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Kazuhiro Hiraki [*]
kazuhiro.hiraki@boj.or.jp

Wataru Hirata[*]
wataru.hirata@boj.or.jp

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Bank of Japan
2-1-1 Nihonbashi-Hongokucho, Chuo-ku, Tokyo 103-0021, Japan

[*] Monetary Affairs Department

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Market-based Long-term Inflation Expectations in Japan: A Refinement on Breakeven Inflation Rates*

Kazuhiro Hiraki† Wataru Hirata‡

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Abstract

In Japan, the breakeven inflation rate (BEI), commonly used as a proxy for market-based long-term inflation expectations, has evolved lower than survey-based measures of long-term inflation expectations. The literature has pointed to three factors, other than long-term inflation expectations, that act as drivers of long-term BEI rates: (i) the deflation protection option premium of inflation-linked bonds, (ii) the liquidity premium of the bonds, and (iii) the spread between nominal and real term premia (the term premium spread). This paper estimates an affine term structure model to decompose Japan's BEI into long-term inflation expectations and these three other driving factors. Our empirical results show that the deflation protection option premium for Japan's Inflation-Indexed Bonds (JGBi) has pushed the BEI up, while the liquidity premium of JGBi and the term premium spread have pulled it down, all having non-negligible contributions to developments in the BEI. This indicates that the evolution of Japan's BEI has been driven by these three factors as well as by the long-term inflation expectations of market participants. Consequently, the estimated long-term inflation expectations have evolved higher than the BEI throughout almost the entire estimation period.

JEL classifications: E31, E43, G12

Keywords: Breakeven inflation rate, Inflation expectations, Liquidity premium, Deflation protection option premium, Term premium

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†Monetary Affairs Department, Bank of Japan, kazuhiro.hiraki@boj.or.jp

‡Monetary Affairs Department, Bank of Japan, wataru.hirata@boj.or.jp

1 Introduction

The breakeven inflation rate (BEI), defined as the spread between yields of nominal government bonds and inflation-linked bonds, has commonly been used as a proxy for market-based inflation expectations. In Japan, the long-term BEI has evolved lower than several measures of survey-based inflation expectations of similar tenor, as shown in Figure 1. This gap between BEI and survey-based measures of inflation expectations could pose a problem when assessing inflation expectations. Yet it has been argued, at least in central bank circles, that BEI can deviate from market-based inflation expectations in the presence of a liquidity premium of inflation-linked bonds, among other factors, and that caution is required in automatically regarding BEI as a proxy for market-based inflation expectations (see e.g., Bernanke 2004 and Yellen 2015).

As drivers of long-term BEI other than long-term inflation expectations, the literature has pointed to three factors: (i) the deflation protection option premium of inflation-linked bonds, (ii) the liquidity premium of the bonds, and (iii) the spread between nominal and real term premia (the term premium spread). As such, one can think of removing these three driving factors from the BEI to gauge inflation expectations. This paper identifies the market-based long-term inflation expectations in Japan by disentangling the other three driving factors from Japan's BEI. In doing so, the paper estimates an affine term structure model, largely following the seminal study on the US BEI by Andreasen, Christensen, and Riddell (2018) (henceforth ACR). However, our study makes several modifications to their model, taking into consideration constraints on the data available for Japan's Inflation-Indexed Bonds (JGBi) and differences between Japan and the US in the structure of their respective inflation-linked bond markets.

Before presenting our empirical results, we begin by confirming that the long-term BEI consists of long-term inflation expectations and the aforementioned other three factors. First, the nominal long-term bond yield can be decomposed as follows:

$$\begin{aligned}
&\textbf{Nominal long-term bond yield} \\
&= \textbf{Average of expected future yields of short-term nominal bonds} \\
&\quad + \textbf{Nominal term premium}.
\end{aligned}$$

Second, the inflation-linked long-term bond yield can be decomposed as follows:

Inflation-linked long-term bond yield
= **Average of expected future yields of short-term real bonds**
+ **Real term premium**
− **Deflation protection option premium**
+ **Liquidity premium**.

The deflation protection option premium and the liquidity premium are factors specific to inflation-linked bonds. For the former premium, many inflation-linked bonds issued in advanced economies, including the US and the "new" JGBi series first issued in 2013, all provide protection against deflation. That is, investors are repaid with the face value (notional amount at issuance) even when the general price level falls from the issuance date to the maturity date. This option may pull down the yields of inflation-linked bonds. On the other hand, the liquidity premium, arising from the fact that the liquidity of inflation-linked bonds is lower than that of nominal government bonds, can push up the yields of inflation-linked bonds (see, e.g., D'Amico et al. 2018 and Yuyama and Moridaira 2017).

Last, the following decomposition of the long-term BEI is obtained by subtracting the inflation-linked long-term bond yield from the nominal long-term bond yield:

Long-term BEI = **Long-term inflation expectations**
+ **Deflation protection option premium (of inflation-linked bonds)**
− **Liquidity premium (of inflation-linked bonds)**
+ **Term premium spread**.

That is, the long-term BEI can be decomposed into long-term inflation expectations and the other three driving factors: the deflation protection option premium of inflation-linked bonds, the liquidity premium of the bonds, and the term premium spread.[1]

[1] Note that the term premium spread is also referred to as the inflation risk premium in the literature. This is because, in standard textbooks, the term premium spread can be interpreted as a compensation

The main findings of our empirical analysis are twofold. First, in addition to the long-term inflation expectations, the other three driving factors of BEI have made non-negligible contributions to developments in Japan's BEI. The deflation protection option premium of JGBi has contributed to pushing the BEI up since the issuance of the new JGBi series in 2013. Its contribution has increased even more during the recent coronavirus pandemic. The liquidity premium of JGBi has pulled the BEI down, except during the transition period from the old to the new JGBi series. While the impact of the liquidity premium on the BEI spiked at the height of the Global Financial Crisis (GFC), it has exerted mild but sustained downward pressure on the BEI since the issuance of the new JGBi series began. The term premium spread has contributed to pulling the BEI down over most of the estimation period, and its effect in absolute terms has been gradually increasing in recent years.

Second, our estimates of the market-based long-term inflation expectations in Japan— which are obtained by removing the other three driving factors from Japan's BEI— have evolved higher than the BEI throughout almost the entire estimation period. Estimated inflation expectations recorded a large negative value during the GFC, when crude oil prices plunged, but the inflation expectations continued to rise thereafter, reaching around 2% in mid 2014, after the Bank of Japan introduced quantitative and qualitative monetary easing (QQE).[2] Since then, as with survey-based measures of inflation expectations, market-based inflation expectations have declined, marking around 1% in the first half of 2016, and remaining around 1% until the beginning of 2020. During the coronavirus pandemic, market-based inflation expectations weakened somewhat amid the rapid downturn in the global economy and a large fall in crude oil prices.

The literature on BEI has developed rapidly in recent years, focusing on the evaluation

for uncertainty about future inflation. However, since the Global Financial Crisis, major central banks' large-scale asset purchases have had the potential to suppress the nominal term premium (see e.g., Sudo and Tanaka 2020). Likewise, the demand for safe assets has had the potential to suppress the nominal term premium (see, e.g., Krishnamurthy and Vissing-Jorgensen 2012 and Ichiue et al. 2012). These factors, *ceteris paribus*, decrease the term premium spread, but may not be considered as changes in investors' preference on the inflation risk. Therefore, except when referring to relevant parts in the literature, the present paper uses the terminology "term premium spread" instead of "inflation risk premium."

[2] Our estimates of market-based long-term inflation expectations in Japan are susceptible to movements in crude oil prices, in line with the findings of recent studies on other advanced economies (see, e.g., Owyang and Shell 2019).

of the other three driving factors of BEI. For the US BEI, there has been a growing number of studies on the liquidity premium after the sudden spike in the yields of Treasury Inflation-Protected Securities (TIPS) during the GFC (e.g., Kajuth and Watzka 2011, Haubrich et al. 2012, Abrahams et al. 2016 and D'Amico et al. 2018). In addition, an increasing amount of research has focused on estimating the deflation protection option premium using series-by-series TIPS data, motivated by the prolonged low inflation environment since the GFC (e.g., Grishchenko et al. 2016 and ACR). Moreover, since the influential work by Campbell and Shiller (1996) (see the literature survey by, for example, Kupfer 2018), there has been a considerable amount of research on the term premium spread (often referred to as the inflation risk premium, as noted above). Among these studies, ACR—which our study closely follows—proposed a novel approach within the affine term structure framework, which enables us to identify the three driving factors of BEI.

The number of studies of Japan's BEI is limited, compared with those on the US TIPS. Yet Yuyama (2016), Yuyama and Moridaira (2017) and Christensen and Spiegel (2019) (henceforth CS) made important contributions focusing on one or two of the three driving factors.[3] Yuyama (2016) employed stochastic interest rate models in modeling inflation dynamics, and estimated the deflation protection option premium based on Monte Carlo simulations. He reported that the estimates of the premium were small in Japan. Yuyama and Moridaira (2017) constructed proxies of the liquidity premium and the inflation risk premium from observed variables, and estimated market-based inflation expectations using a state-space model. They indicated that the average estimated values of the liquidity premium and the inflation risk premium over the period 2005–2014 had non-negligible downward effects on Japan's BEI. CS estimated the deflation protection option premium and the inflation risk premium using an affine term structure model. They pointed out that the deflation protection option premium was considerably large in Japan. In sum, the estimation results presented in these studies exhibit fairly large dispersion in the three driving factors of Japan's BEI. This may be the consequence of

[3]For a study that uses Japanese market data other than BEI, see Imakubo and Nakajima (2015). They estimated an affine term structure model using inflation swap rates and found that their estimates of market-based long-term inflation expectations in Japan were on a rising trend from 2012 to 2014, after recording a large negative value during the GFC, in line with our result.

omitting at least one of the three driving factors in each study. For example, Yuyama and Moridaira (2017) did not estimate the deflation protection option premium, and CS did not consider the liquidity premium. Our paper is the first to estimate simultaneously the market-based long-term inflation expectations and the other three driving factors of BEI in Japan, thereby contributing to more precise evaluations of them.

To estimate simultaneously the long-term inflation expectations and the other three driving factors embedded in BEI, we need to pay particular attention to the identification of the liquidity premium of inflation-linked bonds. This paper proposes an identification strategy that employs the advantages of the following two approaches: studies that show from both theoretical and empirical perspectives that the spread between inflation swap rates and BEI (henceforth the IS-BEI spread) contains information on the liquidity premium of inflation-linked bonds (see, e.g., Haubrich et al. 2012); and the affine term structure model of ACR, which uses information contained in the cross-section of the series-by-series prices of inflation-linked bonds to estimate the liquidity premium.[4] In other words, our paper identifies the liquidity premium using information on inflation-linked bond prices and inflation swap rates, assuming that the liquidity premium of inflation swap rates is less of a concern than that of inflation-linked bonds. In addition, the paper obtains information on the liquidity premium from the cross-section of series-by-series prices of inflation-linked bonds. Taking this new approach, with which our affine term structure model is estimated exploiting the information from both inflation-linked bonds and inflation swap rates, our paper aims to identify the liquidity premium in a stable manner even under data-scarce environments regarding inflation-linked bonds, such as JGBi.

The remainder of the paper is organized as follows. Section 2 overviews the product design and market environment of JGBi, and raises some issues regarding JGBi data. Section 3 describes the model. Section 4 explains the dataset and estimation strategy. Section 5 presents the empirical results. Section 6 concludes.

[4]There are other estimation strategies proposed in the literature. Abrahams et al. (2016), Pflueger and Viceira (2016), and Yuyama and Moridaira (2017) proposed using proxy variables of the liquidity of inflation-linked bonds, such as bid-ask spreads and VIX. D'Amico et al. (2018) combined survey-based inflation expectations measures with BEI to estimate the liquidity premium. These strategies depend largely on the additional variables used in estimation.

2 Overview of JGBi market and some issues regarding JGBi data

2.1 Product design and market size of JGBi

An inflation-linked bond is a government bond whose principal and coupons move in proportion with the price index specified by the treasury authority. In 2004, Japan's Ministry of Finance began issuing JGBi linked to the Consumer Price Index (CPI, all items less fresh food).[5] The issuance of JGBi was suspended in 2008 as demand for JGBi decreased amid the GFC. The Ministry of Finance restarted the issuance of JGBi in October 2013, and there have been quarterly auctions since then. Among all issued JGBi series, the sixteen JGBi series issued before the suspension in 2008 are called the "old JGBi series," while those issued after 2013 are called the "new JGBi series."

There is one key difference in the product design between the old and the new JGBi series: the new series have the deflation protection feature, whereas the old series do not. Therefore, the redemption value of the old JGBi series would be below par if the price level at the redemption date were below that at the issuance date. On the other hand, the new JGBi series would redeem their initial principal value if deflation occurred between the issuance date and maturity date.

Next, we compare the market size of JGBi with that of the US TIPS, as shown in Figure 2-A. The total outstanding value of JGBi as of March 2020 is 12 trillion yen (approximately 110 billion US dollar), amounting to 1.2% of the total outstanding of interest-bearing Japanese Government Bonds (JGB). The total outstanding value of the US TIPS at the same period is 1.5 trillion dollars. This amounts to 8.9% of the marketable US Treasury securities, indicating the market size of JGBi is much smaller. In addition, while the US Treasury has issued 77 series of TIPS since 1997 with four distinct maturities (5, 10, 20 and 30 years), the Ministry of Finance has issued only 24 series of JGBi, all with 10-year maturity, as of March 2020.

[5] The price index referenced by JGBi bonds is called the "Ref Index," and it is approximately equal to the CPI with a three-month lag. See the Ministry of Finance website https://www.mof.go.jp/english/jgbs/topics/bond/10year_inflation/coefficient.htm for more information.

2.2 Market environment and liquidity of JGBi market

The JGBi market has been characterized as having a persistent supply-demand imbalance partly due to a "continuing issue with expansion in the investor base" (Ministry of Finance 2016). In this environment, the JGBi liquidity, roughly measured by the turnover ratio (i.e., the ratio of trading volumes to outstanding traded in the market), has been considerably lower than that of the US TIPS, as shown in Figure 2-B. Against this backdrop, the Ministry of Finance has conducted multiple phases of buyback auctions of JGBi since 2007, especially when investors' demand for JGBi has declined.

Given such a market environment, short-term speculative behavior amid market turmoil, and large-scale trading by a specific type of investor could have a non-negligible impacts on the yields of JGBi. For example, the large-scale buybacks of the old JGBi series during the period when JGBi issuance was resumed (October 2013 to January 2015) might have pushed up the price of the old JGBi series and pulled down their yield.[6] Indeed, the series-by-series yields of the old and the new JGBi series displayed in Figure 3 show that the old series skyrocketed in late 2008 at the height of the GFC. Moreover, there are considerable discrepancies between the old and new series in periods following the restarting of JGBi issuance in 2013, which are difficult to attribute to differences in remaining maturities and product design alone.[7]

2.3 How to address the difficulties

To analyze the JGBi market using an affine term structure model, it is necessary to take appropriate account of the effects of JGBi product design and market environment, as described in Sections 2.1 and 2.2. Further, we must bear in mind the data limitation caused by the suspension of JGBi issuance. We can point to three specific issues in this aspect.[8]

[6] See Ministry of Finance (2013) for details of the special buyback auctions held during the period when JGBi issuance was resumed.

[7] Note that the deflation protection feature of the new JGBi series is likely to pull down their yields relative to those of the old JGBi series. This implies that the yield gaps between the old and new series caused by factors other than the deflation protection, including those induced by the buybacks, would be even larger.

[8] In addition to these issues, note that JGBi and Japanese inflation swaps make reference to CPI (all items less fresh food), which is affected by consumption tax hikes, as the basis for contractual price adjustments. This means that BEI and inflation swap rates in Japan were subject to the upward

First, the deflation protection provided uniquely to the new JGBi series may have caused a discontinuity in Japan's BEI around the transition period from the old to the new JGBi series, because the deflation protection option premium applies only to the new series (see CS and Yuyama 2016). As such, we need to estimate the impact of the deflation protection option premium on yields of the new JGBi series. In our model, we analytically calculate the deflation protection option premium of the new JGBi series based on Christensen et al. (2012) (see Section 3.2 for details).

Second, difficulties in obtaining JGBi yields for a broad range of maturities over a fairly long sample period, the result of the five-year suspension in the bond issuance and the lack of variety in maturity at issuance, prevent us from estimating the real term structure in a stable manner.[9] These difficulties also hamper the identification of the liquidity premium. To overcome these difficulties, we propose using inflation swap rates in addition to JGBi data. An inflation swap is a derivative contract that swaps the realized inflation rate and a predetermined inflation swap rate at the expiry date. Therefore, we can obtain additional information on real interest rates by combining inflation swap rates and nominal interest rates. Because inflation swap data in Japan is available from one-year to ten-year tenor since March 2007, this additional data source helps estimate the real term structure in a stable manner. This approach provides another benefit when extracting information on the liquidity premium, as described in Section 3.3; a number of existing studies show that IS-BEI spreads also contain useful information about the liquidity premium of the inflation-linked bonds (see, e.g., Haubrich et al. 2012 and D'Amico et al. 2018).

Last, the technical formulation of the liquidity premium in ACR, which is based on the US TIPS market environment, may not be suitable for analyzing JGBi mainly because of its low liquidity. To overcome this issue, we conduct a preliminary empirical analysis on the effect on JGBi liquidity of years elapsed since issuance, and we reflect the empirical

pressure from when market participants began to incorporate the phased-in consumption tax hikes into their expectations until the actual CPI (or more precisely, the Ref Index) fully reflected the impact of the tax hikes. Since our data processing takes no account of this impact, we need to pay attention to the potential quantitative impact of consumption tax hikes on our estimation results.

[9] Christensen et al. (2020) compared the deflation protection option premium of inflation-linked bonds between Japan, the US, France, and Canada during the coronavirus pandemic. They reported the estimates based on ACR model for the US, France, and Canada, while for Japan they reported those based on CS model, which takes no account of the liquidity premium. Christensen et al. (2020) mentioned that the "unusual maturity composition" of JGBi prevented them from considering the liquidity premium.

characteristics of JGBi in our model specification (see Section 3.3).

Note that it is possible to employ a method that uses only inflation swap rates as observable data, and avoid estimating the liquidity premium of inflation-linked bonds, as in Imakubo and Nakajima (2015). However, trading volume in the inflation swap market in Japan is more muted than that in the JGBi market, raising the possibility that inflation swap rates take longer time to fully incorporate incoming information on inflation expectations. Technically, this suggests that inflation swap rates contain larger measurement errors than JGBi prices do. Therefore, combining inflation swap rates and JGBi prices would help estimate real interest rates and inflation expectations in a more robust manner, as JGBi prices may incorporate information on inflation and real interest rates faster than the inflation swap rates.[10] Note also that we do not use the data on the old JGBi series for the period after the Ministry of Finance started issuing the new JGBi series. This is because, as mentioned in Section 2.2, the old JGBi series are considered to have been affected by the additional buyback program to a large extent, thereby becoming noisy measures of the real yields by that time.

