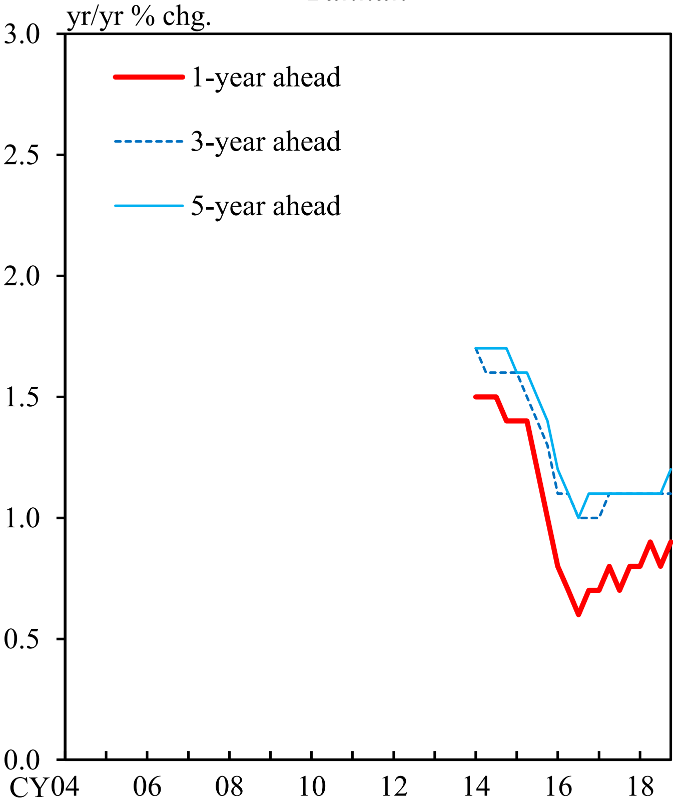
(1) Output Gap (BOJ's Estimate)



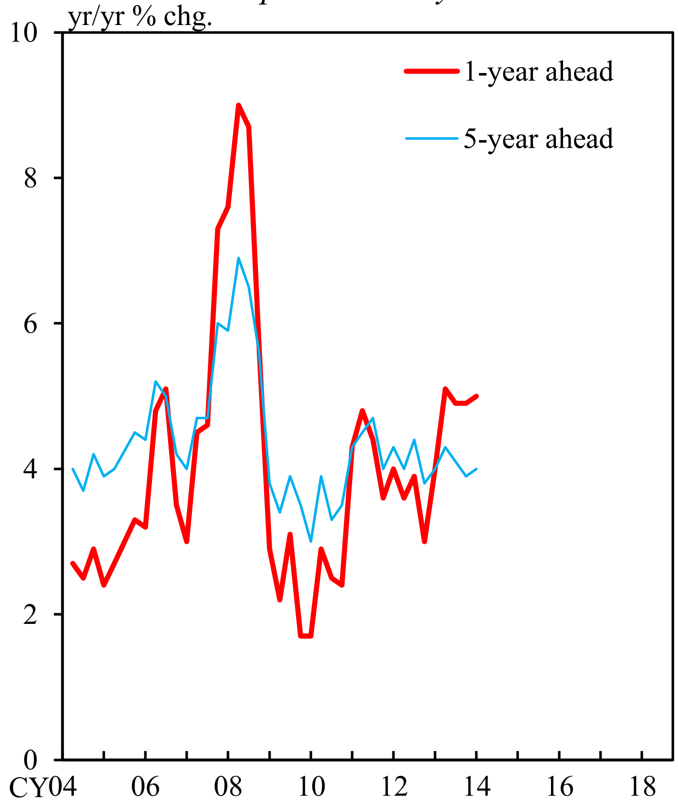
(2) CPI (less Fresh Food)

