Credit Valuation Adjustment (CVA)

Counterparty credit risk pricing, assessment, and dynamic hedging

Citigroup Global Markets, James Lee

Bank of Japan, June 14th, 2010
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1. Introduction
Introduction

- A critical element of the derivative business going forward will be to trade with on an uncollateralised or partially collateralized basis with counterparties.

- Previously, valuation of counterparty credit risk has largely been ignored due to relatively smaller size of the derivative exposures and the high credit rating of the counterparties which were generally AAA or AA rated financial institutions.

- As the size of the derivative exposure increases and the credit quality of the counterparties falls, the valuation of counterparty credit risk can no longer be assumed to be negligible and must be appropriately priced and charged for. Credit Valuation Adjustment or CVA is the process through which counterparty credit is valued, priced and hedged.

- We can no longer assume that derivatives exposures are “credit risk remote”. CVA is the credit reserve process and is analogous to MTM of bonds, loan loss reserves for loan or accounts receivables.

- CVA management involve managing of counterparty credit risk on the “Asset” side as well as “Liability” side risk and funding risk. This is analogous to Asset Liability Management for derivatives.

- CVA is important to create correct incentives for trading and avoid adverse selection. Risky counterparties migrate to banks without CVA. Negative funding trade migrates to non-CVA banks.
2. CVA Methodology
Introduction

• **What is market CVA?**
  – Market CVA is the credit reserve adjustment made to derivatives transactions to account for counterparty risk
  – Market CVA is bilateral at the financial reporting level. Bilateral CVA consists of
    ▪ Asset CVA – this represents the expected cost of Citi’s counterparty exposures (loans)
    ▪ Liability CVA – this is the expected credit costs incurred by the counterparty (deposits)
  – Bilateral market CVA can be thought of as the net market value of an American option by both sides to default on the derivative

• **How is it calculated?**
  – The methodology to calculate both Asset CVA and Liability CVA is similar. In the formula below, we do not differentiate between asset/liability CVA.
  – CVA is the expected value of credit losses over the lifetime of the trade. i.e.
  – CVA at each time bucket = PV (EAD * (1 – Recovery Rate) * Probability of Default) where
    ▪ EAD = Exposure at Default at each time bucket. This is predicted by EPE/ENE profiles
    ▪ EPE/ENE = Expected Positive and Negative Exposures of the portfolio. These are generated using the market implied volatilities of market risk factors
    ▪ Recovery Rate = 50% (Assumed)
    ▪ Probability of Default = Derived through market CDS spreads
  – Bilateral CVA is the sum of the Asset and Liability CVA
Transaction flow of a CVA transaction

Before CVA

- Client
- Swap Trader
- Bank

After CVA

- Client
- Swap Trader
- CVA Option
- Bank
- CVA Trader
- CDS Market
- Underlying Options and Delta

- The Swap Trader will pay a premium to the CVA trader to buy a CVA option
- The CVA option will protect the Swap Trader against any loss due to the default of the Client on the swap
- The CVA trader will hedge the Credit risk in the CDS market
How Do Banks Calculate a Credit Charge?

CVA Premium = -Default Probability * (1 - Recovery) * Max( MTM , 0)

Counterparty Exposure develops as MTM changes over time

- Calculate the expected mark to market (MTM) value of the swap over time by calculating the Asset Profile - The expected MTM value for those situations when the Bank is owed money
Asset vs Liability CVA

Expected Positive Exposure and Expected Negative Exposure (akin to the current PSLE concept) generates the Exposure-at-Default profile for both the counterparty and the bank. The graph below shows the EAD used to calculate CVA at time t, assuming MTM of 0.

Bilateral CVA is the sum of asset and liability CVA at each tenor bucket over the life of the trade.
If MTM (from bank’s perspective) moves in a positive direction, we record an additional CVA charge (loss), assuming everything else remains equal.
Asset vs Liability CVA

If MTM (from bank’s perspective) moves in a negative direction, we record a CVA gain, assuming everything else remains equal.

MTM (Bank)

Discount Factors, CP’s CDS

Discount Factors, Citi’s CDS

Expected Positive Exposure

Exposure at Default for the bank if counterparty defaults at time $t \times 0.5 \times (1 - \text{Recovery Rate})$

Expected Negative Exposure

Exposure at Default for the counterparty if the bank defaults at time $t \times 0.5 \times (1 - \text{Recovery Rate})$

Today

-10 MM

-10 MM
3. Dynamic Hedging
What is Dynamic Hedging?