3 Theoretical framework

This section explains our affine term structure model. First, to fix ideas and notations, Section 3.1 provides general theoretical results of affine term structure models. Section 3.2 presents our affine term structure model. Section 3.3 discusses our identification strategy for the liquidity premium and related studies.

3.1 General results of the affine term structure model

In the affine term structure framework, the term structure of interest rates is expressed by a few factors. Let X_t be a vector of N factors. Among the N factors, the first $N-1$ factors are the *fundamental factors*, that is, these factors determine nominal and real interest rates in the frictionless market. We denote the fundamental factors by X_t^f. The remaining factor, which we denote by X_t^{liq}, determines the liquidity premium of

[10]Indeed, Section 5.1 shows that the estimated measurement errors of JGBi are smaller than those of inflation swap rates.

inflation-linked bonds.

The instantaneous nominal rate is given by

$$r_t^N = \rho_0^N + \left(\rho_x^N\right)^\top X_t^f, \tag{1}$$

where ρ_0^N is a scalar constant, ρ_x^N is a $(N-1)\times 1$ vector constant, and $\top$ denotes the transpose of a vector or a matrix. Note that the liquidity factor X_t^{liq} does not appear in equation (1) because we assume that it never affects the nominal interest rate.

Next, we define instantaneous inflation-linked bond rates. We follow ACR in assuming that the liquidity factor has a heterogeneous effect on each inflation-linked bond series. This means that the instantaneous inflation-linked bond rate is defined for each series. Specifically, the instantaneous inflation-bond rate of the i-th bond series is given by

$$r_t^{R,i} = \underbrace{\rho_0^R + (\rho_x^R)^\top X_t^f}_{r_t^{R,f}} + h^i(t) X_t^{liq}, \tag{2}$$

where ρ_0^R is a scalar constant, and ρ_x^R is a $(N-1)\times 1$ vector constant. We call the sum of the first two terms on the right-hand side, $r_t^{R,f}$, the *fundamental real rate*. The fundamental real rate is the instantaneous real interest rate that would prevail in the frictionless market. The last term on the right-hand side represents the contribution of the liquidity factor to the instantaneous inflation-linked bond rate. Here, $h^i(t)$ controls the impact of the common liquidity factor on the i-th bond series. We call $h^i(t)$ the *liquidity factor coefficient.*

We assume the following dynamics of the vector of the factors X_t under the risk-neutral probability measure $\mathbb{Q}$,

$$dX_t = \mathcal{K}^{\mathbb{Q}}(\theta^{\mathbb{Q}} - X_t)dt + \Sigma dW_t^{\mathbb{Q}}, \tag{3}$$

where $\mathcal{K}^{\mathbb{Q}}$ and Σ are $N \times N$ matrices, $\theta^{\mathbb{Q}}$ is a $N \times 1$ vector, and $W_t^{\mathbb{Q}}$ is a standard Brownian motion under the risk-neutral probability measure $\mathbb{Q}$.

Under this Gaussian affine term structure setup, it is possible to show that the time-t price of nominal and real zero-coupon bonds that mature at time T are given, respectively, by

$$P_t^N(T) = \exp\left(A_t^N(T) + B_t^N(T)^\top X_t^f\right), \tag{4}$$

$$P_t^{R,i}(T) = \exp\left(A_t^{R,i}(T) + B_t^{R,f}(T)^\top X_t^f + B_t^{R,i}(T) X_t^{liq}\right), \tag{5}$$

where $A_t^N(T)$, $A_t^{R,i}(T)$, $B_t^N(T)$, $B_t^{R,f}(T)$, and $B_t^{R,i}(T)$ are given by the solutions of appropriate ordinary differential equations (see Appendix A for the concrete expressions of these coefficients in our model).

By transforming equations (4) and (5), it follows that the nominal and real zero-coupon rate, $R_t^N(T)$ and $R_t^{R,i}(T)$, are given by the following affine functions of the factors,

$$R_t^N(T) = -\frac{1}{T-t}\left(A_t^N(T) + B_t^N(T)^\top X_t^f\right), \tag{6}$$

$$R_t^{R,i}(T) = -\frac{1}{T-t}\left(A_t^{R,i}(T) + B_t^{B,f}(T)^\top X_t^f + B_t^{R,i}(T) X_t^{liq}\right). \tag{7}$$

Next, the inflation rate in our theoretical framework is presented. According to the Fisher equation, the future path of the inflation rate coincides with the path of the difference between the future nominal and real interest rate. Since the liquidity factor of the inflation-linked bond is not pertinent to the price level fluctuation, it follows that the instantaneous inflation rate coincides with the difference between the instantaneous nominal rate and the fundamental real rate, $r_t^N - r_t^{R,f}$. Hence, by denoting the time-t price level by Π_t, the inflation rate from time t to T is given by

$$\frac{\Pi_T}{\Pi_t} = e^{\int_t^T (r_s^N - r_s^{R,f}) ds}. \tag{8}$$

Moreover, the expected inflation rate under the risk-neutral probability measure $\mathbb{Q}$ can be expressed as the exponential affine form

$$\mathbb{E}_t^{\mathbb{Q}}\left[\frac{\Pi_T}{\Pi_t}\right] = \exp\left(A_t^{IS}(T) + B_t^{IS}(T)^\top X_t^f\right), \tag{9}$$

This is because the fundamental interest rate spread, $r_t^N - r_t^{R,f}$, is an affine function of the fundamental factors. Similar to the zero-coupon bond prices, one can obtain $A_t^{IS}(T)$ and $B_t^{IS}(T)$ as the solution of appropriate ordinary differential equations (see Appendix

A for the concrete expressions of these coefficients in our model).

Let us turn to inflation swap rates. An inflation swap is a derivative contract that swaps the fixed inflation swap rate and the realized inflation rate at the expiry of the contract. Specifically, letting $IS_t(n)$ denote the n-year inflation swap rate, the fixed-leg side of a n-year inflation swap contract will pay $(1 + IS_t(n))^n$ at expiry, whereas the floating-leg side pays the realized inflation Π_{t+n}/Π_t. Therefore, the risk-neutral valuation leads to

$$(1 + IS_t(n))^n = \mathbb{E}_t^{\mathbb{Q}} \left[\frac{\Pi_{t+n}}{\Pi_t} \right].$$

Then, taking the logarithm of both sides and applying a log approximation to the left-hand side shows that the inflation swap rate is given approximately by the following affine function of the factors:[11]

$$IS_t(n) = \frac{1}{n} \left(A_t^{IS}(t+n) + B_t^{IS}(t+n)^\top X_t^f \right). \tag{10}$$

3.2 Our affine term structure model

This study employs a five-factor Gaussian affine term structure model. Among the five factors, the first four factors are the fundamental factors, $X_t^f = [L_t^N, S_t^N, L_t^R, S_t^R]^\top$: the level and slope factors of the nominal interest rates (L_t^N, S_t^N) and those of the fundamental real interest rates (L_t^R, S_t^R).[12] Moreover, our paper employs a no-arbitrage Nelson-Siegel model (see Christensen et al. 2011) for the formulation of model parameters. With these specifications, one can derive the observation equations used in our empirical analysis.

First, the instantaneous nominal rate is defined as follows:

$$r_t^N = L_t^N + S_t^N, \tag{11}$$

that is, we set $\rho_t^N = 0$ and $\rho_x^N = [1, 1, 0, 0]^\top$ in equation (1). Next, the instantaneous real

[11] The log approximation error of equation (10), $\log(1 + IS_t(n)) \fallingdotseq IS_t(n)$, is negligible with a typical value of inflation swap rates. Therefore, the subsequent analysis disregards the approximation error.

[12] Our choice of the fundamental factor structure is the same as CS, whose choice is motivated by the result of Kim and Singleton (2012) in which two-factor models capture well the term structure of nominal interest rates in Japan.

rate for the i-th inflation-linked bond is defined as follows:

$$r_t^{R,i} = L_t^R + S_t^R + h^i(t)X_t^{liq}, \tag{12}$$

that is, we set $\rho_t^R = 0$ and $\rho_x^R = [0, 0, 1, 1]^\top$ in equation (2). The formulation of the liquidity factor coefficient, $h^i(t)$, is discussed in Section 3.3.

Regarding the factor dynamics under the risk-neutral probability measure (equation (3)), this paper follows ACR and CS in adopting the no-arbitrage Nelson-Siegel model (Christensen et al. 2011):

$$\mathcal{K}^{\mathbb{Q}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda^N & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda^R & 0 \\ 0 & 0 & 0 & 0 & \kappa^{liq} \end{bmatrix}, \qquad \theta^{\mathbb{Q}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \theta^{liq} \end{bmatrix}.$$

In addition, our paper follows ACR in assuming that Σ is diagonal, $\Sigma = \text{diag}(\sigma_{11}, \sigma_{22}, \ldots, \sigma_{55})$.

Under this formulation, solving equations (6), (7), and (10) based on the theoretical results of Christensen et al. (2011) and ACR yields the following expressions for the nominal zero-coupon rate, the zero-coupon inflation-linked bond rate, and the inflation swap rate:

$$R_t^N(T) = L_t^N + \frac{1 - e^{-\lambda^N(T-t)}}{\lambda^N(T-t)} S_t^N + A_t^N(T), \tag{13}$$

$$R_t^{R,i}(T) = L_t^R + \frac{1 - e^{-\lambda^R(T-t)}}{\lambda^R(T-t)} S_t^R + B_t^{R,i}(T)X_t^{liq} + A_t^{R,i}(T), \tag{14}$$

$$IS_t(T) = \left(L_t^N + \frac{1 - e^{-\lambda^N(T-t)}}{\lambda^N(T-t)} S_t^N\right) - \left(L_t^R + \frac{1 - e^{-\lambda^R(T-t)}}{\lambda^R(T-t)} S_t^R\right) + A_t^{IS}(T). \tag{15}$$

These equations reveal that the four fundamental factors indeed correspond to the level and slope factors in the Nelson-Siegel framework. Note that the last term in each equation is an adjustment term, which makes the model arbitrage-free. Due to their complex expressions, we relegate the exact form of these adjustment terms and the coefficient on the liquidity factor $B_t^{R,i}(T)$ in equation (14) to Appendix A.

Our empirical analysis uses JGBi prices instead of real zero-coupon rates. Under the convention that JGBi bonds pay coupons semi-annually, the theoretical price with the deflation protection feature is given by

$$\begin{aligned} \widehat{P_t^{R,i}}\left(T^i, \frac{\Pi_t}{\Pi_{t_0^i}}\right) &= \frac{c}{2}\frac{\tau_1}{1/2}e^{-\tau_1 R_t^{R,i}(\tau_1)} + \sum_{k=2}^{N_t^i-1} \frac{c}{2}e^{-\tau_k R_t^{R,i}(\tau_k)} \\ &\quad + \left(1+\frac{c}{2}\right)e^{-\tau_{N_t^i} R_t^{R,i}\left(\tau_{N_t^i}\right)} + POP_t\left(T^i, \frac{\Pi_t}{\Pi_{t_0^i}}\right), \end{aligned} \tag{16}$$

where c is the annualized coupon rate, t_0^i is the issuance date of the inflation-linked bond, N_t^i is the number of remaining coupon payments after time t, and τ_k is the time length from time t to the k-th coupon payment date.[13] The last term on the right-hand side, POP_t, denotes the deflation protection option premium; this term needs to be included for the new JGBi series (i.e., $i \geq 17$). Note that POP_t depends on the inflation rate between the issuance date and time t. This is because a higher realized inflation rate up to time t implies a lower probability of deflation over the entire life of the bond, and hence a smaller deflation protection option premium. Our paper estimates the deflation protection option premium on the basis of the theoretical results of Duffie et al. (2000) and Christensen et al. (2012). See Appendix B for details.

Last, the factors X_t are assumed to follow the stochastic differential equation under the physical measure $\mathbb{P}$:

$$dX_t = \mathcal{K}^{\mathbb{P}}(\theta^{\mathbb{P}} - X_t)dt + \Sigma dW_t^{\mathbb{P}}, \tag{17}$$

where $\mathcal{K}^{\mathbb{P}}$ and $\theta^{\mathbb{P}}$ are the functions of the market prices of risk factors, $\mathcal{K}^{\mathbb{Q}}$ and $\theta^{\mathbb{Q}}$. This paper follows ACR in assuming that $\mathcal{K}^{\mathbb{P}}$ is a diagonal matrix $\mathcal{K}^{\mathbb{P}} = \text{diag}(\kappa_{11}, \kappa_{22}, \ldots, \kappa_{55})$ and $\theta^{\mathbb{P}} = [\theta_1, \theta_2, \ldots, \theta_5]^\top$ is an unrestricted vector.

A remark is in order regarding the lower bound on nominal interest rates. As with CS, our Gaussian affine term structure model takes no account of an effective lower bound on the nominal interest rates, because our estimation strategy for the deflation protection option premium requires that nominal interest rates follow a Gaussian process.[14]

[13] The buyer of a bond needs to pay accrued coupon value to sellers. The convention for bond price quotations is the so-called clean price, for which the accrued coupon value is adjusted. The first term in the right-hand side of equation (16) reflects the clean price adjustment, $\tau_1/(1/2)$.

[14] We leave issues on the effective lower bound for future research to develop a computationally feasible

3.3 Identification of liquidity premium

As shown in Sections 3.1 and 3.2, this paper includes both JGBi prices and inflation swap rates as observable variables, and builds a model that assumes that the liquidity factor affects JGBi prices but not inflation swap rates. With this assumption, this paper identifies the liquidity premium of JGBi through IS-BEI spreads, that is, spreads between inflation swap rates and BEI.

Previous studies, both theoretical and empirical, showed that IS-BEI spreads contain useful information to identify the liquidity premium of inflation-linked bonds. Christensen and Gillan (2012) formally show that, when the liquidity premium of nominal bonds is negligible and inflation-linked bonds have no deflation protection feature, the IS-BEI spread coincides with the sum of the liquidity premium of the inflation-linked bond and that of the inflation swap.[15] Under this condition, the intermediary asset pricing model,[16] one of the recent strands of literature on asset pricing, stressed that physical assets (e.g., inflation-linked bonds), which carry large inventory costs from a balance-sheet management and collateral financing point of view, tend to carry a large liquidity premium, whereas derivatives such as inflation swap tend to carry a relatively small liquidity premium mainly due to low margin requirements.[17] Indeed, a number of previous studies on the US documented that the liquidity premium of the inflation swap is much smaller than that of inflation-linked bonds (see, e.g., Campbell et al. 2009, Haubrich et al. 2012, and Fleckenstein et al. 2014). These results suggest that a large

calculation methodology for the deflation protection option premium under non-linear nominal interest rate dynamics.

[15] To see this, let $\widehat{y}^N$, $\widehat{y}^R$, and $\widehat{IS}$ be the observed nominal bond rate, the observed real bond rate, and the observed inflation swap rate, respectively. Moreover, let y^N, y^R, IS be the respective rates in the frictionless market and let δ^R and δ^{IS} be the liquidity premium of the inflation-linked bond and the inflation swap, respectively. Then, the following equation holds.

$$\widehat{IS} - BEI = \widehat{IS} - \left(\widehat{y}^N - \widehat{y}^R\right) = \left(IS + \delta^{IS}\right) - \left(y^N - y^R - \delta^R\right) = \delta^{IS} + \delta^R.$$

[16] In contrast to representative agent models that assume homogeneous market participants, intermediary asset pricing models consider heterogeneous market participants and emphasize the role of financial intermediaries (e.g., broker-dealers and market-makers) in the determination of asset prices. See He and Krishnamurthy (2013), Adrian et al. (2014), He et al. (2017), Brunnermeier and Pedersen (2009), Gârleanu and Pedersen (2011), and Nagel (2012) among others.

[17] For example, Gârleanu and Pedersen (2011) attribute the bond-CDS basis (higher corporate bond yields compared to their corresponding CDS premia) observed during the GFC to the fact that holding corporate bonds typically incurs larger margin costs.

portion of the IS-BEI spread is attributable to the liquidity premium of inflation-linked bonds.

In Japan also, Imakubo and Nakajima (2015) and Suimon and Uchiyama (2019) indicated that JGBi bonds, which are physical assets, are susceptible to the amount of tradable securities available, and the supply-demand conditions. This suggests that the liquidity premium of JGBi can be larger than that of Japanese inflation swaps, as market participants are cautious about the supply-demand conditions in the JGBi market. However, this does not mean that the liquidity premium of inflation swap rates is zero. Low trading activity in the inflation swap market in Japan indicates that inflation swap rates may carry non-negligible liquidity premium under some market conditions. In this regard, our model is estimated assuming that the liquidity premium of the inflation swap rates is contained in the measurement errors of the observation equation. This approach is based on the implications of the intermediary asset pricing model, in which the liquidity premium of inflation swap rates can take both positive and negative values because financial intermediaries occasionally switch from net buyers to net sellers of the derivatives (e.g., Christoffersen et al. 2018).

Next, we discuss the specification of the liquidity factor coefficient $h^i(t)$, (equation (2)). Since the liquidity factor may have heterogeneous impacts on different inflation-linked bond series depending on the elapsed time since the issuance (i.e., their age) and other idiosyncratic reasons, we follow ACR in introducing the liquidity factor coefficient to account for this possible heterogeneity in the determination of the liquidity premium. To fix the specification of the coefficient function, ACR conduct a preliminary panel analysis and find that the bid-ask spreads of TIPS tend to be wider for older series. Based on this finding, ACR adopt the following specification for the liquidity factor coefficient,

$$h^i_{ACR}(t) = \beta^i \left(1 - e^{-\lambda^{liq,i}(t-t_0^i)}\right),$$

where β^i and $\lambda^{liq,i}$ are bond specific parameters. In line with their preliminary analysis, this function is increasing in the elapsed time since issuance.

Because of the vast differences between the market environments of JGBi and TIPS, it may not be appropriate to use exactly the same specification as ACR to analyze the JGBi market. Thus, this paper conducts a preliminary panel analysis, where series-by-series IS-BEI spreads are regressed on the age of JGBi bonds.

Table 1 reports the estimation result. This table shows that the coefficients on the bond age are all statistically significantly negative, regardless of the data frequency and the specification of fixed effects. This result suggests that, in contrast to ACR's result on the TIPS market, older JGBi series tend to have lower IS-BEI spread values, that is a lower liquidity premium. This might reflect a particular feature of the JGBi market, namely, the persistent supply-dominant market condition (see Section 2.2). In such an environment, financial intermediaries' inventory costs and hence the liquidity premium may gradually decrease as supply-demand imbalances improve due to gradual purchases by hold-to-maturity investors or buybacks by the Ministry of Finance.

In light of this preliminary panel result, we adopt the following functional form for the liquidity factor coefficient, which is a decreasing function of the bond age,

$$h^i(t) = \left(1 - e^{-\lambda^{liq,i}(T^i - t)}\right), \tag{18}$$

where T^i is the maturity of i-th bond and $\lambda^{liq,i}$ is a bond-specific parameter.