(3)-(7) Inflation Expectations

< Tankan >



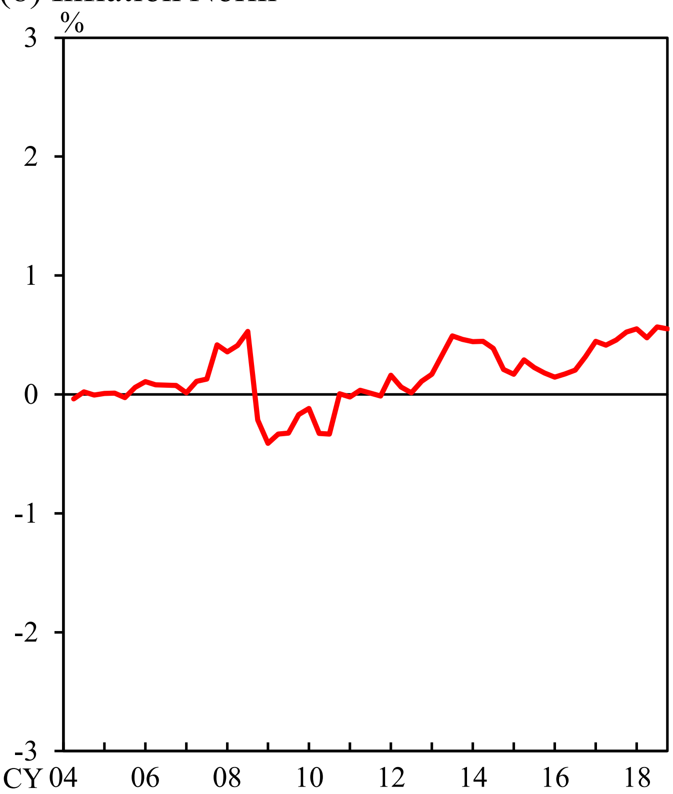
< Opinion Survey >



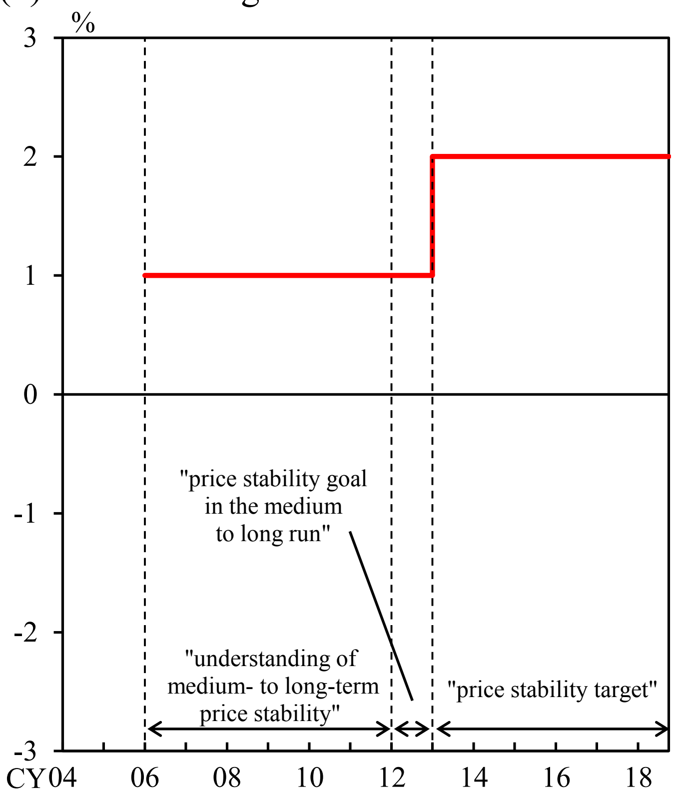
Notes: 1. Series (2) is adjusted for the change in the consumption tax rate.
2. For series (2), we use quarter-on-quarter % changes rather than year-on-year % changes in our estimation.
Sources: Ministry of Internal Affairs and Communications; Bank of Japan

Figure 1. Data Used for Estimation (2)