- CVA is a credit hybrid option on the contingent exposure of a derivative contract or a portfolio of derivative contracts. It is a call option on the MTM of these derivative contracts with an counterparty that can be exercised only upon a credit event of that counterparty.

- Like other options products, CVA can be hedged via dynamic hedging. This attempts to hedge the actual exposure of a derivative portfolio, using a Black Scholes style model.

- The CVA option price is a function of the underlying swap MtM, the counterparty CDS spread, their individual volatilities and the correlation between the two.

- The CVA option can be hedged by delta hedging with the underlying derivative (and/or option on the underlying derivative) and CDS.
Dynamic Hedging

Concepts

Dynamic Hedging
- Hedging the value of expected loss requires hedging its sensitivity to market factors and credit quality
- Because these factors move, and the CVA’s sensitivity to them changes, hedging needs to be rebalanced

Friction
- Each individual market factor is hedged, but correlated moves will cause net losses
- Also, transaction costs *per se*, need to be accounted for

Time Decay
- As time passes, the life of the deal shortens and, all else constant, the expected loss falls
- Thus, like an option premium, all else constant, a CVA’s value will fall over time
Hedging the value of expected loss requires hedging its sensitivity to market factors and credit quality.

- A Cross Currency Swap CVA will be affected by:
  - FX rates and IR (we will focus on FX here) - these affect the expected MTM of the deal
  - Credit Quality – this affects the expected default probability

An example:

- 5 Years / Principal: USD 100mm / Exchange at Inception and Maturity
- Counterparty CDS curve: 1 Year 81bps, 3 Year 85bps, 5 Year 93bps
- EURUSD Spot at inception: 1.3500
Dynamic Hedging - Example

Dynamic Hedging

- The CVA desk measures the sensitivity of expected loss with respect to each factor
- It then creates a hedge to offset that sensitivity
- For FX rate sensitivity:

<table>
<thead>
<tr>
<th>Inception</th>
<th>FX</th>
<th>5y Credit</th>
<th>CVA</th>
</tr>
</thead>
<tbody>
<tr>
<td>T=0</td>
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<th>Sensitivity to FX Moves</th>
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<th>Hedge</th>
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<tr>
<td>1.3600</td>
<td>403</td>
<td>21</td>
<td>2100</td>
<td></td>
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At inception, CVA is 382k
For a +100pip move, CVA increases by 21k
Hedge: Long EUR 2.1mm

- For Credit Curve sensitivity:

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<tr>
<td>103</td>
<td>420</td>
<td>38</td>
<td>7,600</td>
<td></td>
</tr>
</tbody>
</table>

At inception, CVA is 382k
For a +10bp widening, CVA increases by 38k
Hedge: Buy 7.6mm 5Y CDS*

*For illustration purposes, simplistic calculation: PnL = spread*tenor*notional
Dynamic Hedging - Example

Dynamic Hedging

- For any individual move in market factors, its respective hedge will offset changes in expected loss
- For example, if FX moves +100 pips, and Credit Curve stays constant...

Inception

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FX Moves Only

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PnL

<table>
<thead>
<tr>
<th></th>
<th>CVA</th>
<th>FX Hedge</th>
<th>Credit Hedge</th>
<th>Net PnL</th>
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<tbody>
<tr>
<td>Expected Loss</td>
<td>21</td>
<td>21</td>
<td>0</td>
<td>0</td>
</tr>
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</table>

FX Moves +100 pips Expected Loss increases 21k; offset by gain in FX Hedge

- After the shift, sensitivities are recalculated and hedges are rebalanced

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<td>2100</td>
<td>8,000</td>
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FX hedge is unchanged at EUR 2.1mm

CDS hedge is increased to 8mm
Dynamic Hedging - Example

Dynamic Hedging

- Thereafter, if Credit Curve moves +10 bps, and FX stays constant...