4 Data and estimation strategy

4.1 Data

Our estimation period starts in March 2007, because of the availability of inflation swap rates, and ends in May 2020. Our dataset consists of nominal zero-coupon rates for seven maturities (6 months, 1 year, 2 years, 3 years, 5 years, 7 years, and 10 years), inflation swap rates for ten maturities (1 year to 10 years), and the yields and prices of the first to 24th JGBi series all obtained from Bloomberg.[18] The reference values calculated by the Ministry of Finance are used for the Indexation Coefficients of the JGBi bonds.[19]

Our estimation is based on the weekly dataset, where the close prices on Fridays are

[18]Our dataset does not include the 25th JGBi bond series, which was issued on May 11, 2020, because of limited observation numbers in our estimation period.

[19]https://www.mof.go.jp/english/jgbs/topics/bond/10year_inflation/coefficient.htm

used.[20] Moreover, as explained in Section 2.3, this paper does not use the data on the old JGBi series after the introduction of the new series, that is October 10, 2013. Figure 4 displays the data used in estimation.

4.2 Estimation strategy

Our paper estimates the five-factor state-space model presented in Section 3.2 with the Kalman filter. Our state-space model includes nominal zero-coupon rates, JGBi prices, and the inflation swap rates as observable variables. For the state transition equation, we use the following equation, which is a discretized version of the factor dynamics under $\mathbb{P}$, equation (17):

$$X_{t+\Delta t} = \left(I - e^{-\mathcal{K}^{\mathbb{P}}\Delta t}\right)\theta^{\mathbb{P}} + e^{-\mathcal{K}^{\mathbb{P}}\Delta t}X_t + \xi_t, \tag{19}$$

where I is an identity matrix of order five, and the innovation term of the state variables, ξ_t, follows a normal distribution $\xi_t \sim N(0, Q(t, t + \Delta t))$ with the conditional covariance matrix of the state variables

$$Q(t,s) = Var^{\mathbb{P}}(X_s|X_t) = \int_t^s e^{-\mathcal{K}^{\mathbb{P}}(s-u)}\Sigma\Sigma^\top e^{-\left(\mathcal{K}^{\mathbb{P}}\right)^\top(s-u)}du.$$

Since our paper uses a weekly dataset, Δt is set equal to 7/365. This paper also imposes a parameter restriction so that the smallest eigenvalue of $\mathcal{K}^{\mathbb{P}}$ is positive. This restriction ensures the stationarity of the model factors under the physical probability measure. The initial value and the initial error covariance matrix of the factors are set equal to their unconditional mean $\theta^{\mathbb{P}}$ and the covariance $Q(t, \infty)$, respectively. For the calculation of the covariance matrix $Q(t, s)$, this paper relies on the analytical formula in Fisher and Gilles (1996).

Our model has the following observation equations. First, the observed nominal zero-coupon rates (inflation swap rates) are equal to the sum of the theoretical value given by equation (13) (resp., equation (15)) and the measurement error. The standard deviation of the measurement errors is assumed to be constant across different maturities: σ^N

[20] If data are missing on a particular Friday, the data on the last trading day before this Friday are used. This paper treats the data for the week from April 29, 2019 to May 3, 2019 as missing because the Japanese markets were closed throughout this week.

and σ^{IS} denote the constant standard deviations of the nominal zero-coupon rates and inflation swap rates, respectively. As for JGBi, the observation equation is given by

$$P_t^{R,i}\left(T^i, \frac{\Pi_t}{\Pi_{t_0^i}}\right) = \widehat{P_t^{R,i}}\left(T^i, \frac{\Pi_t}{\Pi_{t_0^i}}\right) + \varepsilon_t^{R,i}, \tag{20}$$

where $\widehat{P_t^{R,i}}(\Pi_t/\Pi_{t_0^i})$ is the theoretical price given by equation (16), and $\varepsilon_t^{R,i}$ is the measurement error term. This paper follows ACR in assuming that the standard deviation of the JGBi measurement error is proportional to the bond duration: $\varepsilon_t^{R,i} \sim N(0, dur_t^i \cdot \sigma^R)$,[21] where dur_t^i is the duration of i-th JGBi series at time t, and σ^R is a constant across different JGBi series. We estimate our state-space model based on the extended Kalman filter, because the observation equation for the JGBi, equation (20), is a non-linear function of the state variables (see Appendix C for details).

The model parameters are estimated by maximizing the log-likelihood about the measurement errors. To this end, this paper relies on the EM algorithm proposed by ACR to reduce computational burden (see Appendix D). This is because directly maximizing the log-likelihood of our model is not practically realistic because of the complexity of our model (our model has 43 parameters including 24 JGBi bond-specific parameters) as well as the computational burden of calculating the deflation protection option premium.

Once the estimates of model parameters and state variables (i.e., model factors) are obtained, the estimates of market-based inflation expectations and the other three driving factors of BEI can be obtained as follows. First, we compute the (annualized) inflation expectations from time t to T based on the following equation,

$$\pi_t^*(T) = \frac{1}{T-t}\log \mathbb{E}_t^{\mathbb{P}}\left[\frac{\Pi_T}{\Pi_t}\right] = \frac{1}{T-t}\log \mathbb{E}_t^{\mathbb{P}}\left[e^{\int_t^T \left(r_s^N - r_s^{R,f}\right)ds}\right]. \tag{21}$$

The expectation term on the right-hand side of equation (21) is calculated based on Christensen et al.'s (2012) result (see Appendix E for details). Note that it is possible to compute the inflation expectations for arbitrary durations from equation (21). The subsequent analysis calculates both the constant maturity inflation expectations (e.g., 10-year inflation expectations by setting $T = t + 10$) and those correspond to specific

[21] The durations of JGBi are calculated on the basis of JGBi prices and yields obtained from Bloomberg.

JGBi bond maturities T^i, with the latter enabling us to decompose the bond-specific BEI into the market-based inflation expectation component and the other three driving factors of BEI.

Next, each of the three driving factors of BEI is computed as follows. To this end, this paper defines the correspondence between the price of the inflation-linked bond P_t^i and the yield-to-maturity y_t^i on the basis of the following equation:

$$P_t^i = \frac{c}{2}\frac{\tau_1}{1/2}e^{-\tau_1 y_t^i} + \sum_{k=2}^{N_t^i-1}\frac{c}{2}e^{-\tau_k y_t^i} + \left(1+\frac{c}{2}\right)e^{-\tau_{N_t^i} y_t^i}.$$

In our EM algorithm, the yen value of the deflation protection option premium, POP_t^i, is obtained together with the estimated model parameters (see Appendix D). We transform this yen-valued deflation protection option premium into the contribution of the deflation protection option premium to BEI, $POP_t^{i,y}$, as follows. First, the observed JGBi price $P_t^{i,obs}$ is transformed into the yield-to-maturity $y_t^{i,obs}$. Second, the ex-POP JGBi price, $P_t^{i,obs} - POP_t^i$, is transformed into the yield-to-maturity $y_t^{i,exPOP}$. Last, the contribution of POP to BEI, $POP_t^{i,y}$, is calculated as $POP_t^{i,y} = y_t^{i,exPOP} - y_t^{i,obs}$. Moreover, this paper follows Christensen et al. (2012) in estimating the deflation probability, $\mathbb{P}_t\left(\Pi_T/\Pi_t < 1\right)$ (see Appendix E for our estimation methodology of the deflation probability).

The liquidity premium is calculated as follows. First, the "cum-liquidity factor" bond price, $\widehat{P}_t^i$, is calculated on the basis of equation (16) with the estimated model parameters, state variables, and the yen-valued POP estimate. This theoretical price is then converted into the yield-to-maturity, $\widehat{y}_t^i$. Second, the "ex-liquidity factor" bond price is calculated on the basis of equation (16) again, but with the parameters and factors related to the liquidity premium (i.e., κ^{liq}, θ^{liq}, and $X_{t|t}^{liq}$) replaced with zero. Then, this bond price is converted into the yield-to-maturity, $\widehat{y}_t^{i,exLP}$. Last, our estimates of the liquidity premium are defined as $LP_t^i = \widehat{y}_t^i - \widehat{y}_t^{i,exLP}$.

Once the estimation results above are obtained, the term premium spread, TPD_t^i, can be computed from the decomposition formula of BEI

$$TPD_t^i = BEI_t^i - POP_t^{i,y} + LP_t^i - \pi_t^*(T^i).$$

5 Empirical results

This section reports the estimation result of our affine term structure model as well as the estimates of the three driving factors of BEI and the market-based long-term inflation expectations.

5.1 Model estimation results

Table 2 reports the fitting errors between the observed values and the estimated values of the nominal zero-coupon rates and the inflation swap rates. The second column of the table presents the mean error and the third column presents the root mean squared errors (RMSE). The average RMSE across the seven nominal zero-coupon yield series is about 7 bps, suggesting that our parsimonious two-factor setting for the term structure of nominal yields performs well in reproducing the observed nominal yield curves. On the other hand, the average RMSE of the ten inflation swap rate series is around 18 bps, larger than that of the nominal zero-coupon rate series. A low trading volume in inflation swaps in Japan may have contributed to the large measurement errors, because incorporating information on inflation expectations into inflation swap rates may take time in such a market environment.[22]

Table 3 shows the fitting results for JGBi prices. For ease of comparison with the nominal rates and the inflation swap rates, the fitting error is converted in terms of errors in yield-to-maturity. The table shows that the average RMSE across all JGBi series is about 5 bps, which is about a half of CS's result (about 10 bps). Since CS estimate a model with no liquidity factor, our result implies that adding a liquidity factor improves the fit of the JGBi yield curve. The estimated bond-specific parameters $\lambda^{liq,i}$ are larger for the old series. This may be because there were a number of market events that had a large impact on the liquidity premium, such as the GFC and the announcement of the special buyback program of the old series.

Table 4 reports the estimation result of the remaining model parameters and the

[22]Looking closely at the measurement errors by maturity, the measurement error for one-year tenor is larger than those for other tenors. We conjecture that this is because a shorter tenor is more susceptible to short-term noises that affect CPI, including hikes of the consumption tax rate.

maximum likelihood value.[23] The standard deviations of the measurement error of the nominal zero-coupon rates, inflation swap rates, and JGBi yields, are $\sigma^N = 0.08\%$, $\sigma^{IS} = 0.19\%$, and $\sigma^R = 0.06\%$, respectively. This result is in line with Tables 2 and 3, which show that the observation error of the inflation swap rates is the largest among the three asset classes.

5.2 Estimates of the three driving factors of BEI

This subsection reports the estimates of the three driving factors of Japan's BEI. Our estimates are then compared with those of previous studies.

5.2.1 Estimates of deflation protection option premium

Figure 5-A shows the estimated one-year-ahead deflation probability, which is closely related to the deflation protection option premium. The estimated deflation probability increased sharply in September 2008 amid the GFC. Then, it remained elevated above 80% until 2010. The one-year-ahead deflation probability started to decline rapidly at the end of 2012. Since then, it has maintained a relatively low level. The introduction of the 2% price stability target followed by the implementation of QQE by the Bank of Japan may have contributed to this diminished deflation probability. Yet, the deflation probability has elevated somewhat since the outbreak of the coronavirus pandemic.

Under the above circumstances, the deflation protection option premium of the longest-to-maturity new JGBi series (whose time-to-maturity is about nine to ten years) has been fluctuating around 0.2–0.4%, as shown in Figure 5-B. Since the coronavirus pandemic, it has risen to 0.6%, consistent with the development of the deflation probability.

Our estimates of the deflation protection option premium lie between CS's estimates (on average, 0.5–1.0%) and Yuyama's (2016) indication (less than 0.1%). We conjecture that the differences between our estimates and those in the existing studies arise from the treatment of the other two driving factors of BEI—the liquidity premium and the term premium spread. First, CS take no account of the liquidity premium. This model feature,

[23] Note that the standard errors of parameters regarding the physical measure tend to be larger than those for the risk-neutral measure. ACR report a similar tendency, with the magnitude of the standard errors of the parameters regarding the physical measure in line with those in our estimation.

as ACR pointed out, may result in an overestimation of the deflation protection option premium; disregarding the positive liquidity premium may cause an overestimation of the expected real interest rate, leading to an underestimation of the inflation expectations and hence an overestimation of the deflation protection option premium. Next, in his simulation analysis, Yuyama (2016) sets the expected inflation rate equal to 1.0%, based on actual observations around the introduction of QQE. This setting may have underestimated the deflation protection option premium because the mean parameter of the inflation expectations may be overvalued; according to the option price evaluation theory, the parameter regarding the mean of the expected inflation rate should be the one under the risk-neutral probability measure, not the one under the physical probability measure. Theoretically, the expected inflation rate under the risk-neutral probability measure coincides with the sum of the expected inflation rate under the physical probability measure and the term premium spread. As shown later, the estimated term premium spread in Japan is negative, indicating that the mean inflation rate under the risk-neutral probability measure is lower than the one under the physical probability measure. In contrast to these studies, our paper estimates all three deriving factors of BEI under a unified framework, leading to a different level of estimates in the premium.

5.2.2 Estimates of Liquidity premium

Figure 6-A displays the estimates of the liquidity premium of the old JGBi series from March 2007 to October 2013. This figure shows that the liquidity premium spiked to about 1.5% at the height of the GFC (around October 2008). The liquidity premium decreased steadily thereafter, and stayed below zero from 2012. The estimated negative liquidity premium from 2012 may have been caused by the announcement of the special buyback program at auctions of the new JGBi series: since the special buyback program entitled holders of the old JGBi series the opportunity to sell them (in exchange for successful bidding of the new JGBi series), the old JGBi prices may have been overpriced during this period, relative to inflation swap rates.

Figure 6-B shows the estimates of the liquidity premium of the new JGBi series since October 2013. Apart from movements just after the introduction of the series, the liquidity premium of the new JGBi series has followed a gradually increasing trend; the

estimated value at the end of our estimation period is slightly below 0.5%.[24] As such, our estimated liquidity premium takes positive values except during the switch from the old to new series, contributing to the pulling down of Japan's BEI.

Our estimates of the liquidity premium are in line with Yuyama and Moridaira (2017), who reported that the liquidity premium of Japan's BEI was on average 0.3–0.4%, although it spiked above 1.0% during the GFC. Note that the estimated liquidity premium in the US TIPS market reported in the previous studies is on average 0.1–0.3% under calm market conditions, and 1.0–1.5% during the GFC. This implies that our estimated liquidity premium in the JGBi market is somewhat larger than that in the US TIPS market. This result is consistent with the market condition that the JGBi market faces less liquidity and investors are more cautious about supply-demand conditions compared with the TIPS market.

Note that the correlation between the estimated liquidity premium (the average over all series) and the volatility measure of the stock price index in Japan (Nikkei Stock Average Volatility Index) is reasonably high, amounting to 0.4. This indicates that the estimated liquidity premium for JGBi tends to rise when uncertainty about the market emerges, and the liquidity of financial markets as a whole deteriorates.

A remark is in order regarding the negative BEI observed in Japan since the coronavirus pandemic. In the absence of the liquidity premium, the BEI based on an inflation-linked bond with the deflation protection feature should not become negative.[25] On the contrary, when the liquidity premium pushes up the inflation-linked bond yield, BEI may take a negative value whose absolute value can be as large as the liquidity premium. The lowest BEI observed so far since the coronavirus pandemic, about -0.3%, is smaller in ab-

[24]During the period after the introduction of the new JGBi series, various policy reactions by the Ministry of Finance (e.g., restarting buyback auctions in April 2016 and a special buyback auction in March 2020 in response to the market downturn caused by the coronavirus pandemic) contributed to mitigating supply-demand imbalances in the JGBi market, thereby reducing the liquidity premium. We leave further analysis on the relation between supply-demand conditions in the JGBi market and the liquidity premium for future research.

[25]This can be confirmed by comparing a nominal zero-coupon bond and a zero-coupon inflation-linked bond with the deflation protection feature. First, since the protected inflation-linked bond reimburses the initial principal amount in the case of deflation, its economic value is equivalent to the nominal bond. On the other hand, the inflation-linked bond reimburses more (nominal) amount than the nominal bond in the case of inflation. Therefore, the price of the protected zero-coupon inflation-linked bond never becomes lower than that of the nominal zero-coupon bond, meaning that BEI never takes a negative value. For coupon-bearing bonds, differences in coupon rates may cause departure from the above reasoning, but this effect is negligible in the current low interest rate environment.

solute term than the estimated value of the liquidity premium in the same period, about 0.5%. This suggests that the negative BEI observed during the coronavirus pandemic is still within the range admissible in the theoretical framework with the liquidity factor.

5.2.3 Estimates of term premium spread

Figure 7 displays the estimates of the term premium spread. This figure illustrates that the term premium spread takes a negative value over almost the entire estimation period, pulling down Japan's BEI. Moreover, the absolute size of the negative values has been increasing in recent years, bringing the estimated value at the end of our estimation period down to around -0.9%.

The negative term premium spread is in line with Yuyama and Moridaira (2017) and CS, who reported negative inflation risk premia (i.e., term premium spread). In addition, its secular decline in recent years may reflect the declining trend in the estimated nominal term premium.[26] This is consistent with Ueno (2017) and Sudo and Tanaka (2020), who pointed to the possibility that the large-scale purchase of nominal JGB by the Bank of Japan has exerted downward pressure on the nominal term premium. It is also consistent with the view that secular increases in demand for safe assets may have contributed to lowering the nominal term premium (see e.g., Ichiue et al. 2012).[27] While our estimation result does not identify the cause of the recent decline in the nominal term premium, our findings suggest that it is important to consider the various factors that affect the nominal term premium when evaluating recent developments in the BEI.[28]

[26] According to the consumption CAPM theory, term premium spread, which is also interpreted as inflation risk premium, can take both positive and negative values because it is the negative of the covariance between the real stochastic discount factor and the reciprocal of inflation expectations (see CS and Imakubo and Nakajima 2015).

[27] Our result indicates that the decline in the nominal term premium was larger than that in the real term premium, and that the associated flattening of the nominal yield curve outweighed that of the real yield curve. Still, we should not jump to conclusions on the effect on the real term premium of large-scale asset purchases and increased demand for safe assets without considering other factors. We leave this to a future research agenda.

[28] Our model labels the portion of the nominal interest rates unexplained by the expectation hypothesis as movements in the nominal term premium, because the model assumes that the fundamental factors alone explain the nominal yields. If, however, the decline of the nominal yields associated with safe asset demand should be interpreted as the "negative liquidity premium" of nominal bonds, as indicated by Fleckenstein et al. (2014), then our estimation result would be prone to overestimate the liquidity premium of inflation-linked bonds, and it would also be prone to underestimate the term premium spread, though all in all, the effect on the estimates of the market-based inflation expectations would be minor. Incorporating the liquidity premium of nominal bonds and the effect of safe asset demand in

5.3 Estimates of market-based long-term inflation expectations

The empirical results shown so far confirm that each of the three driving factors has made a quantitatively non-negligible contribution to developments in Japan's BEI. Figure 8-A displays the estimates of ten-year market-based inflation expectations, which are obtained by removing the other three driving factors from the BEI. In addition, Figure 8-B compares our estimates of market-based inflation expectations with several measures of survey-based inflation expectations, specifically the *Consensus Forecast* of the Consensus Economics, the *Inflation Outlook of Enterprises* of the Bank of Japan TANKAN survey, and the *ESP forecast* of the Japan Center for Economic Research.