(8) Inflation Norm



(9) Inflation Target



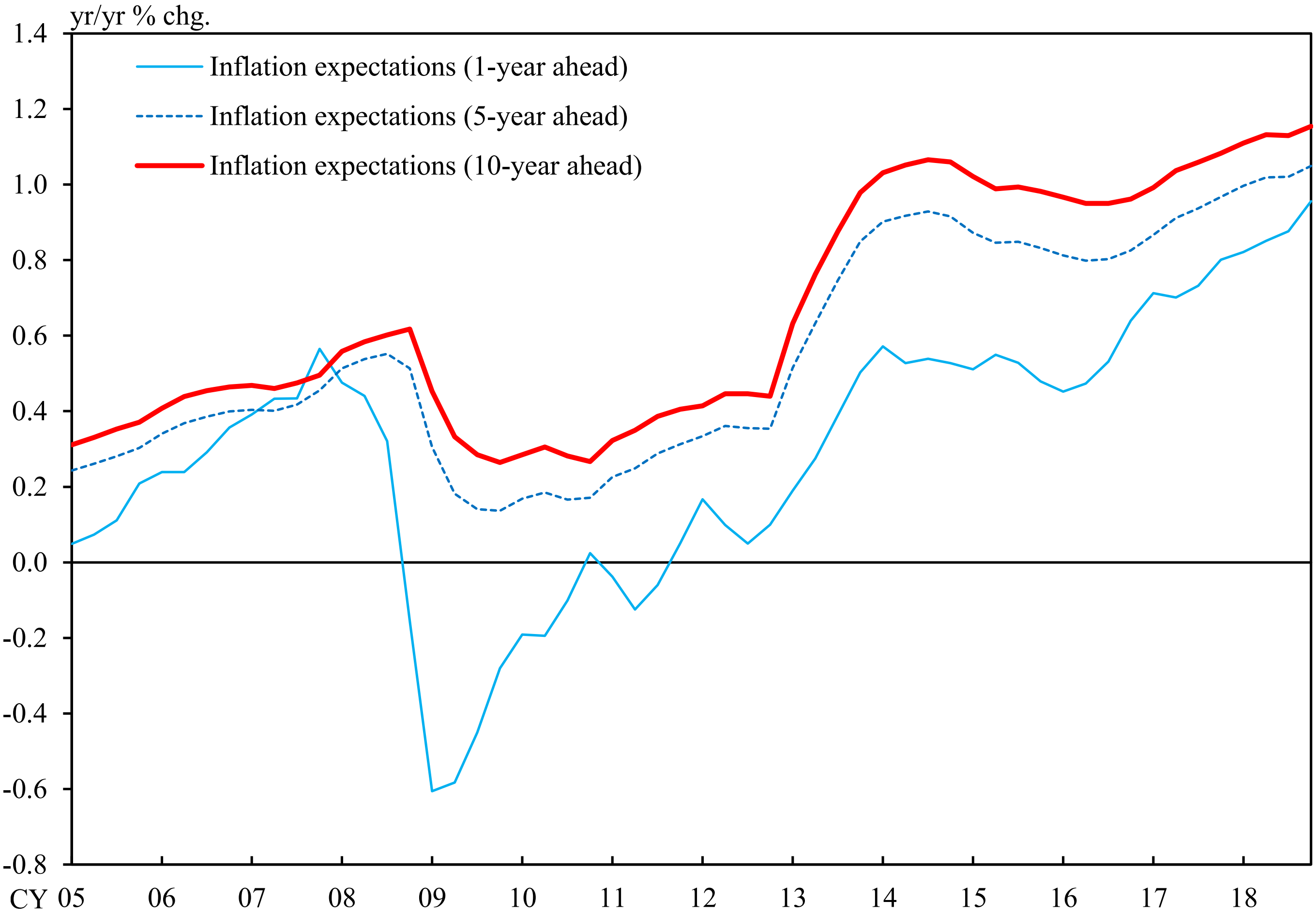
(10) Import Price Index

Notes: 1. Series (8) is the permanent component of series (2) extracted using the BN decomposition. For the methodology of the BN decomposition we use, see Takahashi (2016).

2. For series (10), we use deviations from the sample avarage of quarter-on-quarter % changes in our estimation, rather than year-on-year % changes.