After an FX Move

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Now Credit Curve Moves

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- Expected Loss increases 40k; offset by gain in Credit Hedge

Credit Curve Moves +10 bps

- After the shift, sensitivities are recalculated and hedges are rebalanced

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<tr>
<td>466</td>
<td>483</td>
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<tr>
<td>23</td>
<td>40</td>
</tr>
<tr>
<td>2300</td>
<td>8,000</td>
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FX hedge is increased to EUR 2.3mm

CDS hedge is unchanged at 8mm
Credit Valuation Adjustment

Dynamic Hedging

- The aim of dynamic hedging is to ensure that changes in expected loss are neutralised
- That is:

  \[
  \text{Initial CVA + Dynamic Hedging} = \text{Prevailing CVA}
  \]

- If the underlying deal is unwound, the Prevailing CVA is returned to the Sales Desk

- Note that, at any time, the purpose of a CDS hedge is to offset changes in CVA due to the change in credit quality
- If credit worsens such that default becomes almost certain, then CDS hedges done up to that point should have covered the increase in expected loss
- The outstanding CDS hedge by itself is not a hedge against loss from actual default
- In a Jump to Default (JTD) scenario, the lost deal MTM will be covered by the sum of the Prevailing CVA and the outstanding CDS position:

  \[
  \text{Deal MTM} \times (1-R) = \text{Prevailing CVA} + \text{CDS Hedge} \times (1-R)
  \]
Friction

- Dynamic hedging for individual, separate market moves works as long as there is time to rebalance the hedges
- However, **simultaneous** market moves will not be covered by the sum of individual hedges
- This is because CVA sensitivity to one factor is changes, if another factor moves
- And at the time of hedging, such an effect cannot be assumed
- This is a cross gamma/correlation effect
Credit Valuation Adjustment

Concepts

Friction

– Recall the example used above
– FX and then the Credit Curve had moved one after the other, and the latest position was:

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– The sensitivities were recalculated and the hedges rebalanced:

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– Now assume that FX and the Credit Curve move by the amount hedged for (+100pips/+10bps)
– But assume that they both move together, simultaneously

FX and Credit Curve Move Simultaneously

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The increase in Expected Loss is more than the sum of the individual hedge PnL

This results in a net loss
Friction and Cross Gamma

- The amount of loss due to simultaneous market moves depends on correlation
- In addition, higher correlation requires more frequent rebalancing of hedges
- And this increases the total transaction costs *per se*

- Thus, for deals involving correlation, a ‘Friction’ adjustment is made
- That is:

  \[
  \text{Friction adjustment} = \text{Net loss from Correlation} + \text{Transaction costs}
  \]

- Usually, it is priced via a shift in the credit curve assumption
- For example, a 100bps credit spread adjusted by 25% for Friction, would become 125bps
Credit Valuation Adjustment

Concepts

Time Decay
- CVA is forward looking, it measures *expected* loss
- Time to maturity has significant importance
- Longer tenor means a larger range of possible outcomes and a higher expected loss

- The flipside to this: after inception, CVA value is subject to time decay
**Credit Valuation Adjustment**

**Concepts**

**Time Decay**

- When Sales desks transfer CVA value to the CVA desk, it is like paying an option premium.
- The option protects Sales desks from risk of counterparty default, paying the MTM of a deal if default occurs.
- At inception, CVA value is mostly time value — all else constant, this will fall over time.
- In the example used above, time decay profile is:

```
  Inception

  Maturity
```

- Time decay consists of (a) positive carry from asset side credit risk & liability benefit and (b) vega of the underlying market factors.
- These are offset from (a) buying CDS and (b) buying options on underlying market factors.
- As the MTM goes deep in-the-money positive (or deep out-of-the-money negative, the time decay split shifts from (b) to (a).
Credit Valuation Adjustment

Appendix – Dynamic Hedging and Cross Gamma Risk

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After an FX Move

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Now Credit Curve Moves

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FX and Credit Curve Move Simultaneously

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FX and Credit Curve Moves Simultaneously

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Sensitivity to FX Moves

Sensitivity to Credit Curve Moves

PnL

Citi
# Credit Valuation Adjustment

## Appendix – Time Decay

### Effect of Time - Decreasing CVA

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</table>
4. Risk and reserves
Residual Risks

- Liquidity Risk – Covers transaction cost including bid/offer spread and the sudden widening of bid/offer spread due to lack of liquidity.

- Recovery Risk – Covers the risk of the obligation’s recovery value upon a counterparty default. E.g. Recovery locks.

- Gap Risk (Cross Gamma) – Covers the risk of a simultaneous move in credit and the underlying FX, IR or Equity rates.

- Correlation Risk – Covers the correlation assumed in the model and the change in correlation between credit and the underlying.

- Model Risk - Covers uncertainty in the model vs the actual market for unwind.

- Legal, netting and Documentation risk – Covers the legal and netting effectiveness of the CDS hedge and the enforceability of the ISDA swap documentation in various jurisdictions.
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