The estimates of market-based long-term inflation expectations have evolved higher than Japan's BEI throughout almost the entire estimation period. The inflation expectations recorded a large negative value during the GFC, when crude oil prices plunged, but continued to rise thereafter, reaching around 2% in mid 2014, after the Bank of Japan introduced QQE.[29] Since then, as with survey-based expectations, market-based long-term inflation expectations have declined, marking around 1% in the first half of 2016, and remaining around 1% until the beginning of 2020. During the coronavirus pandemic, market-based inflation expectations weakened somewhat amid a rapid downturn in the global economy and a large fall in crude oil prices.

It is worth noting that recent studies on advanced economies have reported that market-based long-term inflation expectations are susceptible to movements in crude oil prices (see, e.g., Owyang and Shell 2019). Our estimates of inflation expectations in Japan share the same feature. It is particularly noticeable during the periods when crude oil prices have been especially volatile: during the GFC period, between mid-2014 and 2016, and during the coronavirus pandemic, as illustrated in Figures 8-A and 9-A.

To confirm this observation quantitatively, we conduct a parsimonious three-variable vector autoregression (VAR) model analysis with the Dubai crude oil price, our estimates

term structure models is an important agenda for future research.

[29]The consumption tax hikes likely affected the estimated inflation expectations. In particular, the steady increase in inflation expectations, which started in early 2012 when the legislation related to the tax hikes was approved by the Noda Cabinet, may reflect market participants' views on the government policy regarding the consumption tax hikes. Note that the effect of the consumption tax hikes on the estimated ten-year inflation expectations is at most 0.5% (i.e., the scheduled consumption tax hikes from 5% to 10% divided by ten years). Therefore, the increase in the estimated inflation expectations between 2012 and 2014 incorporates various other factors including the introduction of QQE.

of market-based long-term inflation expectations, and the CPI (all items less fresh food, year-on-year change). Identification of structural shocks is based on the Cholesky decomposition of the aforementioned order (exogenous to endogenous).[30] The impulse response of market-based long-term inflation expectations to a 10% positive shock to crude oil prices shows that the shock significantly increases the long-term inflation expectations for about one year after the occurrence of the shock (Figure 9-B). Moreover, this impulse response is also economically significant; the peak of the impulse response reaches about 0.2 percentage points six months after the shock. While movements in crude oil prices pass through to the CPI via energy-related items (e.g., gasoline) and via intermediary input costs, previous studies documented that the comovements between crude oil prices and market-based long-term inflation expectations in advanced economies were puzzlingly large (e.g., Lumsdaine, 2009, Cette and de Jong, 2013, and Owyang and Shell, 2019). Our VAR result suggests that, in line with other advanced economies, the market-based inflation expectations extracted from JGBi also exhibit strong comovement with crude oil prices. Although it is beyond the scope of our paper to shed light on the reasons behind these comovements, several conjectures have been advanced. For example, economists argue the possibility that crude oil price movements are regarded as a proxy of global demand shocks, while market participants point to the effect of short-term trading activity by international investors in inflation-linked bond markets.

5.4 Decomposition of the longest-to-maturity BEI

Figure 10 decomposes the BEI computed from the longest-to-maturity JGBi into the market-based inflation expectations and the three driving factors of BEI. First, this figure shows that the inflation expectation component has been above the BEI through most of the estimation period, even though the deflation protection option premium of the new JGBi series has made a positive contribution to the BEI. Second, the relatively low level of the BEI in recent periods is attributed to the liquidity premium (the negative

[30] Despite its parsimonious identification scheme, our VAR model captures well the propagation of crude oil price shocks to the CPI. That is, the impulse response of the CPI to a 10% positive shock to crude oil prices reaches its peak of 0.2 percentage points in 10–12 months after the shock occurred. This quantitative result is in line with the simulation result obtained from the Japanese macroeconomic model of Hirakata et al. (2019).

sign of the estimated value shown in Section 5.2.2) and (the negative values of) the term premium spread. Specifically, recent declines in the term premium spread have been bringing the BEI even lower when compared with the inflation expectations.

Before proceeding to the concluding section, we add some observations on the impact of the coronavirus pandemic on market-based inflation expectations in advanced economies. First, in Japan, the longest-to-maturity BEI at the end of March 2020 decreased by 0.2 percentage points compared with the beginning of January 2020. This change is attributable to a 0.4% point decrease in our estimates of market-based inflation expectations and a 0.2% point increase in those of the deflation protection option premium, while the liquidity premium and the term premium spread remained largely unchanged.

Compared with Japan, market-based inflation expectations in the US and the euro area experienced even larger declines in the same period, as shown in Figure 11. The US ten-year current BEI dropped from 1.8% at the beginning of January 2020 to 0.6% in mid-March. It rebounded to 0.9% at the end of March, yet this means that the US ten-year BEI decreased by almost 1 percentage point from the beginning of 2020. Regarding this sharp movement in the US BEI, there are a few preliminary results which indicate that most part of this movement was caused by a decrease in inflation expectations, while the liquidity premium was largely unchanged.[31] Similarly, the five-year forward five-year inflation swap rate in the euro area decreased by 0.4 percentage points between the beginning of January 2020 (1.3%) and the end of March (0.9%). Broeders et al. (2020), economists at De Nederlandsche Bank (the central bank of the Netherlands), analyzed recent developments in the euro area inflation swap market and reported that estimated inflation expectations declined at least 0.3 percentage points (and possibly 1 percentage point, depending on the estimation approach).

Even though the results for the US and the euro area are preliminary and hence need to be viewed with reservations, it is highly likely that the coronavirus pandemic has exerted downward pressure on market-based inflation expectations in advanced economies,

[31] On March 19, 2020, Professor Stefan Nagel of the University of Chicago posted a Twitter thread, asking whether the decline in the US BEI was caused by the elevated liquidity premium as in the GFC. Several researchers replied that there was no visible increase in the liquidity premium based on, for example, Fleckenstein et al.'s (2014) approach extended to the recent period. See the Twitter thread (https://twitter.com/ProfStefanNagel/status/1240637482355621888) and replies.

through common shocks, including sharp movements in crude oil prices.

6 Conclusions

This paper decomposed Japan's BEI into long-term inflation expectations and the three other driving factors: (i) the deflation protection option premium of JGBi, (ii) the liquidity premium of JGBi, and (iii) the spread between the nominal and real term premia (the term premium spread). This paper has constructed an affine term structure model that includes inflation swap rates in addition to nominal government bond yields and JGBi prices as observable variables, taking into consideration the constraints on data of JGBi and the unique environment of the JGBi market compared with the US TIPS market.

Our empirical result has shown that the deflation protection option premium for the new JGBi series has pushed up Japan's BEI, while the liquidity premium of JGBi and the term premium spread have pulled it down, all having non-negligible contributions to developments in the BEI. This indicates that the evolution of Japan's BEI has been driven by the three factors as well as by the long-term inflation expectations of market participants. Consequently, the estimated long-term inflation expectations have evolved higher than the BEI throughout almost the entire estimation period, and they have traced a similar trajectory to the survey-based measures of long-term inflation expectations, for the last five years at least. Our analysis has also pointed out that market-based long-term inflation expectations are susceptible to movements in crude oil prices. This was particularly apparent during the GFC, from the latter half of 2014 to 2016, and during the coronavirus pandemic.

Our analysis has indicated that market-based inflation expectations can be successfully extracted from BEI through the appropriate use of affine term structure models. However, there are questions still to be addressed. First, there is room to examine further the mechanism behind developments in the estimated inflation expectations and the other three driving factors. For example, although beyond the scope of this paper, a thorough examination remains to be made of the mechanism behind the high degree of comovement between market-based long-term inflation expectations and crude oil prices. Second, because of technical constraints in the estimation of the deflation protection option premium, this paper does not take into account an effective lower bound on nominal interest rates. These issues are left for future research.

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A Observation equations in our model

We provide the concrete expressions of the observation equations for the nominal zero-coupon rate, the real zero-coupon rate, and the inflation swap rate (equations (6), (7), (10), respectively). First, the observation equation for the nominal zero-coupon rate is as follows.

$$R_t^N(T) = L_t^N + \frac{1-e^{-\lambda^N(T-t)}}{\lambda^N(T-t)}S_t^N + A_t^N(T),$$

where $A_t^N(T)$ is given by

$$A_t^N(T) = -\frac{1}{2}\left(\sigma_{11}^2\frac{(T-t)^2}{3} + \frac{\sigma_{22}^2}{(\lambda^N)^2}\left[1-2\frac{1-e^{-\lambda^N(T-t)}}{\lambda^N(T-t)} + \frac{1-e^{-2\lambda^N(T-t)}}{2\lambda^N(T-t)}\right]\right).$$

Next, the observation equation for the real zero-coupon rate is as follows.

$$\begin{aligned}
R_t^{R,i}(T) &= L_t^R + \frac{1-e^{-\lambda^R(T-t)}}{\lambda^R(T-t)}S_t^R + B_t^{R,i}(T)X_t^{liq} + A_t^{R,i}(T),\\
B_t^{R,i}(T) &= -\left(\frac{1-e^{-\kappa_{liq}^{\mathbb{Q}}(T-t)}}{\kappa_{liq}^{\mathbb{Q}}(T-t)} + \frac{e^{-\lambda^{liq,i}(T-t)}-e^{-\kappa^{\mathbb{Q}}(T-t)}}{(\lambda^{liq,i}-\kappa_{liq}^{\mathbb{Q}})(T-t)}\right),\\
A_t^{R,i}(T) &= \theta_{liq}^{\mathbb{Q}}\left(1-\frac{1-e^{-\kappa^{\mathbb{Q}}(T-t)}}{\kappa_{liq}^{\mathbb{Q}}(T-t)}\right) - \frac{\kappa_{liq}^{\mathbb{Q}}\theta_{liq}^{\mathbb{Q}}}{\kappa^{\mathbb{Q}}-\lambda^{liq,i}}\left(\frac{1-e^{-\lambda^{liq,i}(T-t)}}{\lambda^{liq,i}(T-t)} - \frac{1-e^{-\kappa_{liq}^{\mathbb{Q}}(T-t)}}{\kappa_{liq}^{\mathbb{Q}}(T-t)}\right)\\
&\quad -\frac{1}{2}\left(\sigma_{33}^2\frac{(T-t)^2}{3} + \frac{\sigma_{44}^2}{(\lambda^R)^2}\left[1-2\frac{1-e^{-\lambda^R(T-t)}}{\lambda^R(T-t)} + \frac{1-e^{-2\lambda^R(T-t)}}{2\lambda^R(T-t)}\right] + \sigma_{55}^2 C\right),
\end{aligned}$$

where C in $A_t^{R,i}(T)$ is e given by

$$\begin{aligned}
C &= \frac{1}{(\kappa_{liq}^{\mathbb{Q}})^2}\left(1-2\frac{1-e^{-\kappa_{liq}^{\mathbb{Q}}(T-t)}}{\kappa_{liq}^{\mathbb{Q}}(T-t)} + \frac{1-e^{-2\kappa_{liq}^{\mathbb{Q}}(T-t)}}{2\kappa_{liq}^{\mathbb{Q}}(T-t)}\right)\\
&\quad - \frac{2}{\kappa_{liq}^{\mathbb{Q}}(\kappa_{liq}^{\mathbb{Q}}-\lambda^{liq,i})}\left(\frac{1-e^{-\lambda^{liq,i}(T-t)}}{\lambda^{liq,i}(T-t)} - \frac{1-e^{-\kappa_{liq}^{\mathbb{Q}}(T-t)}}{\kappa_{liq}^{\mathbb{Q}}(T-t)}\right.\\
&\qquad\qquad \left. + \frac{1-e^{-2\kappa_{liq}^{\mathbb{Q}}(T-t)}}{2\kappa_{liq}^{\mathbb{Q}}(T-t)} - \frac{1-e^{-(\kappa_{liq}^{\mathbb{Q}}+\lambda^{liq,i})(T-t)}}{(\kappa_{liq}^{\mathbb{Q}}+\lambda^{liq,i})(T-t)}\right)\\
&\quad + \frac{1}{(\kappa_{liq}^{\mathbb{Q}}-\lambda^{liq,i})^2}\left(\frac{1-e^{-2\lambda^{liq,i}(T-t)}}{2\lambda^{liq,i}(T-t)} - 2\frac{1-e^{-(\kappa_{liq}^{\mathbb{Q}}+\lambda^{liq,i})(T-t)}}{(\kappa_{liq}^{\mathbb{Q}}+\lambda^{liq,i})(T-t)} + \frac{1-e^{-2\kappa_{liq}^{\mathbb{Q}}(T-t)}}{2\kappa_{liq}^{\mathbb{Q}}(T-t)}\right).
\end{aligned}$$

Finally, the observation equation for the inflation swap rate is as follows.

$$IS_t(T) = L_t^N + \frac{1-e^{-\lambda^N(T-t)}}{\lambda^N(T-t)}S_t^N - L_t^R - \frac{1-e^{-\lambda^R(T-t)}}{\lambda^R(T-t)}S_t^R + A_t^{IS}(T),$$

where $A_t^{IS}(T)$ is given by

$$\begin{aligned} A_t^{IS}(T) = \frac{1}{2}\Bigg(&(\sigma_{11}^2+\sigma_{33}^2)\frac{(T-t)^2}{3} + \frac{\sigma_{22}^2}{(\lambda^N)^2}\left[1-2\frac{1-e^{-\lambda^N(T-t)}}{\lambda^N(T-t)} + \frac{1-e^{-2\lambda^N(T-t)}}{2\lambda^N(T-t)}\right] \\ &+ \frac{\sigma_{44}^2}{(\lambda^R)^2}\left[1-2\frac{1-e^{-\lambda^R(T-t)}}{\lambda^R(T-t)} + \frac{1-e^{-2\lambda^R(T-t)}}{2\lambda^R(T-t)}\right]\Bigg). \end{aligned}$$

B Estimation of the deflation protection option premium

For simplicity, our explanation below is for a real zero-coupon bond. This simplification does not affect our estimation, because the deflation protection feature of the JGBi does not apply to coupon payments.

Let $P_t^p(T, \Pi_t/\Pi_{t_0})$ and $P_t^u(T, \Pi_t/\Pi_{t_0})$ be the price of a *protected* and an *unprotected* real zero-coupon bond, respectively, where T is the maturity of the real bonds, t_0 is the issuance date of the real bonds, and Π_t/Π_{t_0} is the *Indexation Coefficient*, that is, the ratio of the price levels today to the price level on the issuance date.[32]

Since the redemption value of an unprotected bond equals Π_T/Π_{t_0}, the risk-neutral evaluation formula yields the following equation,

$$P_t^u\left(T, \frac{\Pi_t}{\Pi_{t_0}}\right) = \mathbb{E}_t^{\mathbb{Q}}\left[\frac{\Pi_T}{\Pi_{t_0}}e^{-\int_t^T r_s^N ds}\right] = \frac{\Pi_t}{\Pi_{t_0}}\mathbb{E}_t^{\mathbb{Q}}\left[e^{\int_t^T (r_s^N - r_s^R)ds}e^{-\int_t^T r_s^N ds}\right] = \frac{\Pi_t}{\Pi_{t_0}}\mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^T r_s^R ds}\right]. \tag{B.1}$$

On the other hand, a protected bond redeems Π_T/Π_{t_0} if the link coefficient is greater than one, and redeems one if the link coefficient is below or equal to one. Therefore, we

[32] Strictly speaking, the Indexation Coefficient is the ratio of the *Ref Index* at time t to the Ref Index at the issuance date. Since there is approximately three months' lag between the Ref Index and the CPI, this indexation lag may result in a bias in estimation results. However, we do not consider this issue in our estimation because existing studies show that the indexation lag bias is typically negligible.

obtain the following equation.

$$
\begin{aligned}
P_t^P\left(T, \frac{\Pi_t}{\Pi_{t_0}}\right) &= \mathbb{E}_t^{\mathbb{Q}}\left[\left(\frac{\Pi_T}{\Pi_{t_0}} \cdot \mathbb{1}_{\left\{\frac{\Pi_T}{\Pi_{t_0}}>1\right\}} + 1 \cdot \mathbb{1}_{\left\{\frac{\Pi_T}{\Pi_{t_0}}\leq 1\right\}}\right) e^{-\int_t^T r_s^N ds}\right] \\
&= \mathbb{E}_t^{\mathbb{Q}}\left[\frac{\Pi_T}{\Pi_{t_0}} e^{-\int_t^T r_s^N ds}\right] + \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^T r_s^N ds} \mathbb{1}_{\left\{\frac{\Pi_T}{\Pi_{t_0}}\leq 1\right\}}\right] - \mathbb{E}_t^{\mathbb{Q}}\left[\frac{\Pi_T}{\Pi_{t_0}} e^{-\int_t^T r_s^N ds} \mathbb{1}_{\left\{\frac{\Pi_T}{\Pi_{t_0}}\leq 1\right\}}\right] \\
&= P_t^u\left(T, \frac{\Pi_t}{\Pi_{t_0}}\right) + \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^T r_s^N ds} \mathbb{1}_{\left\{\frac{\Pi_T}{\Pi_{t_0}}\leq 1\right\}}\right] - \frac{\Pi_t}{\Pi_{t_0}} \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^T r_s^R ds} \mathbb{1}_{\left\{\frac{\Pi_T}{\Pi_{t_0}}\leq 1\right\}}\right],
\end{aligned}
\tag{B.2}
$$

where $\mathbb{1}_{\{\cdot\}}$ is the indicator function, which equals to one when the condition inside the parenthesis holds, and equals to zero otherwise.