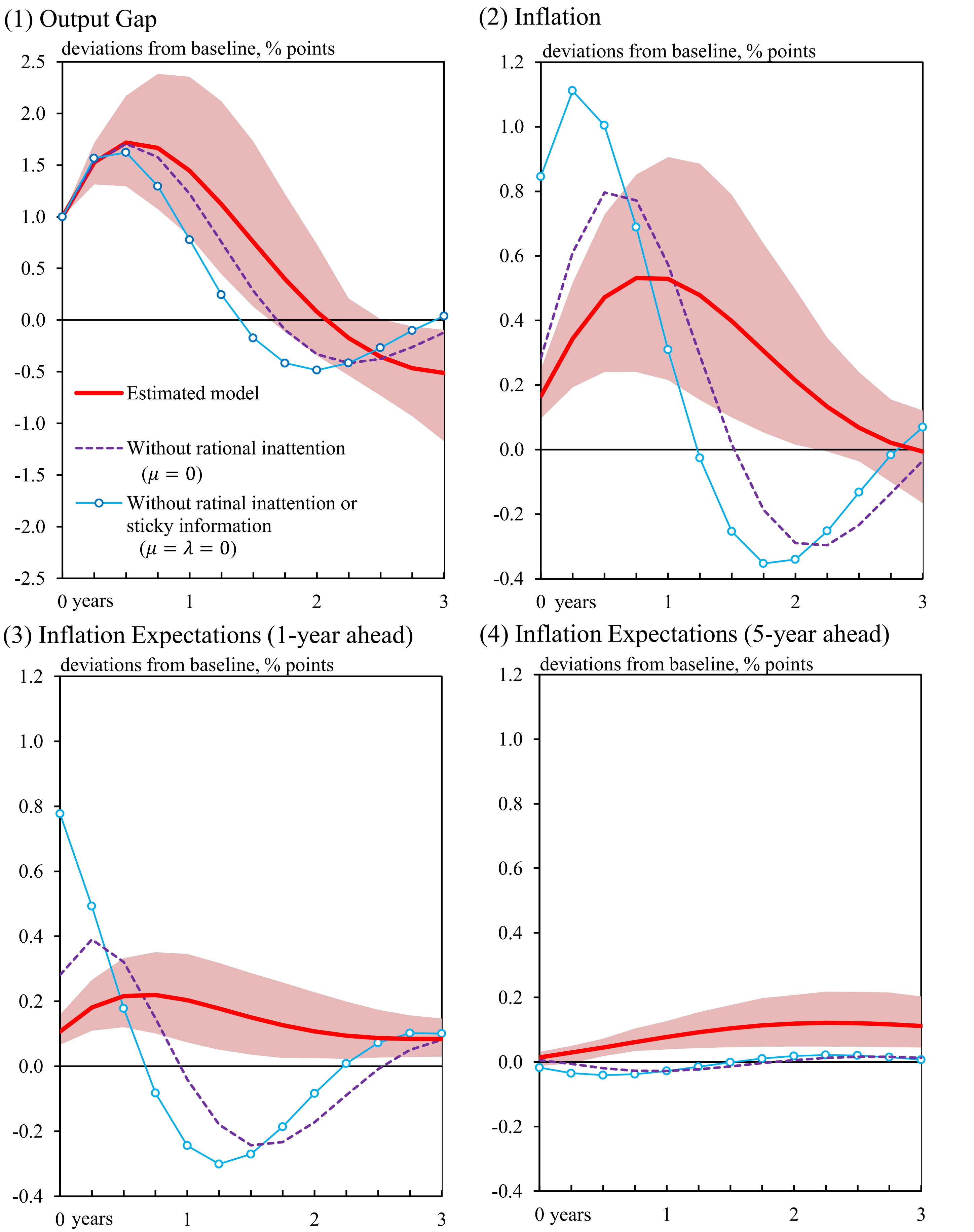
Sources: Bank of Japan

Figure 2. Term Structure of Firms' Inflation Expectations in Japan