Comparing equations (B.1) and (B.2) shows that the deflation protection option premium is given by the following equation,[33]

$$
POP_t\left(T, \frac{\Pi_t}{\Pi_{t_0}}\right) = \underbrace{\mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^T r_s^N ds} \mathbb{1}_{\left\{\frac{\Pi_T}{\Pi_t}\leq \frac{\Pi_{t_0}}{\Pi_t}\right\}}\right]}_{F_t^N\left(T, \frac{\Pi_t}{\Pi_{t_0}}\right)} - \frac{\Pi_t}{\Pi_{t_0}} \underbrace{\mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^T r_s^R ds} \mathbb{1}_{\left\{\frac{\Pi_T}{\Pi_t}\leq \frac{\Pi_{t_0}}{\Pi_t}\right\}}\right]}_{F_t^R\left(T, \frac{\Pi_t}{\Pi_{t_0}}\right)}. \tag{B.3}
$$

Equation (B.3) shows that the estimation of the deflation protection option premium is attained by calculating the two expectation terms, F_t^N and F_t^R. Although we omit the detailed derivation process of these two terms, one can prove the following results using theoretical results in Duffie et al. (2000) and Christensen et al. (2012).

$$
F_t^X\left(T, \frac{\Pi_t}{\Pi_{t_0}}\right) = \frac{\varphi^X(\mathbf{0}, t, T)}{2} - \frac{1}{\pi}\int_0^\infty \frac{Im\{e^{-ivz}\varphi^X(\mathbf{0} + iv\mathbf{b}, t, T)\}}{v} dv, \qquad X \in \{N, R\}, \tag{B.4}
$$

where $\mathbf{0}$ is a 5×1 vector of zeros, i is the imaginary unit, $\mathbf{b} = [0, 0, 0, 0, 1]^\top$, $z = \log(\Pi_t/\Pi_{t_0})$, and $Im\{\cdot\}$ denotes the imaginary part of the argument. The auxiliary

[33] Note that the corresponding equations in Christensen et al. (2012) and CS miss the coefficient Π_t/Π_{t_0} on the second expectation term on the right-hand side of equation (B.3).

function, φ^X ($X \in \{N, R\}$), in equation (B.4) is given as follows,

$$\begin{aligned}
\varphi^N(\mathbf{a}, t, T) = \exp\Bigg(& (a-1)(T-t)L_t^N + (a-1)\frac{1-e^{-\lambda^N(T-t)}}{\lambda^N} S_t^N \\
& - a(T-t)L_t^R - a\frac{1-e^{-\lambda^R(T-t)}}{\lambda^R} S_t^R + \widehat{A}_t^N(T) \Bigg), \\
\widehat{A}_t^N(T) = & \frac{\sigma_{11}^2}{2}(a-1)^2\frac{(T-t)^3}{3} + \frac{\sigma_{22}^2}{2}\frac{(a-1)^2}{(\lambda^N)^2}\left((T-t) - 2\frac{1-e^{-\lambda^N(T-t)}}{\lambda^N} + \frac{1-e^{-2\lambda^N(T-t)}}{2\lambda^N}\right) \\
& + \frac{\sigma_{33}^2}{2}a^2\frac{(T-t)^3}{3} + \frac{\sigma_{44}^2}{2}\frac{a^2}{(\lambda^R)^2}\left((T-t) - 2\frac{1-e^{-\lambda^R(T-t)}}{\lambda^R} + \frac{1-e^{-2\lambda^R(T-t)}}{2\lambda^R}\right),
\end{aligned}$$

$$\begin{aligned}
\varphi^R(\mathbf{a}, t, T) = \exp\Bigg(& a(T-t)L_t^N + a\frac{1-e^{-\lambda^N(T-t)}}{\lambda^N} S_t^N \\
& - (1+a)(T-t)L_t^R - (1+a)\frac{1-e^{-\lambda^R(T-t)}}{\lambda^R} S_t^R + \widehat{A}_t^R(T) \Bigg), \\
\widehat{A}_t^R(T) = & \frac{\sigma_{11}^2}{2}a^2\frac{(T-t)^3}{3} + \frac{\sigma_{22}^2}{2}\frac{a^2}{(\lambda^N)^2}\left((T-t) - 2\frac{1-e^{-\lambda^N(T-t)}}{\lambda^N} + \frac{1-e^{-2\lambda^N(T-t)}}{2\lambda^N}\right) \\
& + \frac{\sigma_{33}^2}{2}(1+a)^2\frac{(T-t)^3}{3} + \frac{\sigma_{44}^2}{2}\frac{(1+a)^2}{(\lambda^R)^2}\left((T-t) - 2\frac{1-e^{-\lambda^R(T-t)}}{\lambda^R} + \frac{1-e^{-2\lambda^R(T-t)}}{2\lambda^R}\right),
\end{aligned}$$

where $\mathbf{a} = [0, 0, 0, 0, a]^\top$.

C Application of the extended Kalman filter for inflation-linked bond prices

For simplicity, our discussion below assumes an inflation-linked bond without the deflation protection feature. This does not affect our actual estimation, because our EM algorithm (see Appendix D) estimates the state-space model given the estimate of the deflation protection option premium.

In our state-space model, we use coupon-bearing inflation-indexed bonds as one of the observed variables. Since the coupon-bearing inflation-indexed bond price in equation (20) is a non-linear function of the model factors, we estimate our state-space model using the extended Kalman filter. The extended Kalman filter uses the first-order approximation of a non-linear function of the factors, $f_t^i(X_t)$, around the projected state, $X_{t|t-1}$,

$$P_t^{R,i} \approx f_t^i(X_{t|t-1}) + \left(\frac{\partial f_t^i}{\partial X}_{|X=X_{t|t-1}}\right)^\top (X_t - X_{t|t-1}) + \varepsilon_t.$$

Since this equation is approximately equivalent to the following linear observation equation, one can apply the standard Kalman filter to it.

$$P_t^{R,i} = \underbrace{f_t^i(X_{t|t-1}) - \left(\frac{\partial f_t^i}{\partial X}_{|X=X_{t|t-1}}\right)^\top X_{t|t-1}}_{A_t^{EKF,i}} + \underbrace{\left(\frac{\partial f_t^i}{\partial X}_{|X=X_{t|t-1}}\right)^\top}_{B_t^{EKF,i}} X_t + \varepsilon_t.$$

By explicitly calculating the derivatives in this equation from the formula of the theoretical price of the inflation-linked bond, equation (16), we can see that each element of $B_t^{EKF,i}$ is given by the following equation,

$$\begin{aligned}
[B_t^{EKF,i}]_j = &-\frac{c}{2}\frac{\tau_1}{1/2}e^{-\tau_1 R_t^{R,i}(\tau_1)}\tau_1[B_t(t+\tau_1)]_j - \sum_{k=2}^{N_t^i-1}\frac{c}{2}e^{-\tau_k R_t^{R,i}(\tau_k)}\tau_k[B_t(t+\tau_k)]_j \\
&- \left(1+\frac{c}{2}\right)e^{-\tau_{N_t^i} R_t^{R,i}\left(\tau_{N_t^i}\right)}\tau_{N_t^i}[B_t(t+\tau_{N_t^i})]_j,
\end{aligned}$$

where $[B_t^{EKF,i}]_j$ $(j = 1, 2, \ldots, 5)$ denotes the j-th element of $B_t^{EKF,i}$ and $B_t(T)$ is the vector of the factor coefficients, which appears in the theoretical price of the zero-coupon inflation-linked bond,

$$B_t(T) = \begin{bmatrix} B_t^{R,f}(T) \\ B_t^{R,i}(T) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -(T-t) \\ -\frac{1-e^{-\lambda^R(T-t)}}{\lambda^R} \\ \left(\frac{1-e^{-\kappa_{liq}^{\mathbb{Q}}(T-t)}}{\kappa_{liq}^{\mathbb{Q}}} + \frac{e^{-\lambda^{liq,i}(T-t)} - e^{-\kappa_{liq}^{\mathbb{Q}}(T-t)}}{\lambda^{liq,i} - \kappa_{liq}^{\mathbb{Q}}}\right) \end{bmatrix}.$$

D Details of the EM algorithm

Due to the heavy computational burden, it is not practical to calculate the deflation protection option premium at each step of the maximum likelihood estimation. To this end, we follow ACR in employing an EM algorithm for the estimation of the model parameters and the deflation protection option premium. Let $\boldsymbol{\theta}^{(i)}$ be the estimated value of the model parameters at the end of i-th step, and let $\boldsymbol{POP}^{(i)}$ be the matrix of the estimated deflation protection option premium (across different series and time periods). Let $\mathcal{L}(\boldsymbol{\theta}, \boldsymbol{POP})$ be the log-likelihood of the state-space model, given the model parameters $\boldsymbol{\theta}$ and the deflation protection option premium matrix $\boldsymbol{POP}$. Finally, let ε be a small number of the stopping criterion (we set it to 0.1). Then, our EM algorithm is as follows.

Initialization: Set the initial value of the model parameters, $\boldsymbol{\theta}^{(0)}$. Set the estimated value of the deflation protection option premium to zero (i.e., $\boldsymbol{POP}^{(0)} \equiv 0$).

Step 1: Given the estimate of the deflation protection option premium, $\boldsymbol{POP}^{(i-1)}$, estimate the model parameters, $\boldsymbol{\theta}^{(i)}$, starting from the initial value, $\boldsymbol{\theta}^{(i-1)}$.

Step 2: Update the estimate of the deflation protection option premium based on the following procedures, where $\boldsymbol{POP}^{(i-1,0)} = \boldsymbol{POP}^{(i-1)}$.

Step 2–1: Given $\boldsymbol{\theta}^{(i)}$ and $\boldsymbol{POP}^{(i-1,j-1)}$, run the Kalman filter estimation to get the estimated state variables, $\{X_t\}^{(j)}$.

Step 2–2: Given $\boldsymbol{\theta}^{(i)}\{X_t\}^{(j)}$, estimate the deflation protection option premium $\boldsymbol{POP}^{(i-1,j)}$.

Step 2–3: If $\left|\mathcal{L}\left(\boldsymbol{\theta}^{(i)}, \boldsymbol{POP}^{(i-1,j-1)}\right) - \mathcal{L}\left(\boldsymbol{\theta}^{(i)}, \boldsymbol{POP}^{(i-1,j)}\right)\right| < \varepsilon$, set $\boldsymbol{POP}^{(i)} = \boldsymbol{POP}^{(i-1,j)}$ and proceed to Step 3. Otherwise, set $j = j + 1$ and return to Step 2–1.

Step 3: If $\left|\mathcal{L}\left(\boldsymbol{\theta}^{(i-1)}, \boldsymbol{POP}^{(i-1)}\right) - \mathcal{L}\left(\boldsymbol{\theta}^{(i)}, \boldsymbol{POP}^{(i)}\right)\right| < \varepsilon$, stop with the final estimation result $\boldsymbol{\theta}^{(i)}$ and $\boldsymbol{POP}^{(i)}$. Otherwise, set $i = i + 1$ and return to Step 1.

In addition, we follow ACR in employing a two-step estimation approach for the parameter estimation in Step 1. Namely, we divide the parameters into two groups, the inflation-linked bond-specific parameters, $\boldsymbol{\theta}_{BS}$ (i.e., $\{\lambda^{liq,i}\}$), and the common parameters $\boldsymbol{\theta}_{CP}$. Then, we conduct the following procedure.

Initialization: Initialize the parameters as $\boldsymbol{\theta}^{(i-1)} = \left[\boldsymbol{\theta}_{CP}^{(i-1,0)}, \boldsymbol{\theta}_{BS}^{(i-1,0)}\right]$.

Step A: Given $\boldsymbol{\theta}_{BS}^{(i-1,j-1)}$, estimate $\boldsymbol{\theta}_{CP}^{(i-1,j)}$.

Step B: Given $\boldsymbol{\theta}_{CP}^{(i-1,j)}$, estimate $\boldsymbol{\theta}_{BS}^{(i-1,j)}$.

Step C: If $\left|\mathcal{L}\left(\boldsymbol{\theta}_{CP}^{(i-1,j-1)}, \boldsymbol{\theta}_{BS}^{(i-1,j-1)}\right) - \mathcal{L}\left(\boldsymbol{\theta}_{CP}^{(i-1,j)}, \boldsymbol{\theta}_{BS}^{(i-1,j)}\right)\right| < \varepsilon$, set $\boldsymbol{\theta}^{(i)} = \left[\boldsymbol{\theta}_{CP}^{(i-1,j)}, \boldsymbol{\theta}_{BS}^{(i-1,j)}\right]$ and finish this two-step estimation procedure (i.e., return to Step 2 above). Otherwise, set $j = j + 1$ and return to Step A.

Note that this type of two-step estimations is commonly used in maximum likelihood estimations with many parameters (see e.g., DeJong and Dave (2007)).

E Estimation of the market-based inflation expectations and the deflation probability

We explain how we estimate the market-based inflation expectations, equation (21), and the deflation probability. To this end, we follow Christensen et al. (2012) and introduce an auxiliary variable $Y_{t,T}$ as follows,

$$Y_{t,T} = \int_t^T (r_s^N - r_s^{R,f})ds.$$

Because our term structure model is a Gaussian affine term structure model, the auxiliary variable $Y_{t,T}$ follows a normal distribution. Therefore, by denoting the mean and the standard deviation of $Y_{t,T}$ by $m_{t,T}^Y$ and $\sigma_{t,T}^Y$, respectively, the market-based inflation expectations equal

$$\pi_t^*(T) = \frac{1}{T-t} \log \mathbb{E}_t^{\mathbb{P}} \left[e^{Y_{t,T}}\right] = \frac{1}{T-t}\left(m_{t,T}^Y + \frac{1}{2}(\sigma_{t,T}^Y)^2\right),$$

thanks to the moment generating function formula for normal distributions. Moreover, the deflation probability satisfies the following equation,

$$\mathbb{P}\left(\frac{\Pi_T}{\Pi_t} < 1\right) = \mathbb{P}\left(Y_{t,T} < 0\right) = \Phi\left(\frac{-m_{t,T}^Y}{\sigma_{t,T}^Y}\right),$$

where $\Phi(\cdot)$ is the cumulative standard normal density function. Therefore, the calculation of the market-based inflation expectations and the deflation probability is attained by calculating $m^Y_{t,T}$ and $\sigma^Y_{t,T}$.

We numerically estimate $m^Y_{t,T}$ and $\sigma^Y_{t,T}$ based on the following procedure. First, note that $Y_{t,s}$ satisfies the following dynamics under the specification of the fundamental factors in our model.[34]

$$dY_{t,s} = \left(r^N_s - r^{R,f}_s\right) ds = \left(L^N_s + S^N_s - L^R_s - S^R_s\right) ds, \qquad Y_{t,t} = 0$$

Hence, the vector of the fundamental factors augmented by the auxiliary variable, $Z_s = \left[L^N_s, S^N_s, L^R_s, S^R_s, Y_{t,s}\right]$, follows the following stochastic differential equation,

$$dZ_s = (A + BZ_s)ds + \overline{\Sigma} dW^{\mathbb{P}}_s, \qquad Z_t = \left[L^N_t, S^N_t, L^R_t, S^R_t, 0\right],$$

where

$$A = \begin{bmatrix} \kappa^{\mathbb{P}}_{11} & 0 & 0 & 0 & 0 \\ 0 & \kappa^{\mathbb{P}}_{22} & 0 & 0 & 0 \\ 0 & 0 & \kappa^{\mathbb{P}}_{33} & 0 & 0 \\ 0 & 0 & 0 & \kappa^{\mathbb{P}}_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta^{\mathbb{P}}_1 \\ \theta^{\mathbb{P}}_2 \\ \theta^{\mathbb{P}}_3 \\ \theta^{\mathbb{P}}_4 \\ 0 \end{bmatrix}, \qquad B = -\begin{bmatrix} \kappa^{\mathbb{P}}_{11} & 0 & 0 & 0 & 0 \\ 0 & \kappa^{\mathbb{P}}_{22} & 0 & 0 & 0 \\ 0 & 0 & \kappa^{\mathbb{P}}_{33} & 0 & 0 \\ 0 & 0 & 0 & \kappa^{\mathbb{P}}_{44} & 0 \\ -1 & -1 & 1 & 1 & 0 \end{bmatrix},$$

and $\overline{\Sigma} = \mathrm{diag}(\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{44}, 0)$. Christensen et al. (2012) show that the conditional expectation of Z_s, $m^{\mathbb{P}}_t(s) = \mathbb{E}^{\mathbb{P}}_t[Z_{t,s}]$, and the conditional covariance matrix $V^{\mathbb{P}}_t(s) = Var^{\mathbb{P}}_t(Z_{t,s})$ satisfy the following ordinary differential equations, respectively,

$$\frac{dm^{\mathbb{P}}_t(s)}{ds} = A + Bm^{\mathbb{P}}_t(s), \quad m^{\mathbb{P}}_t(t) = Z_t,$$

$$\frac{dV^{\mathbb{P}}_t(s)}{ds} = BV^{\mathbb{P}}_t(s) + V^{\mathbb{P}}_t(s)B^\top + \overline{\Sigma}\overline{\Sigma}^\top, \quad V^{\mathbb{P}}_t(t) = 0.$$

We solve these ordinary differential equations by the fourth-order Runge-Kutta method. Then, we obtain $m^Y_{t,T}$ and $\sigma^Y_{t,T}$ by extracting the elements of $m^{\mathbb{P}}_t(T)$ and $V^{\mathbb{P}}_t(T)$ that

[34] We treat t as a fixed time point and view $Y_{t,s}$ as a function of s.

correspond to the mean and the variance of $Y_{t,T}$. Finally, we compute the market-based inflation expectations and the deflation probability using the aforementioned formulae.

Table 1. Panel analysis on the liquidity of JGBi and duration since the issuance

	(1)	(2)	(3)	(4)	(5)
Age	-17.34**	-19.63**	-22.27**	-22.40**	-17.20**
DP dummy		-37.32*			
R^2	0.32	0.36	0.47	0.47	0.86
Obs	39,238	39,238	39,238	8,056	8,056
Frequency	Daily	Daily	Daily	Weekly	Weekly
Series FE	No	No	Yes	Yes	Yes
Time FE	No	No	No	No	Yes

Notes: 1. Dependent variable: spread between inflation swap rate and series-by-series BEI (IS-BEI spread, bps).

2. Estimation period: March 2, 2007—January 31, 2020.

3. Data on Fridays are used for the weekly frequency estimations.

4. ** and * represents a 1% and 5% statistical significance, respectively.

5. DP dummy stands for the deflation protection dummy.

Table 2. Fit of nominal zero-coupon rates and inflation swap rates

Nominal zero-coupon rates		
Matuirty	Mean error	RMSE
6 month	5.44	8.85
1 year	3.19	5.61
2 years	-1.02	2.98
3 years	-4.08	6.00
5 years	-5.71	8.48
7 years	-5.69	8.11
10 years	6.90	9.05
Average	-0.14	7.01
Inflation swap rates		
Maturity	Mean error	RMSE
1 year	-12.63	32.34
2 years	2.23	18.19
3 years	9.63	21.20
4 years	10.74	21.54
5 years	4.59	13.99
6 years	3.79	12.67
7 years	0.25	10.47
8 years	-3.59	13.18
9 years	-5.85	17.07
10 years	-7.91	17.28
Average	0.12	17.79

Note: Unit of fitting errors is basis points.

Table 3. Fit of JGBi

JGBi specifications					Fit		Bond-specific parameter	
Series	First issuance	Maturity	Coupon rate	DP	Mean error	RMSE	$\lambda^{liq,i}$	s.e.
1	10 Mar, 2004	10 Mar, 2014	1.2%	No	-2.60	16.79	0.51	(0.05)
2	10 Jun, 2004	10 Jun, 2014	1.1%	No	2.45	10.41	0.29	(0.01)
3	10 Dec, 2004	10 Dec, 2014	0.5%	No	2.21	6.75	0.52	(0.02)
4	10 Jun, 2005	10 Jun, 2015	0.5%	No	2.40	9.38	0.29	(0.01)
5	12 Sep, 2005	10 Sep, 2015	0.8%	No	-1.22	8.79	0.32	(0.01)
6	12 Dec, 2005	10 Dec, 2015	0.8%	No	-2.79	8.19	0.48	(0.02)
7	10 Mar, 2006	10 Mar, 2016	0.8%	No	-1.89	6.93	0.50	(0.02)
8	12 Jun, 2006	10 Jun, 2016	1.0%	No	3.84	8.39	0.80	(0.02)
9	11 Oct, 2006	10 Sep, 2016	1.1%	No	-0.48	2.78	1.05	(0.04)
10	12 Dec, 2006	10 Dec, 2016	1.1%	No	-0.42	2.71	1.22	(0.05)
11	10 Apr, 2007	10 Mar, 2017	1.2%	No	-0.10	3.83	1.28	(0.05)
12	12 Jun, 2007	10 Jun, 2017	1.2%	No	0.54	3.71	1.21	(0.06)
13	10 Oct, 2007	10 Sep, 2017	1.3%	No	-0.56	3.97	1.53	(0.07)
14	11 Dec, 2007	10 Dec, 2017	1.2%	No	-0.04	4.52	1.69	(0.07)
15	10 Apr, 2008	10 Mar, 2018	1.4%	No	-0.67	5.23	1.81	(0.08)
16	10 Jun, 2008	10 Jun, 2018	1.4%	No	0.64	6.78	1.51	(0.10)
17	10 Oct, 2013	10 Sep, 2023	0.1%	Yes	-0.37	2.83	0.09	(0.02)
18	10 Apr, 2014	10 Mar, 2024	0.1%	Yes	-0.14	2.31	0.08	(0.02)
19	10 Oct, 2014	10 Sep, 2024	0.1%	Yes	0.39	3.07	0.07	(0.02)
20	12 May, 2015	10 Mar, 2025	0.1%	Yes	0.20	2.15	0.06	(0.01)
21	14 Apr, 2016	10 Mar, 2026	0.1%	Yes	-0.09	1.73	0.06	(0.02)
22	13 Apr, 2017	10 Mar, 2027	0.1%	Yes	-0.53	2.52	0.06	(0.02)
23	11 May, 2018	10 Mar, 2028	0.1%	Yes	-0.07	1.83	0.05	(0.01)
24	13 May, 2019	10 Mar, 2029	0.1%	Yes	1.06	4.01	0.02	(0.01)
Ave.					0.07	5.40		

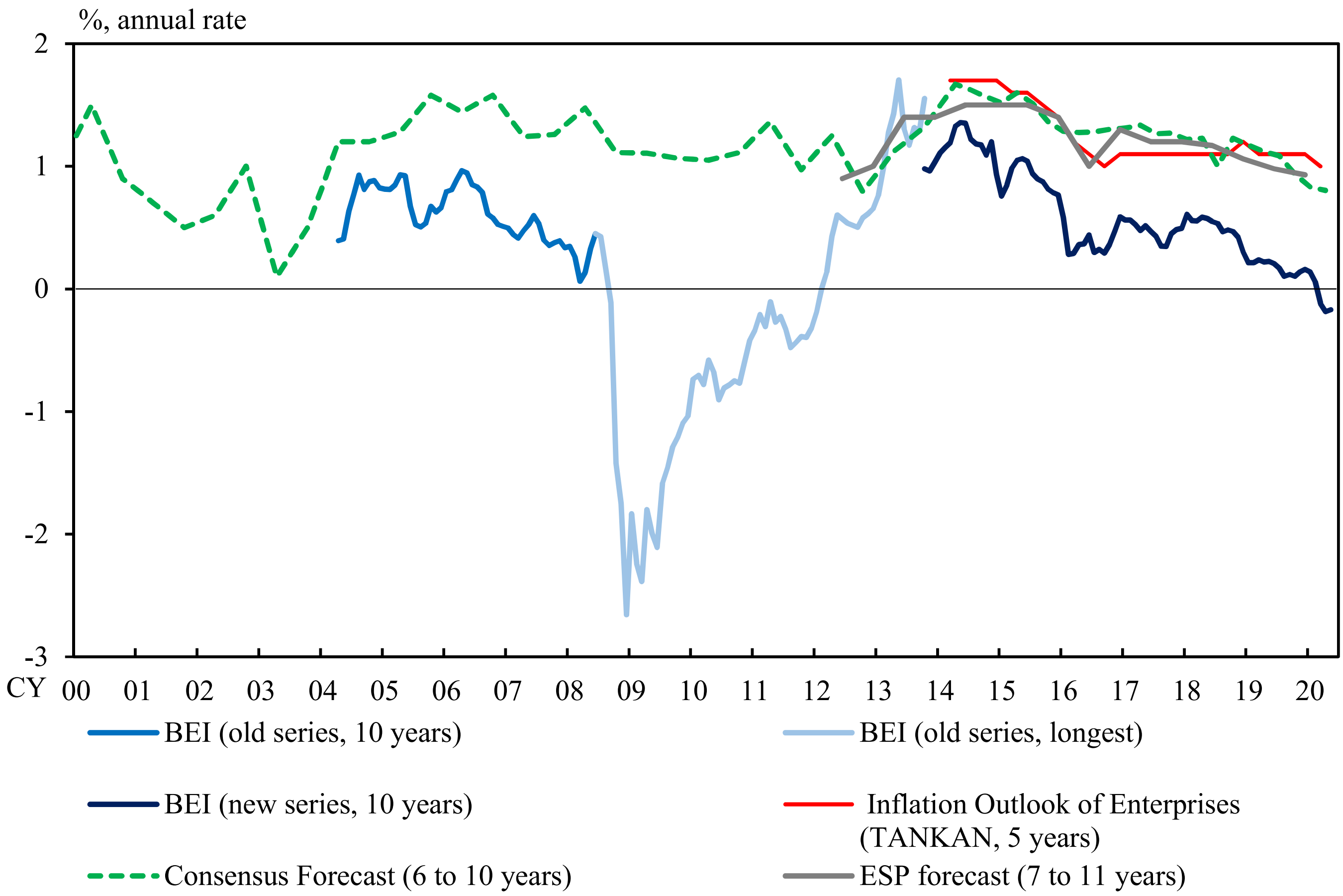
Notes: 1. Unit of fitting error is basis points.

2. DP stands for "deflation protection."

Table 4. Estimates of model parameters

	Estimated values	s.e.
λ^N	0.0641	(0.0029)
λ^R	0.2833	(0.0124)
κ^{liq}	0.6540	(0.0147)
θ^{liq}	0.0103	(0.0005)
θ_1	0.0388	(0.0649)
θ_2	0.0233	(0.0117)
θ_3	0.0133	(0.0254)
θ_4	-0.0170	(0.0196)
θ_5	-0.0060	(0.0232)
σ_{11}	0.0022	(0.0001)
σ_{22}	0.0022	(0.0001)
σ_{33}	0.0043	(0.0002)
σ_{44}	0.0071	(0.0003)
σ_{55}	0.0290	(0.0006)
κ_{11}	0.0006	(0.0011)
κ_{22}	0.0033	(0.0020)
κ_{33}	0.0113	(0.0161)
κ_{44}	0.1408	(0.2560)
κ_{55}	0.2280	(0.1048)
σ^N	0.0008	(9.32E-6)
σ^{IS}	0.0019	(1.88E-5)
σ^R	0.0006	(6.51E-6)
log LLH	102,119.3	

Figure 1. BEI and survey-based long-term inflation expectations



Notes: 1. BEI figures are the monthly average of daily data. The latest data as of May 2020.
2. JGBi issued since October 2013 are designated as "new," while the rest are designated as "old." The longest BEI refers to the 16th JGBi series, which matured in June 2018.

Sources: Bloomberg; Consensus Economics, "Consensus Forecast"; Japan Center for Economic Research; Bank of Japan.

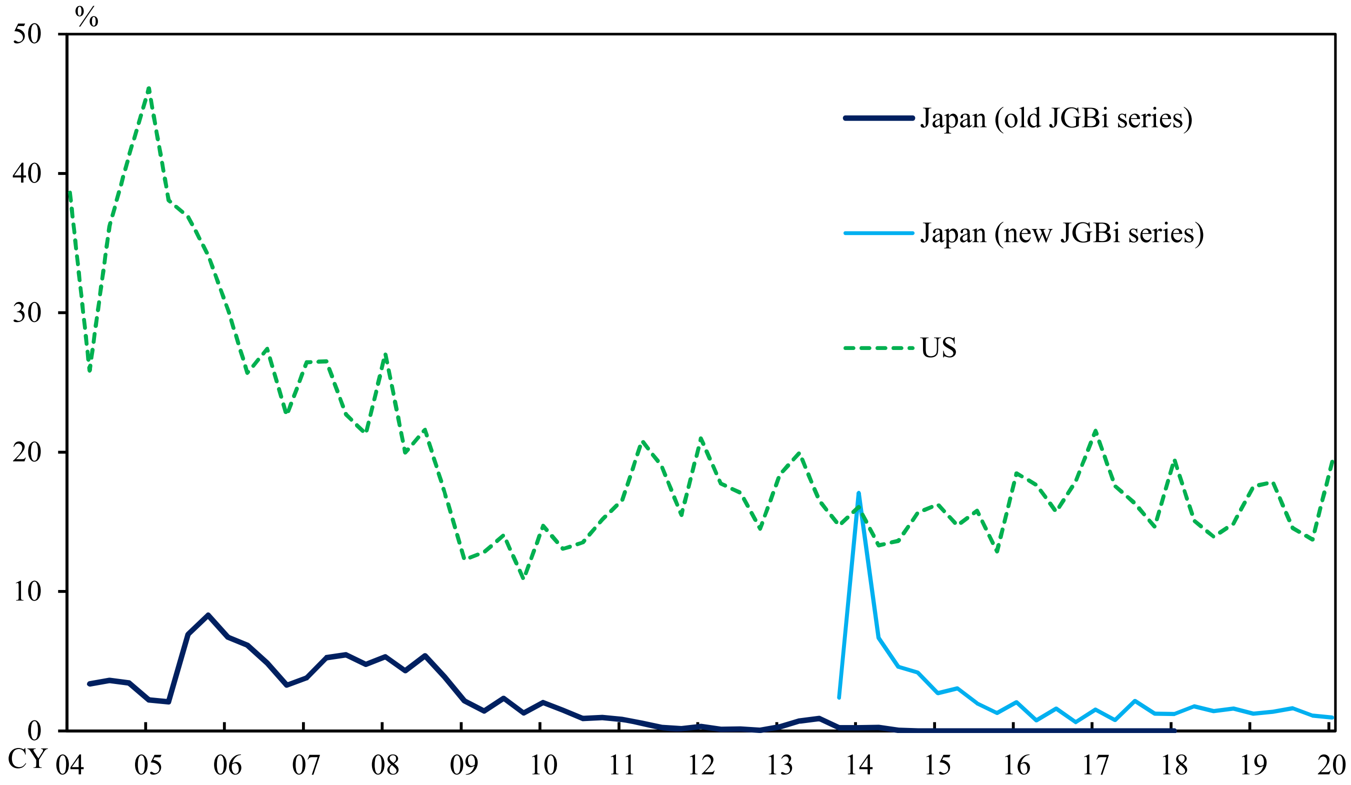
Figure 2. Product specifications and market environments of inflation-linked bonds

(A) Product specifications of inflation-linked bonds

		Period	Number of series	DP	Refernce Price Index	Term	Proportion to all outstanding government bonds
Japan	old	2004--08	16	No	CPI all items (less fresh food)	10 years only	1.2%
	new	2013--	8	Yes			
US		1997--	77	Yes	CPI-U	5, 10, 20, 30 years	8.9%

Note: "DP" in the fourth column refers to the deflation protection.

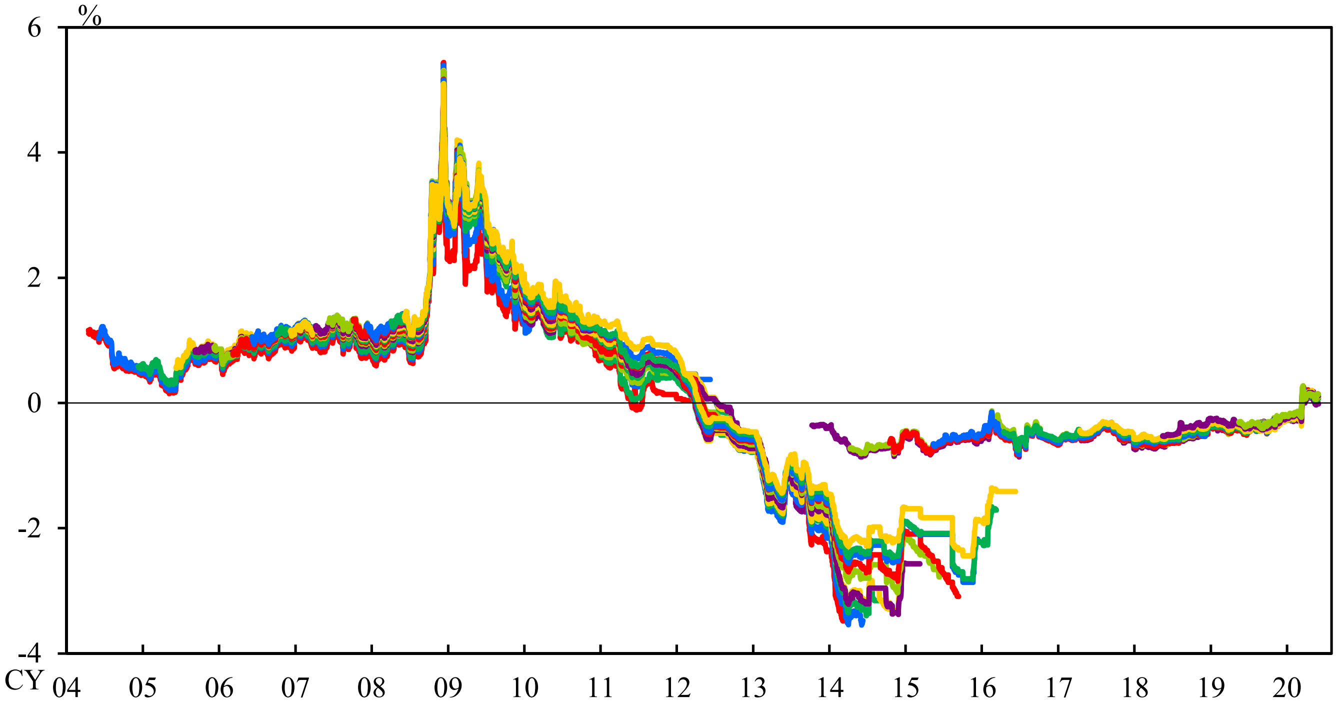
(B) Turnover ratio of inflation-linked bonds



Notes: 1. The number of issued series and the proportion to all outstanding government bonds in (A) are as of the end of March 2020.
2. The denominator of the proportion to all government bonds for Japan is the sum of general bonds + FILP bonds, and for the US is the marketable Treasury.
3. Turnover ratio in (B) is the ratio of aggregated quarterly trading volume to the end-of-quarter outstanding inflation-linked bonds. The latest data are as of the first quarter of 2020.
4. Outstanding inflation-linked bonds for Japan is that traded in the market, and for the US is total outstanding TIPS.

Sources: Treasury Direct; Federal Reserve Bank of New York; QUICK; Ministry of Finance; Bank of Japan.

Figure 3. Yields of JGBi by series

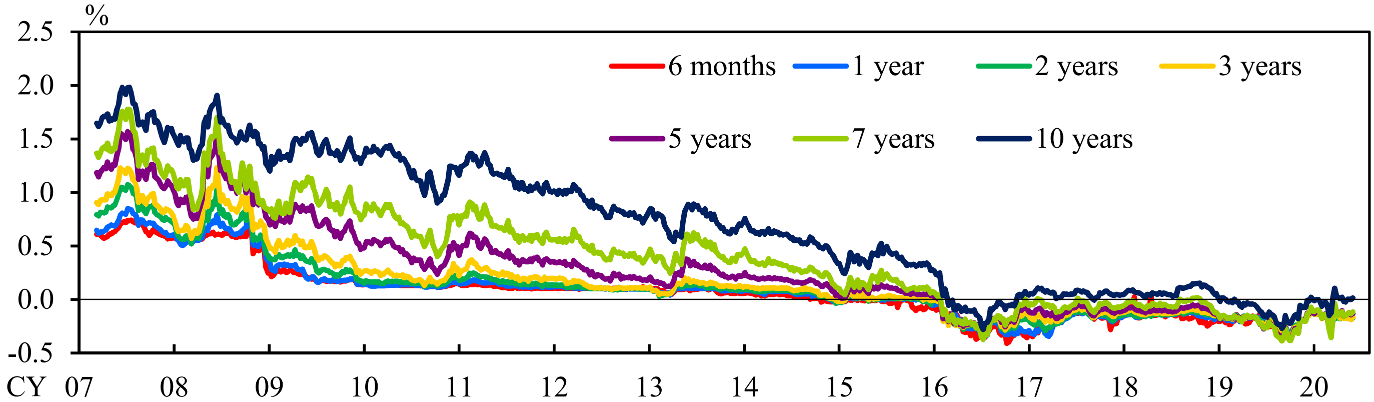


Notes: 1. Each series (each colored line) represents the yields of one of the first to 24th JGBi series.
2. The initial (left) point of each series corresponds to the issuance date of that series.
3. The end (right) point of the old JGBi series (first to 16th series) corresponds to the date on which the time-to-maturity equals two years. The end (right) point of the new JGBi series (17th to 24th series) is May 29, 2020, as the time-to-maturity of all new JGBi series is longer than two years as of this date.

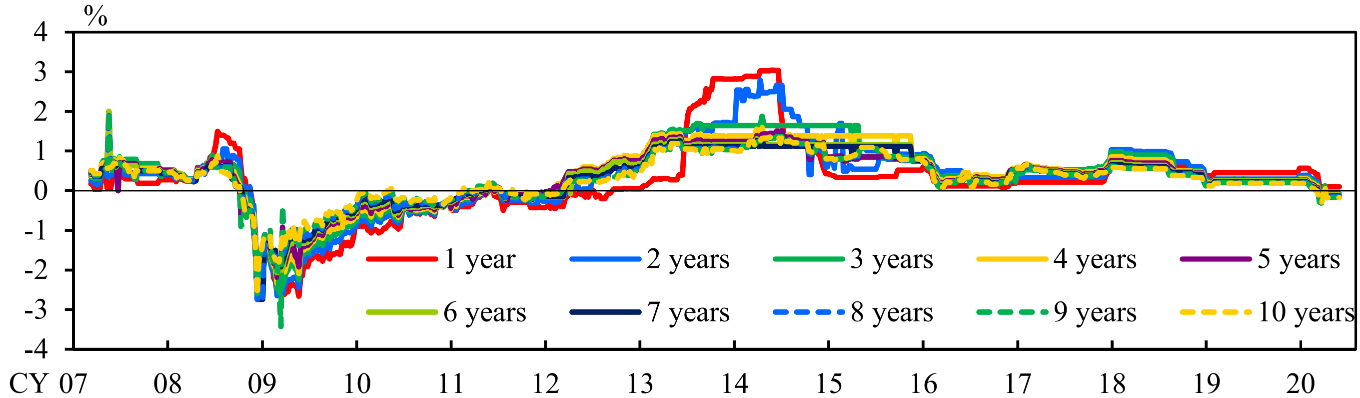
Sources: Bloomberg; Ministry of Finance.