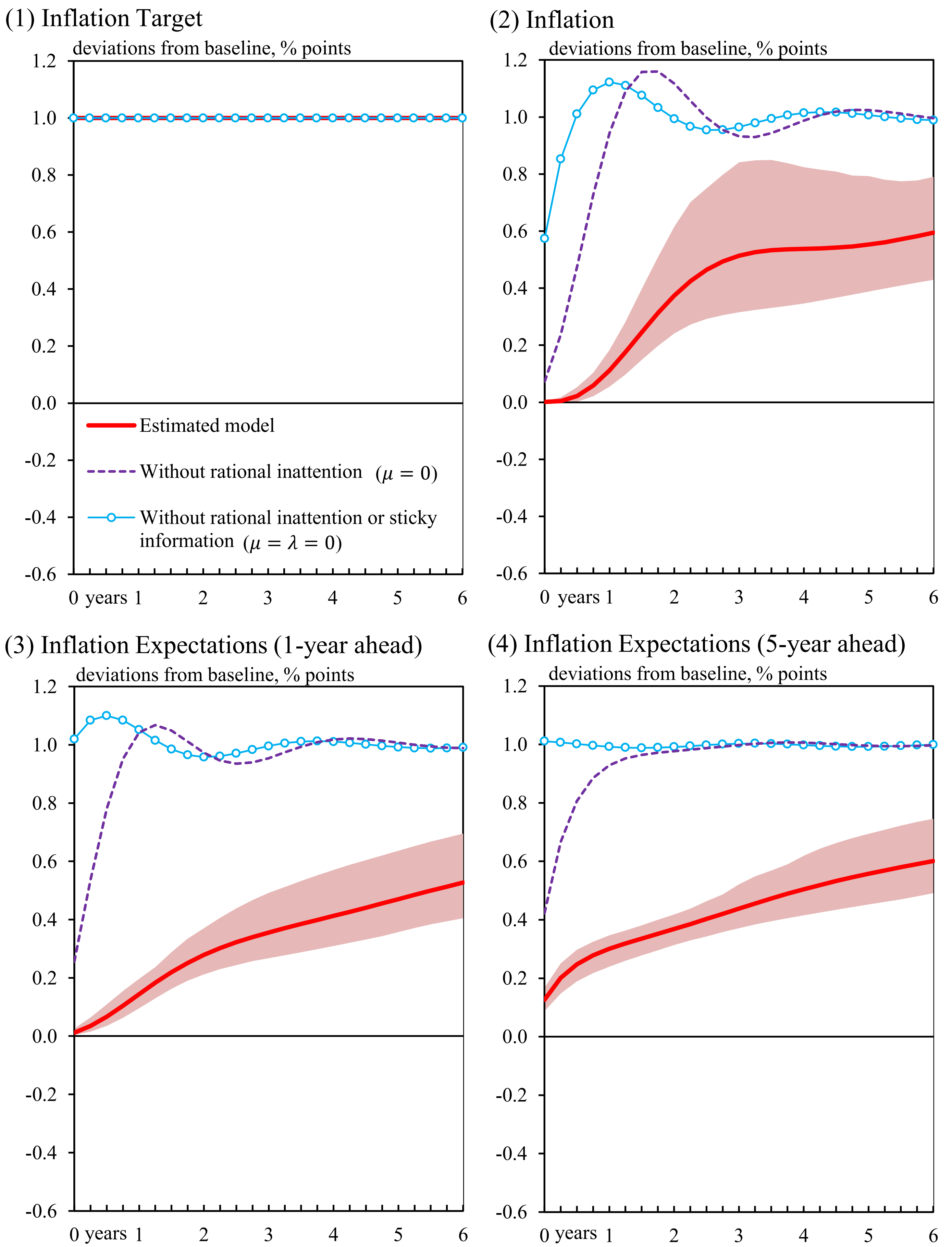
Note: The figures are the posterior means of the estimates of the aggregate inflation expectations.