Figure 4. Input data for the estimation

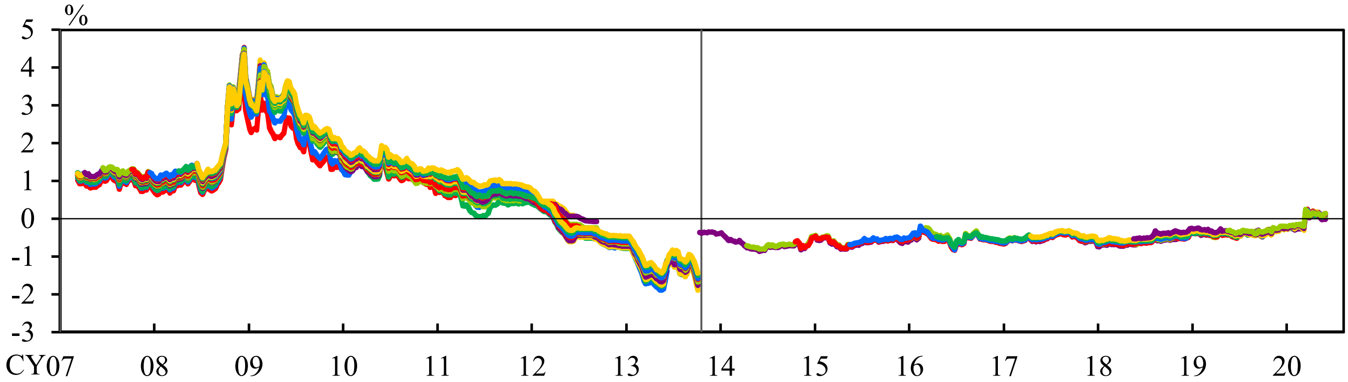
(A) Nominal zero-coupon rates



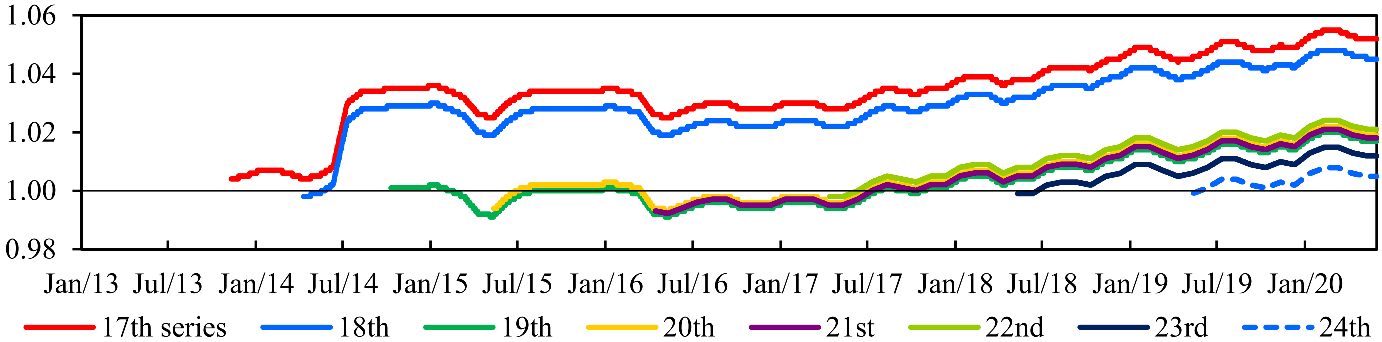
(B) Inflation swap rates



(C) JGBi yields by series



(D) Indexation coefficients

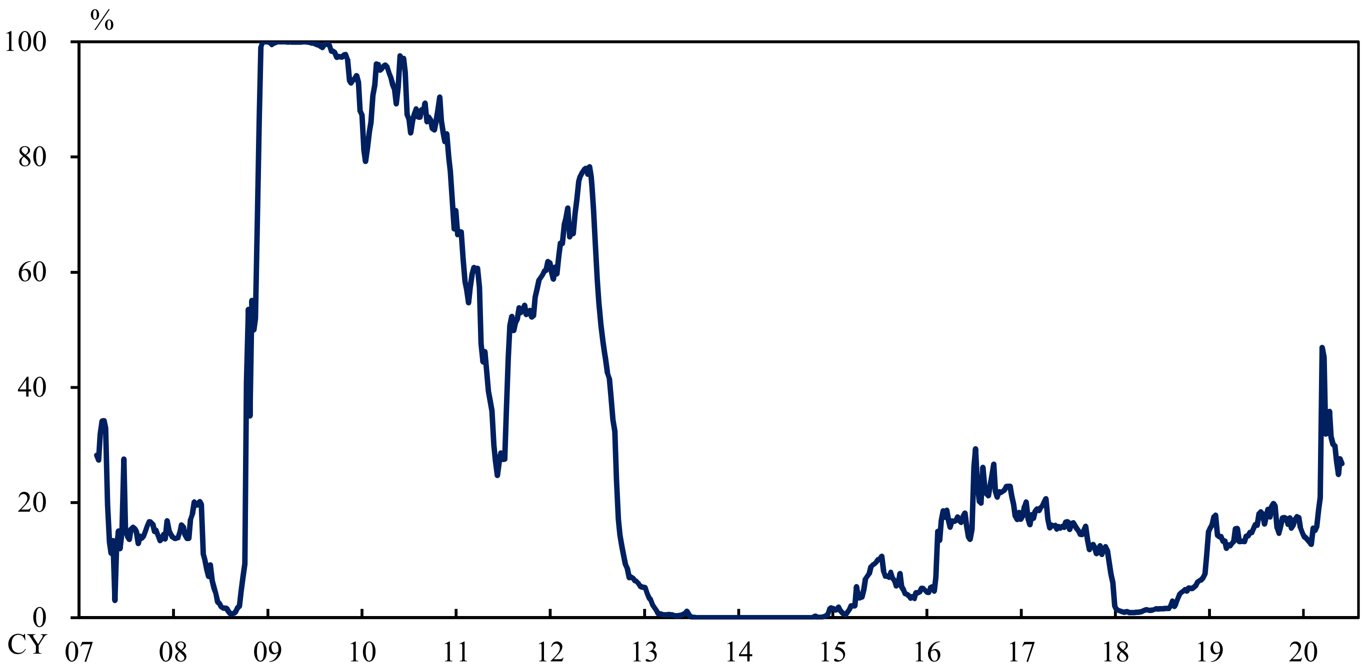


Notes: 1. Each series (each colored line) in (C) represents the yields of one of the first to 24th JGBi series.
The vertical line in (C) denotes the date of the issuance start of the new JGBi series.
2. The latest data as of the week ending May 29, 2020.

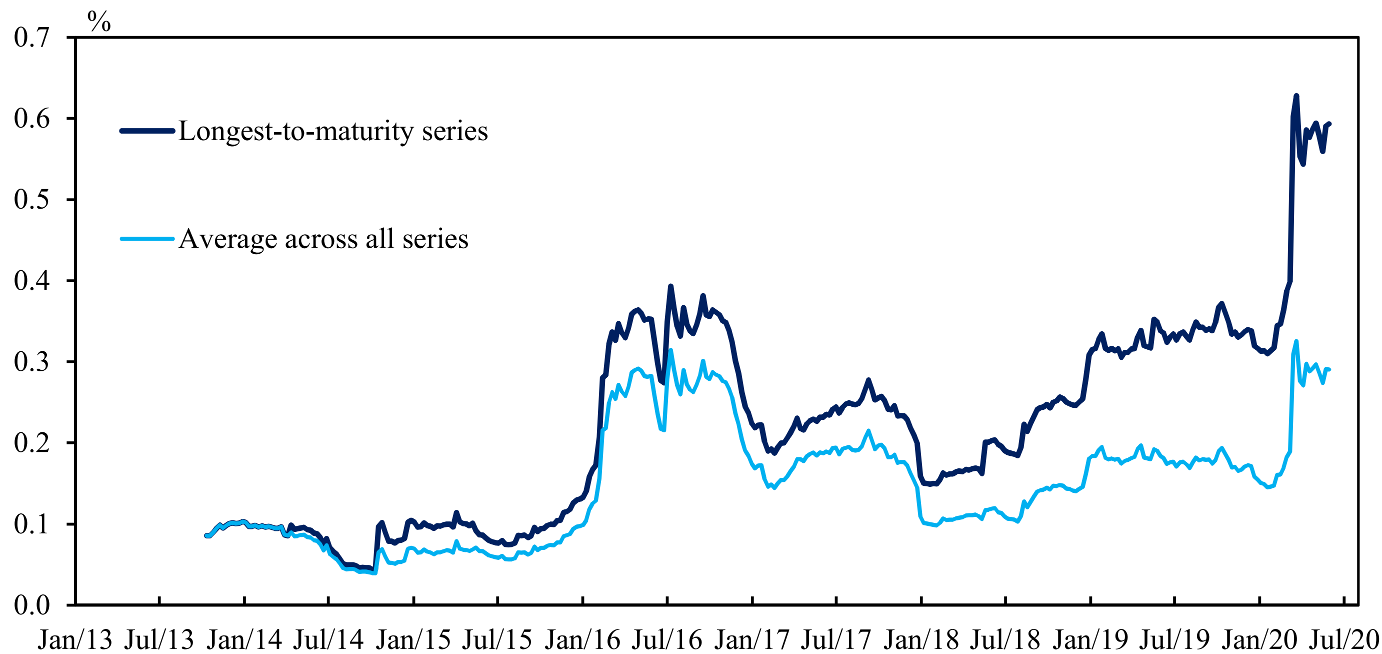
Sources: Bloomberg; Ministry of Finance.

Figure 5. Estimates of deflation protection option premium

(A) One-year-ahead deflation probability



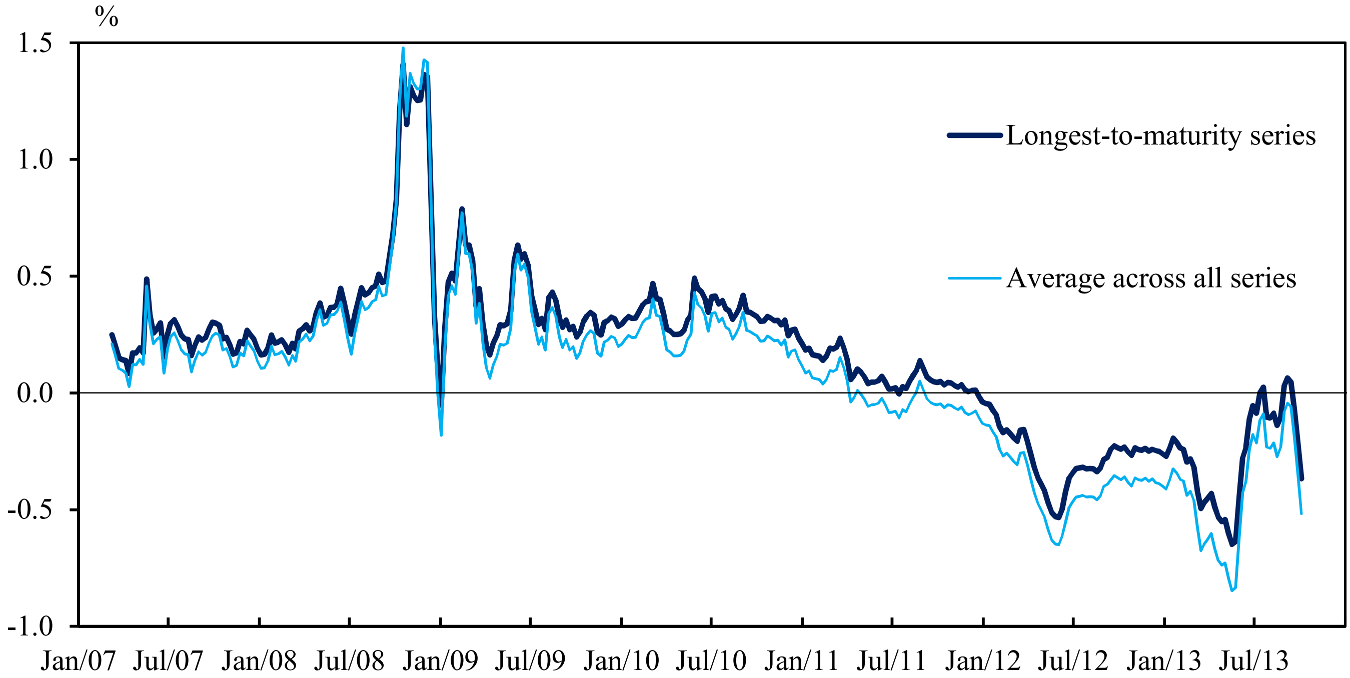
(B) Deflation protection option premium of new JGBi series



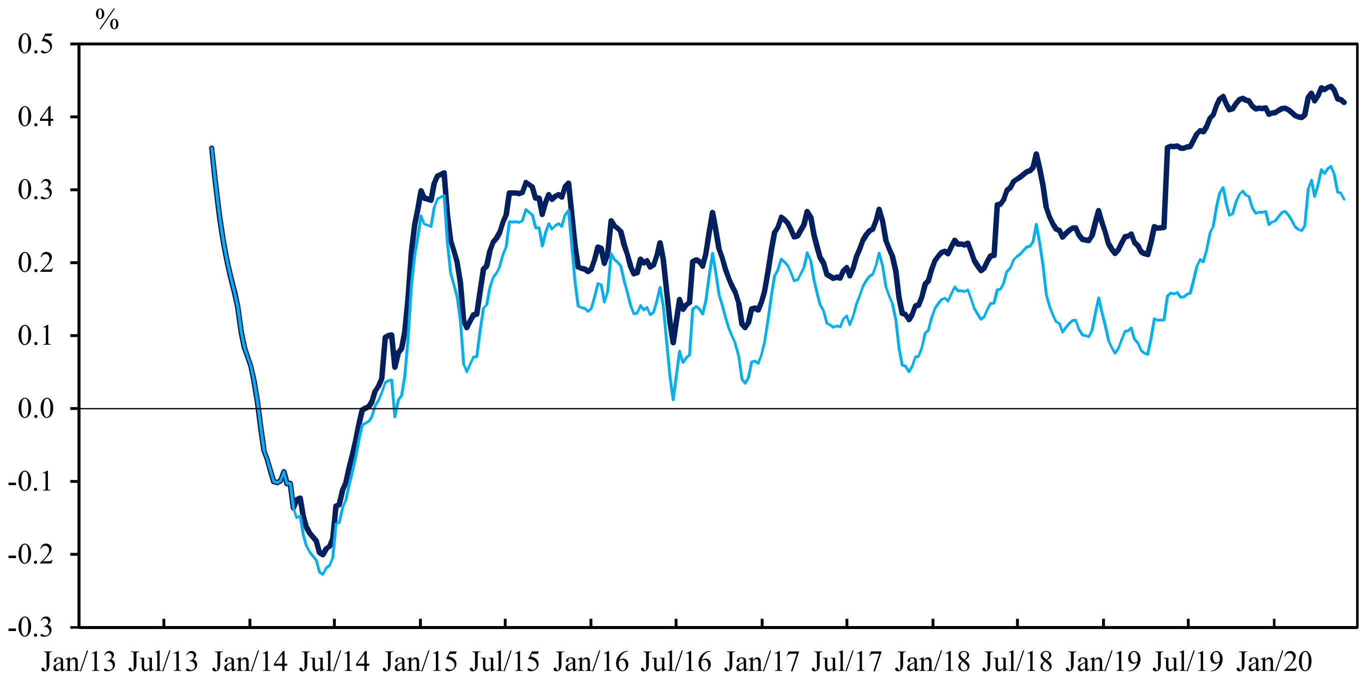
Note: The latest data as of the week ending May 29, 2020.

Figure 6. Estimates of liquidity premium

(A) Old JGBi series

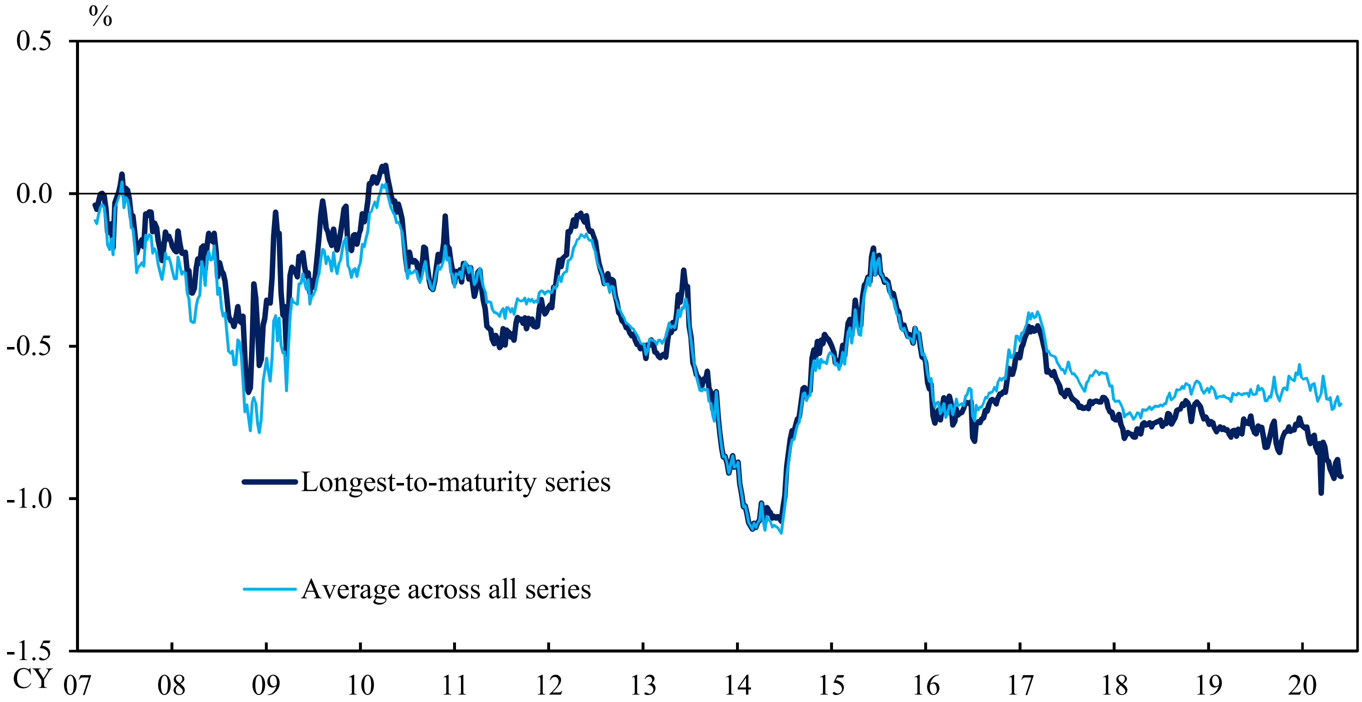


(B) New JGBi series



Note: The latest data of (B) as of the week ending May 29, 2020.