Figure 3. Impulse Responses to a Temporary Output Gap Shock



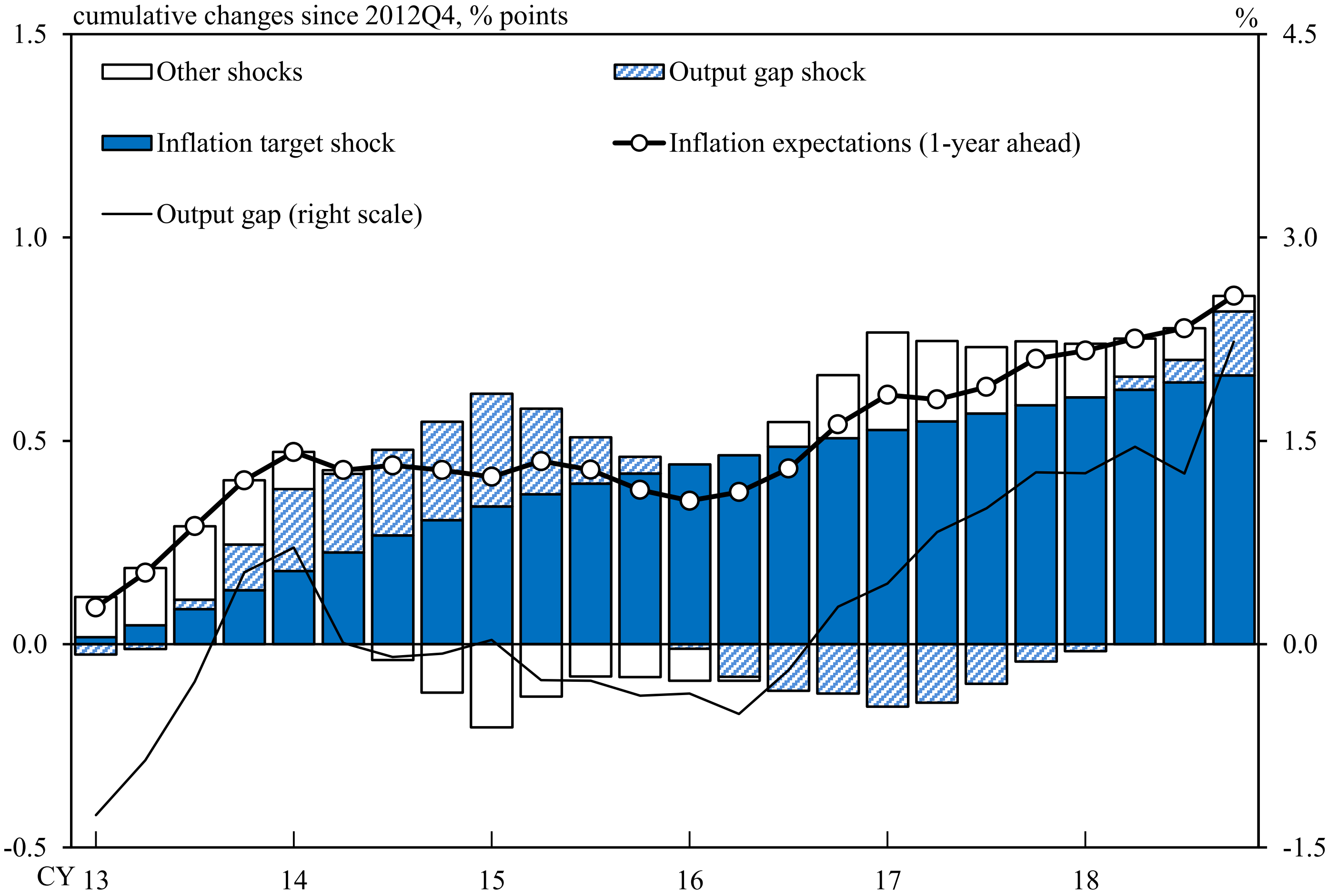
Note: Each line represents the mean of the impulse response. The shaded areas represent the 90% intervals of the responses in the estimated model (depicted as red solid lines).

Figure 4. Impulse Responses to a Permanent Inflation Target Shock



Note: Each line represents the mean of the impulse response. The shaded areas represent the 90% intervals of the responses in the estimated model (depicted as red solid lines).

Figure 5. Historical Decomposition of Inflation Expectations



Notes: 1. The values of the parameters are set to their posterior means.
2. The contributions of "Other shocks" include those of the initial values.

Source: Bank of Japan