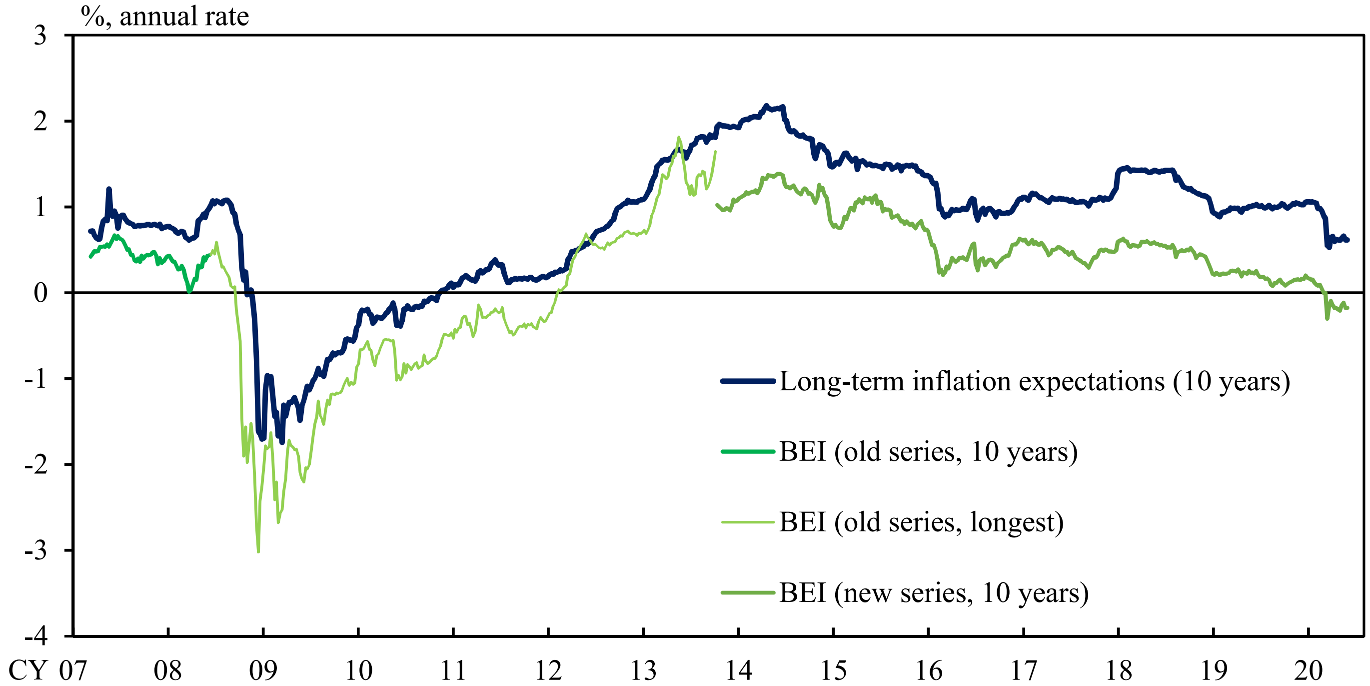
Figure 7. Estimates of term premium spread



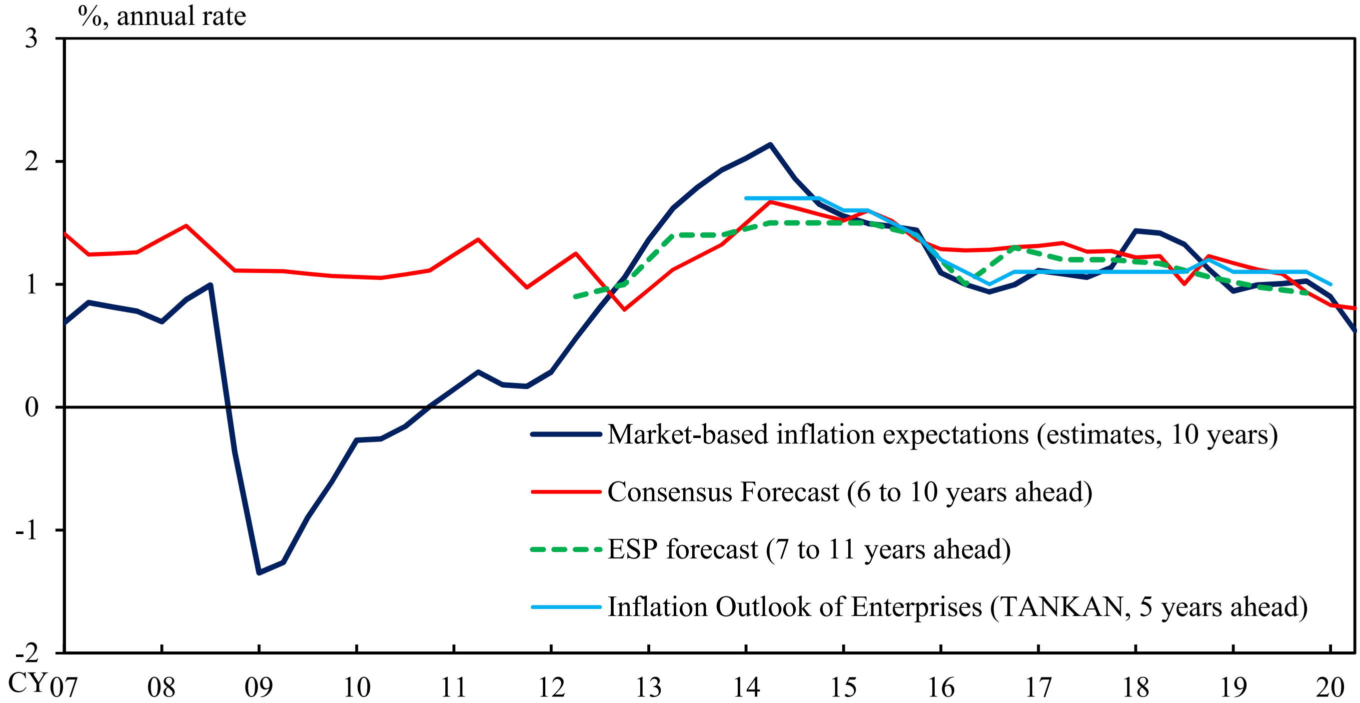
Note: The latest data as of the week ending May 29, 2020.

Figure 8. Estimates of market-based long-term inflation expectations

(A) Market-based long-term inflation expectations (10 years)



(B) Comparison with survey-based measures of inflation expectations

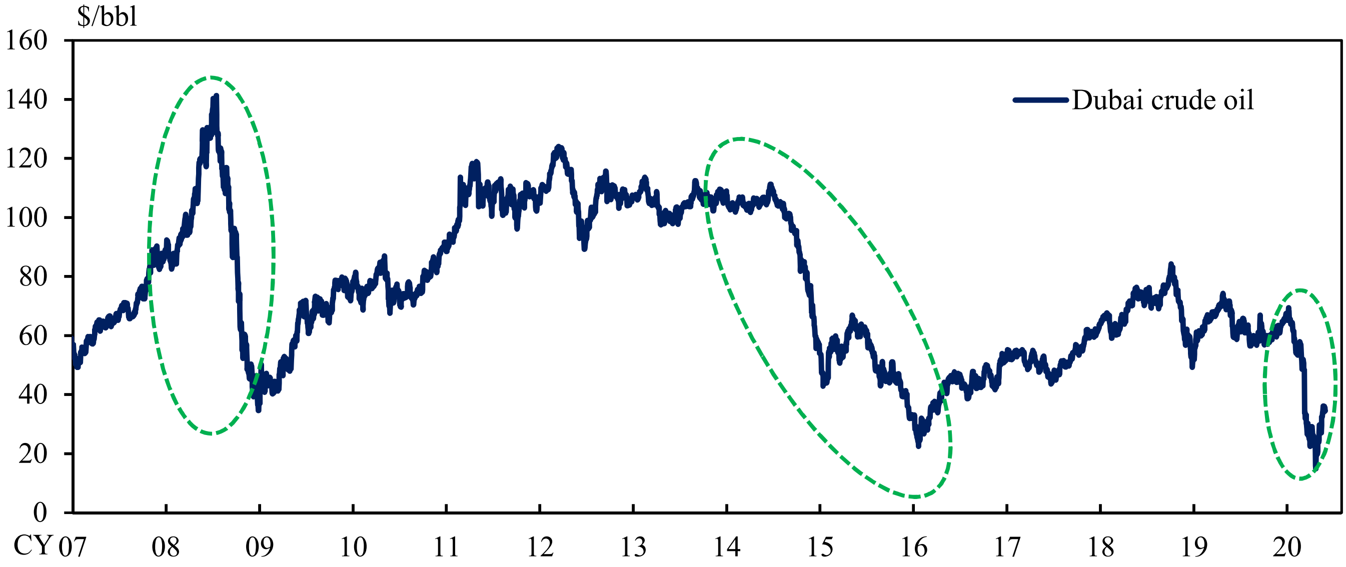


Notes: 1. The latest data in (A) as of the week ending May 29, 2020.
2. "Market-based inflation expectations" in (B) is the quarterly average of the estimates of the weekly market-based inflation expectations. The number for 2020/Q2 is the average between April and May 2020.

Sources: Bloomberg; Consensus Economics, "Consensus Forecast"; Japan Center for Economic Research; Bank of Japan.

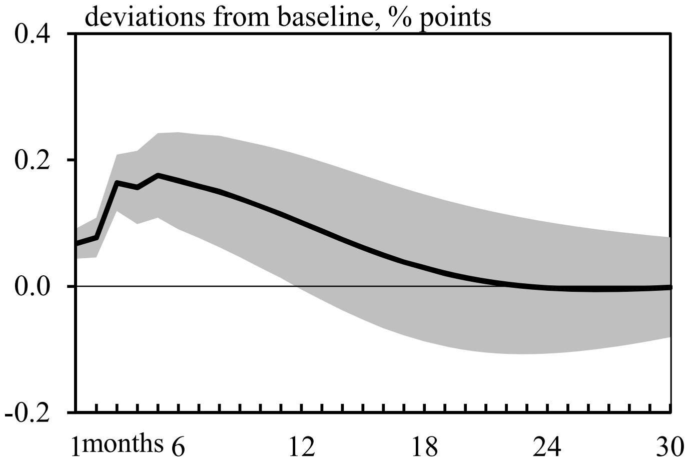
Figure 9. Market-based inflation expectations and crude oil prices

(A) Crude oil prices

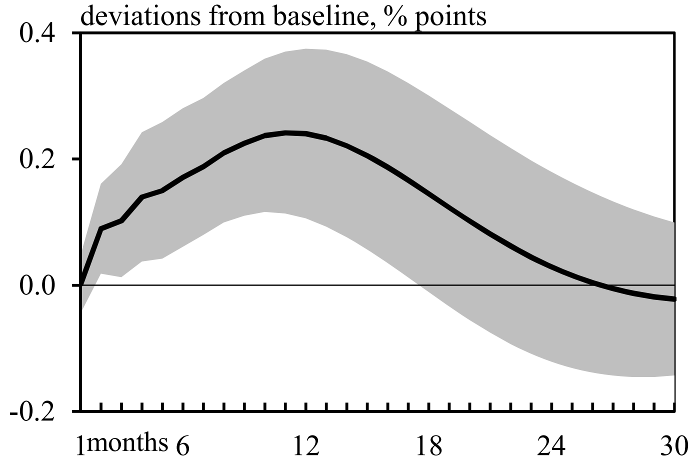


(B) Impulse response to a 10% increase in crude oil prices

(a) Market-based long-term inflation expectations



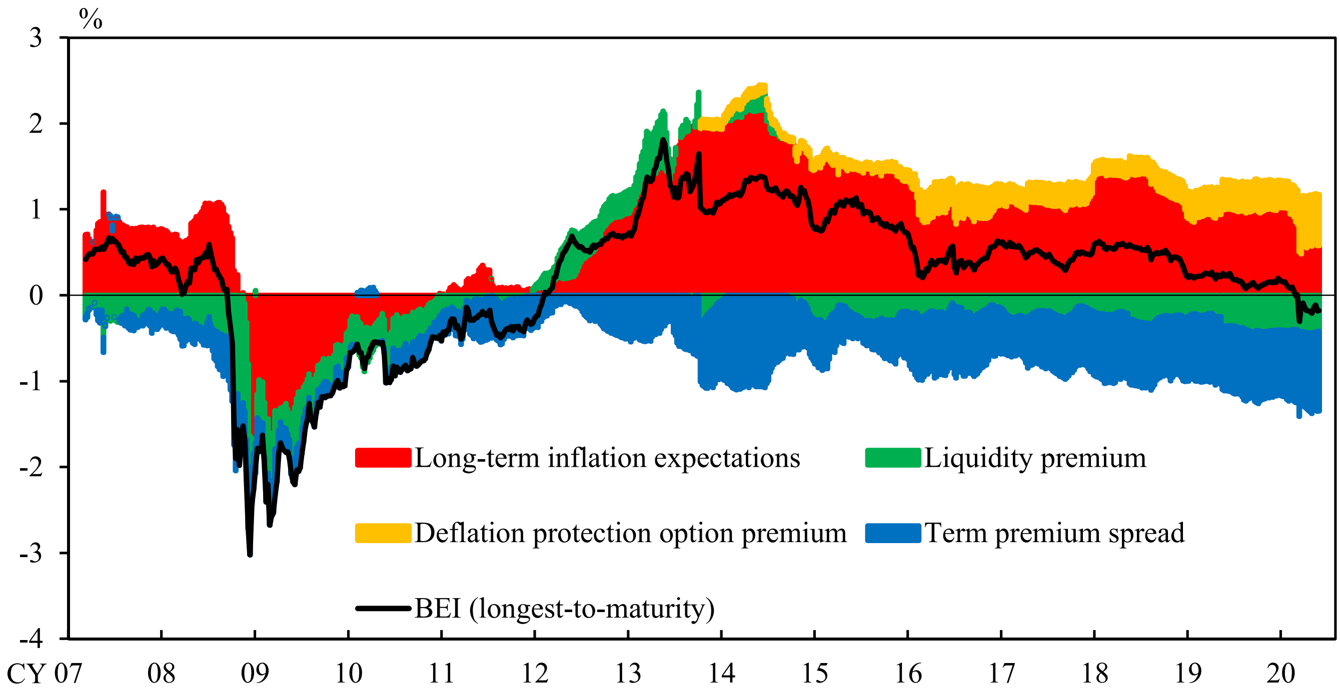
(b) CPI (year-on-year change)



Estimation period: March 2007 to December 2019 (monthly, end-of-month value)
Variables: 1. Dubai crude oil price (logarithmic value)
2. Estimated market-based long-term inflation expectations (10 years)
3. CPI (All items less fresh food), year-on-year change
Specifications: The lag length is set to 4 based on the AIC.
Structural shocks are identified based on the Cholesky decomposition.
The order of the decomposition is variable 1, variable 2 and then variable 3.

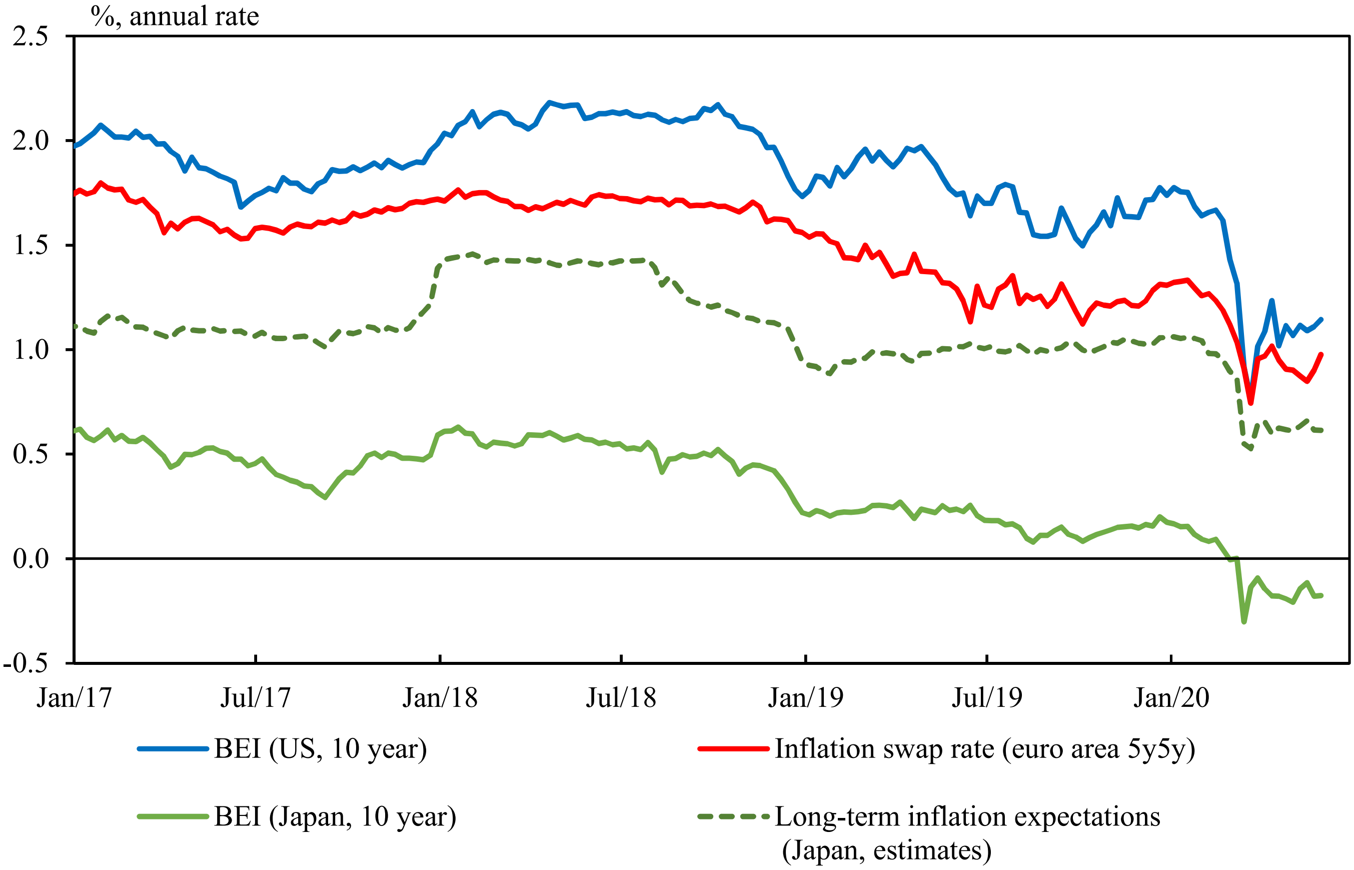
Notes: 1. The latest data in (A) as of May 29, 2020.
2. The shaded areas in (B) represent the two standard deviation range.
Sources: Bloomberg; Ministry of Internal Affairs and Communications.

Figure 10. Decomposition of the longest-to-maturity BEI



Note: The latest data as of the week ending May 29, 2020.

Figure 11. Observed market-based inflation expectations for Japan, US, and Euro Area



Note: The latest data as of the week ending May 29, 2020.
Source: Bloomberg.